Načrtovanje nizkodimenzionalnih spinskih struktur z azafulerenskimi radikali

DENIS ARČON

Fakulteta za matematiko in fiziko, Univerza v Ljubljani,
Jadranska 19, SI-1000 Ljubljana, Slovenija
Institut Jožef Stefan, Jamova 39, SI-1000 Ljubljana, Slovenija
denis.arcon@fmf.uni-lj.si

Molekularni radikali ponujajo kemijsko prilagodljiv pristop k pripravi nizkodimenzionalnih kvantnih spinskih sistemov, vendar njihovo urejanje na trdnih površinah spremljata dva ključna izziva: ohranitev nesparjenega elektrona in nadzor geometrije spinske mreže. Predstavil bom, kako lahko azafulerenski radikali $C_{59}N^{\bullet}$ služijo kot gradniki s spinom 1/2, katerih stabilnost, prostorsko razporeditev in sklopitev z okolico lahko nadzorujemo z izbiro površine in supramolekularnim načrtovanjem.

Z združitvijo vrstične tunelske mikroskopije, sinhrotronske spektroskopije in DFT izračunov smo razvili dva dopolnjujoča se pristopa k stabilizaciji radikalov: izolacijo v molekularni plasti brez neposrednega stika s podlago Au(111) [1] in ujetje v nanobroče [10]cikloparafenilena [2]. Slednji preprečujejo dimerizacijo azafulerenov, zmanjšujejo sklopitev s kovinsko podlago in omogočajo spontano urejanje v razsežne dvodimenzionalne trikotne mreže. Naši zadnji rezultati tudi kažejo, da je to molekularno ogrodje dinamično: kinetično ujetje, migracija radikalov in rekonstrukcija ogrodja ob robovih vodijo do drugačnih struktur s kvazi-enodimenzionalnimi verigami [3]. Te strukture ponujajo zanimive geometrije za preučevanje magnetne frustracije in vpliva zmanjšane dimenzionalnosti, določitev njihovih magnetnih interakcij pa ostaja osrednji izziv.

Predstavil bom tudi napovedi DFT izračunov o sklopitvi površinsko vezanih radikalov s plitvimi centri dušik–vrzel (NV) v diamantu. Zaključni atomi na površini določajo prenos naboja: vodikova terminacija destabilizira želeno spinsko aktivno konfiguracijo, medtem ko fluorova terminacija ohranja tako nevtralni radikal kot negativno nabiti center NV [4]. Nazadnje se bom dotaknil trenutnih raziskav Kondovega pojava [5] in uporabe teorije pakiranja krogov kot geometrijskega ogrodja za načrtovanje molekularnih mrež. Ti pristopi povezujejo zaščito molekularnih spinov, nadzor geometrije spinskih mrež in možnosti lokalnega branja spinskih stanj v skupno platformo za načrtovanje kvantnih spinskih struktur.

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Engineering low-dimensional spin architectures with azafullerene radicals

DENIS ARČON

Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia
Jožef Stefan Institute, Jamova 39, SI-1000 Ljubljana, Slovenia
denis.arcon@fmf.uni-lj.si

Molecular radicals offer a chemically versatile route to low-dimensional quantum spin systems, but their assembly on solid surfaces faces two competing challenges: preserving the unpaired electron and controlling the geometry of the spin network. I will discuss how azafullerene radicals, $C_{59}N^{\bullet}$, can serve as spin-1/2 building blocks whose stability, spatial organization, and interaction with their environment are engineered through surface and supramolecular design.

Combining scanning tunnelling microscopy, synchrotron spectroscopy, and first-principles calculations, we establish two complementary strategies for radical stabilization: isolation in a noncontact molecular layer on Au(111) [1] and encapsulation within [10]cycloparaphenylene nanohoops [2]. The latter protects against dimerization, reduces coupling to the metallic substrate, and enables self-assembly into extended two-dimensional triangular arrays. Recent work reveals that this molecular template is also a dynamic framework: kinetic trapping, radical migration, and edge-driven template reconstruction produce alternative architectures with quasi-one-dimensional chain motifs [3]. These structures offer geometries for exploring frustration and reduced dimensionality, while establishing their magnetic interactions remains a central challenge.

I will also discuss first-principles predictions for coupling surface-bound radicals to shallow nitrogen-vacancy centres in diamond. Surface termination controls charge transfer: hydrogenation destabilizes the desired spin-active configuration, whereas fluorination preserves both the neutral radical and the negatively charged NV centre [4]. Finally, I will outline ongoing work on Kondo physics [5] and the use of circle packing theory as a geometric framework for designing molecular arrays. Together, these directions connect molecular spin protection, tunable network geometry, and prospective local spin readout in a common platform for quantum spin architectures.

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Ali obstaja perturbativna teorija poenotenja E6?

BORUT BAJC

Institut Jožef Stefan
Jamova 39, 1000 Ljubljana, Slovenia
borut.bajc@ijs.si • www-f1.ijs.si/∼bajc

Teorije velikega poenotenja z umeritveno grupo E_6 tipično vključujejo nerazcepne reprezentacije z visoko dimenzionalnostjo. Zaradi tega se te teorije v ultravijoličnem režimu ne obnašajo dobro, saj imajo običajno že en red velikosti nad skalo velikega poenotenja težave s perturbativnostjo in/ali unitarnostjo. Opisal bom ta problem in poskušal zasnovati realistično teorijo, ki bi lahko bila perturbativna pri višjih energijah. Takšna teorija nujno temelji izključno na 27- in 78-dimenzionalnih reprezentacijah, hkrati pa izkazuje nekatere posebne lastnosti vakuuma.

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Is there a perturbative E6 GUT?

BORUT BAJC

Jožef Stefan Institute
Jamova 39, 1000 Ljubljana, Slovenia
borut.bajc@ijs.si • www-f1.ijs.si/∼bajc

Grand unified theories with gauge group E_6 have usually irreducible representations with large dimensionality. Because of that these theories are not well behaved in the ultraviolet, having tipically problems with perturbativity and/or unitarity already one order of magnitude above the grand unification scale. I will describe this problem and try to construct a realistic theory which has some chance to be perturbative at higher energies. It necessarily consists of 27 and 78 dimensional representations only but has some peculiar vacuum properties.

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Opportunities and challenges of quantum batteries

GIULIANO BENENTI

Center for Nonlinear and Complex Systems,
Dipartimento di Scienza e Alta Tecnologia
Università degli Studi dell'Insubria,
via Valleggio 11, 22100 Como, Italy
giuliano.benenti@uninsubria.it

Quantum batteries harness the principles of quantum mechanics to transfer, store and release energy within quantum systems on demand. Emerging from foundational research at the intersection of quantum physics, thermodynamics and information theory, the field of quantum batteries introduces new principles for energy manipulation rooted in quantum mechanics.

This talk will present the fundamental concepts underlying quantum batteries, review the current theoretical and experimental progress, and discuss the main opportunities and challenges associated with the realization of efficient, scalable, and practically relevant quantum energy-storage devices.

Finally, I will show how coupling a quantum battery to an environment that can be continuously monitored can enhance work extraction, revealing the potential of measurement-based strategies for improving quantum energy transfer and utilization.

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Podpisi elektronskega parjenja brez ozadja v dvofotonski dvoelektronski koincidenčni fotoemisijii

JANEZ BONČA

Fakulteta za matematiko in fiziko, Univerza v Ljubljani, SI-1000 Ljubljana, Slovenija
Institut J. Stefan , SI-1000 Ljubljana, Slovenija
janez.bonca@fmf.uni-lj.si

Z metodo renormalizacijske grupe gostotne matrike (DMRG) in variacijsko eksaktno diagonalizacijo (VED) izračunamo eno-elektronsko spektralno funkcijo Hubbard-Holsteinovega modela pri nizkih elektronskih gostotah. Polaronske tekočine imajo spektralno težo vse do Fermijeve energije, medtem ko kovinske bipolaronske tekočine kažejo energijsko vrzel, določeno z vezavno energijo. Slednje lahko $k-$prostoru približno opišemo s plinom bozonov s trdo sredico [1]. Izračuni spektra dvoelektronske kotno ločljive foto emisije z metodo VED predstavljajo približek za t.i. 2e-ARPES. Emisija elektronov iz istega para poteka pri manjši vezavni energiji in ima drugačno simetrijo v $k-$prostoru kot emisija elektronov iz različnih parov. Ti splošni spektralni podpisi omogočajo zaznavanje predhodno tvorjenih parov ter razlikovanje med koherentnim in nekoherentnim parjenjem [2]. Ločljivost 2e-ARPES po posameznih energijah (ω_1, ω_2) razkrije območja 'brez ozadja', v katerih je dovoljena emisija iz istega para, emisija iz različnih parov pa je prepovedana. Ta robustna spektralna krila določa kemijski potencial μ in ne vezavna energija Δ. Izračuni DMRG in VED potrjujejo takšno obnašanje ter kažejo, da je mogoče neposredno izmeriti spekter bozonov, odgovornih za tvorbo parov. Poleg tega zaznavanje enega samega elektrona razkrije spektralno težo, ki izvira iz parov, nad μ, kar omogoča zaznavanje parjenja z običajnim ARPES [3].

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Background-Free Signatures of Electronic Pairing in Two-Photon Two-Electron Coincidence Photoemission

JANEZ BONČA

Faculty of Mathematics and Physics, University of Ljubljana, SI-1000 Ljubljana, Slovenia
J. Stefan Institute, SI-1000 Ljubljana, Slovenia
janez.bonca@fmf.uni-lj.si

Using density matrix renormalization group (DMRG) and variational exact diagonalization (VED), we calculate electron-removal spectra of the Hubbard-Holstein model at low densities. Polaron liquids show spectral weight up to the Fermi energy, whereas metallic bipolaron liquids exhibit a gap determined by the binding energy and can be approximated by hard-core bosons in momentum space [1]. VED calculations of the two-electron removal spectrum provide a proxy for 2e-ARPES. Emission from the same pair occurs at lower binding energy and has a distinct momentum symmetry compared with emission from different pairs. These generic signatures can reveal preformed pairs and distinguish coherent from incoherent pairing [2]. Resolving 2e-ARPES in the individual energies (ω_1, ω_2) reveals "background-free" regions where same-pair emission is allowed but different-pair emission is forbidden. These robust spectral wings are determined by μ, rather than the binding energy Δ. DMRG and VED calculations confirm this behavior and show that the pairing-boson spectrum can be measured directly. Moreover, single-electron detection yields pair-derived spectral weight above μ, providing a pairing signature accessible with conventional ARPES [3].

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Spatio-Temporal Entanglement in Double Ionization of Helium by Attosecond Pulses

JOACHIM BURGDÖRFER*

Institute for Theoretical Physics
TU Wien, Wiedner Hauptstrasse 8 - 10, AT-1040 Wien, Austria
burg@concord.itp.tuwien.ac.at • https://www.tuwien.at/phy/itp/seite/1

Double ionization by attosecond XUV pulses provides a unique platform for studying electron-electron correlation and entanglement in atomic systems on ultrashort time scales. One-photon double ionization of helium by a time-delayed pair of such pulses opens up novel opportunities to study time-dependent correlations and bi-partite entanglement of two massive particles with continuous spectra. We show that two complementary entanglement sources, momentum correlations in the two-particle continuum of stationary two-particle scattering states [1] and time entanglement between an early and a late emitted two-particle wavepacket [2] can be combined to yield spatio-temporal entanglement. Such a combination allows to tune and modify the continuously distributed one-particle density matrix and its purity where the partner electron, taking on the role of the environment, is also continuously distributed. We find pronounced time-delay dependent interference structures in the reduced density matrix and marginal distributions [3] signatures of which should become experimentally accessible [4].

* Work in collaboration with Wei-Chao Jiang (Shenzhen University) and Liang-You Peng (Peking University)

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Classical-to-Quantum Transition in Blackbody Radiation Thermodynamics

GIULIO CASATI

Department of Science and Technology
University of Insubria and Lake Como School of Advanced Studies, 22100 Como, Italy
gcgiuliocasati@gmail.com

Inspired by Fermi's model of radiation–matter interaction, we investigate the classical dynamics of a closed, finite-energy system, in which matter is coupled to an electromagnetic field. We find that energy is progressively transferred to increasingly high-frequency field modes, so that the effective temperature of both matter and radiation approaches zero in the long-time limit — in sharp contradiction with the finite-temperature equilibrium assumed by classical blackbody thermodynamics. Requiring consistency with the Stefan–Boltzmann and Wien laws identifies a characteristic time t_L, at which both laws are simultaneously satisfied. Strikingly, classical spectra obtained for different energies and cavity sizes collapse onto the universal Planck distribution at t_L. We conjecture that quantum dynamical localization arrests the classical energy cascade at t_L, and that the Planck distribution emerges as a dynamical consequence of this arrest — without invoking any a priori statistical assumption.

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String Theory, Quantum Field Theory and Higher Symmetries

MIRJAM CVETIČ

Department of Physics and Astronomy
University of Pennsyslvania, Philadelphia, USA
CAMTP - Center for Applied Mathematics and Theoretical Physics
University of Maribor, Mladinska 3, SI-2000 Maribor, Slovenia

In the past decades, String Theory has emerged as the prime candidate for the unification of particle physics with quantum gravity. It has shed light on important fundamental questions of theoretical physics, such as the microscopic structure of black holes and the geometric origin of particle physics. We focus on the important geometric role that branes, extended objects in String Theory play in deriving the Standard Model of particle physics. We further highlight developments in deriving particle physics from F-theory, a geometric domain of String Theory at finite string coupling, and systematic exploration of the landscape of the (quadrillion) Standard Models with three families of quarks and leptons. Time permitting, we also point out recent insights into the geometric origin of higher symmetry structures for quantum field theories derived from String Theory.

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Kaskada operatorjev kot pot v mnogodelčni kvantni kaos

URBAN DUH

Oddelek za fiziko, Fakulteta za matematiko in fiziko
Univerza v Ljubljani, 1000 Ljubljana, Slovenija
urban.duh@fmf.uni-lj.si • uduh.gitlab.io

Opisal bom scenarij kaosa, ki velja za mnogodelčne kvantne sisteme, in pojasnil njegov pomen za klasične simulacije s trunkacijo. S preučevanjem spektralnih lastnosti trunkiranega propagatorja smo prvič identificirali fraktalne strukture, odgovorne za mnogodelčni kvantni kaos, in analogne fraktalnosti, ki v klasičnem kaosu nastane, ko se sprva gladke dolgovalovne porazdelitve s časom razvijajo v vse finejše kratkovalovne. Vodilni, najpočasneje relaksirajoči Ruelle-Pollicottovi lastni operatorji imajo namreč netrivialno fraktalno dimenzijo, ki kvantificira njihovo nelokalnost, kar se kaže tudi v divergenci njihovih števil občutljivosti (oz. pogojenostni). Poleg tega unitarnost narekuje asimptotsko enakost med hitrostjo časovnega pojemanja lokalnih korelacij in to mnogodelčno fraktalno dimenzijo. S tem spoznanjem lahko razumemo scenarij mnogodelčnega kvantnega kaosa: lokalni operatorji se s časom razvijajo v vse bolj nelokalne z dobro definirano fraktalno strukturo, kar vodi do efektivne neunitarne relaksacije v podprostoru lokalnih operatorjev, torej nekakšne mnogodelčne Kolmogorove kaskade v prostoru operatorjev. Naše napovedi numerično potrdimo na različnih eno- in dvodimenzionalnih kvantnih vezjih, z ohranitvenimi zakoni in brez njih, pa tudi z eksaktnimi rezultati za dvojno unitarna vezja in CMV matrike.

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Quantum many-body operator cascade as a route to chaos

URBAN DUH

Physics Department, Faculty of Mathematics and Physics
University of Ljubljana, 1000 Ljubljana, Slovenia
urban.duh@fmf.uni-lj.si • uduh.gitlab.io

I will describe a chaos scenario valid for many-body quantum systems and clarify its importance for classical truncation-based simulations. By studying the spectral properties of the truncated propagator, we, for the first time, identified fractal structures responsible for many-body quantum chaos, analogous to the fractality in classical chaos emerging from the time evolution of initially smooth long-wavelength densities into ever finer short-wavelength densities. Namely, the leading slowest-decaying Ruelle-Pollicott eigenoperators have a non-trivial fractal dimension quantifying their non-locality, visible also in the divergence of their condition numbers. Furthermore, unitarity imposes an asymptotic equality between the temporal decay rate of local correlations and this many-body fractal dimension. With this insight, the scenario for many-body quantum chaos becomes clear: over time, local operators evolve toward increasingly non-local ones with a quantifiable fractal structure, thereby naturally leading to effective non-unitary relaxation on the subspace of local operators – a kind of many-body Kolmogorov cascade in the space of operators – which enables accurate truncation-based simulations. Our predictions are demonstrated numerically in various one- and two-dimensional quantum circuits, with and without conservation laws, as well as by exact results for dual-unitary circuits and CMV matrices.

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Resonant chains in planetary systems

RUDOLF DVORAK

Inst. of Astrophysics, University of Vienna

We investigate the non-linear stability of first-order, three-body mean motion resonant chains in planetary systems. Our results are based on analytical studies of a Hamiltonian model which are supported by long-term simulations of the full gravitational equations of motion. We will discuss our results with the aid of respective stability diagramms; our main result is that a resonant chain 2:1-2:1 (4:2:1) mean-motion resonance of three planets is stable up to a treshhold of 0.0001 of the masses involved.

Semiclassical phase-space simulation of quantum tunneling

PAVEL CEJNAR

Institute of Particle and Nuclear Physics, Faculty of Mathematics and Physics, Charles University, V Holešovičkách 2, 18000 Prague, Czechia
pavel.cejnar@matfyz.cuni.cz • https://ipnp.cz/

Using classical solutions to the tunneling problem in complex time, we develop an extension of the truncated Wigner approximation suitable for efficient simulations of the quantum tunneling processes in the phase space. We show that the approximation yields satisfactory results for the under-barrier transmission in simple and resonant potentials, and sheds light on the issue of superluminal tunneling times.

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Ko čas nihanja ni več enak: kritične periode v nelinearnih oscilatorjih

BRIGITA FERČEC

Fakulteta za energetiko, Univerza v Mariboru
CAMTP – Center za uporabno matematiko in teoretično fiziko, Univerza v Mariboru

Pri harmonskem oscilatorju je perioda nihanja neodvisna od amplitude. Pri nelinearnih oscilatorjih pa se perioda lahko spreminja z amplitudo nihanja. Točke, v katerih periodna funkcija doseže lokalne ekstreme, imenujemo kritične periode. V predavanju bom predstavila povezavo med centrom ravninskega dinamičnega sistema in njegovo periodno funkcijo ter pokazala, kako lahko majhne perturbacije sistema povzročijo bifurkacijo kritičnih period. Na primerih polinomskih diferencialnih sistemov bom predstavila pristop za njihovo preučevanje, ki temelji na kompleksifikaciji sistema in metodah računske algebre.

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When the period of oscillation is no longer constant: critical periods in nonlinear oscillators

BRIGITA FERČEC

Faculty of Energy Technology, University of Maribor
CAMTP – Center for Applied Mathematics and Theoretical Physics, University of Maribor

For a harmonic oscillator, the period of oscillation is independent of the amplitude. In nonlinear oscillators, however, the period may vary with the amplitude. Points at which the period function has local extrema are called critical periods. I will explain the connection between a center of a planar dynamical system and its period function, and show how small perturbations of the system can lead to bifurcations of critical periods. Using examples of polynomial differential systems, I will present an approach to studying these bifurcations based on the complexification of the system and methods of computational algebra.

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Flatband Physics

SERGEJ FLACH

Institute for Basic Science, Daejeon, South Korea

Flat bands (FB) – single-particle dispersionless energy bands – in tight-binding networks (TBN) have attracted attention due to the presence of macroscopic degeneracies and compact localized states (CLS), and their extreme sensitivity to perturbations. This makes them natural candidates for emerging exotic phases and unconventional orders. FBs come in three classes, fully defined by the algebraic properties of their CLS set. Using CLS based FB model generators we tailor TBNs with predefined real space FB projector properties. We will then discuss how the fine-tuning constructions can be further extended, adapted or exploited in presence of perturbations, both single-particle and many-body. This strategy has lead to the discovery of non-perturbative metal-insulator transitions, fractal phases, nonlinear and quantum caging and many-body nonergodic quantum models. Starting from one CLS defined through a few complex numbers, we will derive degenerate condensate groundstates for interacting Bose-Einstein condensates, π-Josephson junction networks and XY spin lattices. We will match the above by reviewing a plethora of experimental applications.

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Razumevanje kompleksne dinamike mrež celic beta z večceličnimi matematičnimi modeli

DAVID RISTIČ[1], UROŠ BARAĆ[1], MARKO GOSAK[1,2,3]

[1] *Oddelek za fiziko, Fakulteta za naravoslovje in matematiko, Univerza v Mariboru, Koroška cesta 160, 2000 Maribor, Slovenija*
[2] *Inštitut za fiziologijo, Medicinska fakulteta, Univerza v Mariboru, Taborska ulica 8, 2000 Maribor, Slovenija*
[3] *Univerza Alma Mater Europaea, Slovenska ulica 17, 2000 Maribor, Slovenija*

Celice beta v Langerhansovih otočkih tvorijo električno sklopljen sincicij, ki izkazuje kompleksno kolektivno dinamiko na več časovnih skalah. Mrežne analize večceličnih posnetkov kalcijeve aktivnosti omogočajo kvantitativen opis te organizacije ter razkrivajo modularno funkcionalno povezanost in obstoj centralnih ter iniciatorskih celic, ki so pomembne pri mediaciji medceličnih signalov [1,2]. Vendar eksperimentalne meritve običajno zajemajo le omejen nabor spremenljivk, zato težko pojasnijo, kako znotrajcelični procesi, heterogenost celic in medcelična sklopitev oblikujejo opažene mrežne vzorce. V prispevku predstavimo večcelični matematični model, ki temelji na integriranem oscilatorskem modelu (angl. Integrated Oscillator Model, IOM) [3] in povezuje presnovno, električno ter kalcijevo dinamiko heterogenih celic beta. Primerjamo funkcionalne mreže, rekonstruirane iz različnih modelnih spremenljivk ter iz počasne in hitre komponente kalcijevih signalov. Pokažemo, da počasna komponenta odraža presnovno koordinacijo s povezavami tudi med prostorsko oddaljenimi celicami, hitra pa električno aktivnost s pretežno lokalnimi funkcionalnimi povezavami. Lastnosti centralnih celic in iniciatorjev kalcijevih valov so prav tako odvisne od časovne skale ter se različno povezujejo z vzdražnostjo in jakostjo medcelične sklopitve. Prostorsko korelirana heterogenost vzdražnosti omogoči nastanek območij s podobno aktivnostjo in njihovo delno ujemanje s funkcionalnimi skupnostmi, kot se opazi v eksperimentih [4]. Rezultati ponazarjajo, kako večcelični matematični modeli omogočajo mehanistično razlago eksperimentalno zaznanih funkcionalnih mrež ter globlji vpogled v povezavo med znotrajceličnimi procesi in kolektivno dinamiko celic beta [5].

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Understanding Complex Beta-Cell Network Dynamics through Multicellular Mathematical Models

DAVID RISTIČ[1], UROŠ BARAĆ[1], MARKO GOSAK[1,2,3]

[1]*Department of Physics, Faculty of Natural Sciences and Mathematics, University of Maribor, Koroška cesta 160, 2000 Maribor, Slovenia*
[2]*Institute of Physiology, Faculty of Medicine, University of Maribor, Taborska ulica 8, 2000 Maribor, Slovenia*
[3]*Alma Mater Europaea University, Slovenska ulica 17, 2000 Maribor, Slovenia*

Beta cells within pancreatic islets form an electrically coupled syncytium exhibiting complex collective dynamics across multiple timescales. Network analyses of multicellular calcium recordings provide a powerful framework for characterizing this organization, revealing modular functional connectivity and the presence of hub and wave initiators cells that mediate intercellular signals [1,2]. However, experimental measurements typically capture only a limited set of variables, making it difficult to disentangle how intracellular processes, cellular heterogeneity, and intercellular coupling shape the observed network patterns. Here, we present a multicellular mathematical model based on the Integrated Oscillator Model (IOM) [3], combining metabolic, electrical, and calcium dynamics in heterogeneous beta cells. We compare functional networks reconstructed from different model variables and from the slow and fast components of calcium signals. Slow calcium dynamics reflect metabolic coordination, including functional connections between spatially distant cells, whereas fast calcium dynamics reflect electrical activity with predominantly local connectivity. The properties of hub cells and calcium wave initiators also depend on the timescale, exhibiting different relationships with intrinsic excitability and intercellular coupling strength. Spatially correlated excitability heterogeneity promotes regions of similar cellular activity and their partial correspondence with functional communities, as observed in experiments [4]. These findings demonstrate how multicellular mathematical models support mechanistic interpretation of experimentally inferred functional networks and provide deeper insight into the links between intracellular processes and collective beta-cell dynamics [5].

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Kako lahko opazujemo notranjost črnih lukenj?

SAŠO GROZDANOV

Higgs Centre for Theoretical Physics, University of Edinburgh, Edinburgh, EH8 9YL, Scotland
FMF, University of Ljubljana, Jadranska ulica 19, SI-1000 Ljubljana, Slovenia
saso.grozdanov@fmf.uni-lj.si

Dogodkovno obzorje črne luknje nam preprečuje neposredno opazovanje njene notranjosti. Kljub temu se lahko informacije o geometriji v bližini singularnosti odražajo v korelacijskih funkcijah in spektrih kvazinormalnih nihanj, ki jih določimo zunaj obzorja. V predavanju bom predstavil tri nedavna dela [1–3], v katerih smo to povezavo postopoma razvili. Najprej bom pojasnil, kako svetlobne geodetke, ki prodrejo globoko v notranjost črne luknje, povzročijo singularnosti v retardiranih korelacijskih funkcijah. Nato bom obravnaval asimptotsko ravne črne luknje in črne luknje v prostoru de Sitter ter pokazal, kako se v prisotnosti odbojne meje informacija o notranjosti zapiše v spekter kvazinormalnih nihanj. Ključni rezultat je univerzalna asimptotska zveza med visokimi kvazinormalnimi načini in geodetkami, ki se približajo singularnosti. Nazadnje bom opisal možen astrofizikalni scenarij, v katerem akrecijska plazma deluje kot delno prepustno zrcalo, elektromagnetni signali hipotetičnih majhnih prvobitnih črnih lukenj pa bi lahko razkrili lastnosti njihove notranjosti.

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How can we observe the interiors of black holes?

SAŠO GROZDANOV

Higgs Centre for Theoretical Physics, University of Edinburgh, Edinburgh, EH8 9YL, Scotland
FMF, University of Ljubljana, Jadranska ulica 19, SI-1000 Ljubljana, Slovenia
saso.grozdanov@fmf.uni-lj.si

A black hole's event horizon prevents us from directly observing its interior. Nevertheless, information about the geometry near the singularity can be encoded in correlation functions and quasinormal mode spectra accessible outside the horizon. In this talk, I will discuss three recent works [1–3] in which we developed this connection. I will first explain how null geodesics probing the black hole interior give rise to singularities in retarded correlation functions. I will then turn to asymptotically flat and de Sitter black holes and show how, in the presence of a reflecting boundary, information about the interior is encoded in the quasinormal mode spectrum. The central result is a universal asymptotic relation between highly damped quasinormal modes and geodesics probing the vicinity of the singularity. Finally, I will outline a possible astrophysical setup in which accreting plasma acts as a partially transmissive mirror and electromagnetic signals from hypothetical small primordial black holes could reveal properties of their interiors.

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Exact Finite-$\hbar$ Path Representation of Quantum States in One Dimension

LIANG HUANG

Lanzhou Center for Theoretical Physics, School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China
huangl@lzu.edu.cn • *https://icpcs.lzu.edu.cn/lhuang/*

A central question in quantum–classical correspondence is whether quantum states can be represented exactly in terms of classical dynamical quantities beyond the semiclassical limit. We develop an exact finite-$\hbar$ formulation for stationary quantum states in one dimension, expressing the wave function as an absolutely convergent pseudo-path expansion constructed from classical momentum and action. The expansion is organized by reflection order, with analytic upper bounds on the contribution of each order. The formulation also yields eigenenergies and eigenstates for bound-state problems and admits an efficient numerical implementation. The reflection-order decomposition reveals quantitatively how different path contributions build a quantum state, providing an exact finite-$\hbar$ connection between quantum states and classical momentum and action.

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Večskalna dinamika in kolektivna organizacija mrež celic β trebušne slinavke

LANA KRALJ[1,3] in MARKO GOSAK[1,2,3]

[1] *Inštitut za fiziologijo, Medicinska fakulteta, Univerza v Mariboru, Taborska ulica 8, 2000 Maribor, Slovenija*
[2] *Oddelek za fiziko, Fakulteta za naravoslovje in matematiko, Univerza v Mariboru, Koroška cesta 160, 2000 Maribor, Slovenija*
[3] *Alma Mater Europaea, Slovenska ulica 17, 2000 Maribor, Slovenija*
lana.kralj1@um.si, marko.gosak@um.si

Celice β v Langerhansovih otočkih predstavljajo značilen primer kompleksnega večceličnega sistema, v katerem usklajena aktivnost nastaja kot posledica prepletanja procesov na različnih časovnih in prostorskih skalah. Za njihovo dinamiko so značilne počasne presnovne oscilacije, ki so tesno povezane s hitrejšo električno aktivnostjo in oscilacijami znotrajcelične koncentracije Ca^{2+}. Usklajeno delovanje med seboj povezanih celic omogoča nastanek kolektivnih vzorcev aktivnosti, ki imajo pomembno vlogo pri pulznem izločanju inzulina [1,2]. V prvem delu predavanja se bomo osredotočili na večskalno organizacijo dinamike celic β, ki jo lahko preučujemo s kombinacijo eksperimentalnega slikanja kalcijeve aktivnosti in naprednih časovno-frekvenčnih metod. Konfokalno slikanje v akutnih rezinah trebušne slinavke omogoča sočasno spremljanje aktivnosti velikega števila celic β v njihovem naravnem večceličnem okolju [3]. Za podrobnejšo analizo tako pridobljenih signalov lahko uporabimo valčno analizo, ki omogoča vpogled v večskalno dinamiko kalcijeve aktivnosti, časovni razvoj posameznih oscilacijskih komponent ter interakcije med počasnimi in hitrimi oscilacijami [4]. V nadaljevanju bomo preučili funkcionalno povezanost med celicami in organizacijo funkcionalnih mrež na različnih časovnih skalah. Pokazali bomo, da so hitre oscilacijske komponente povezane predvsem z lokalno organiziranimi funkcionalnimi mrežami, medtem ko počasne komponente odražajo prostorsko bolj razširjeno povezanost [4,5]. Posebno pozornost bomo namenili tudi metodološkim vidikom rekonstrukcije funkcionalnih mrež, predvsem vplivu predobdelave signalov, izbire analitičnih parametrov in metod določanja funkcionalnih povezav na topologijo mrež ter njihovo fiziološko interpretacijo [5]. Čeprav eksperimentalni in analitični pristopi omogočajo dragocen vpogled v večskalno dinamiko in funkcionalno organizacijo celic β trebušne slinavke, ostaja razumevanje mehanizmov, na katerih temelji njihovo kolektivno delovanje, še vedno nepopolno. Pomemben izziv za prihodnje raziskave zato predstavlja razvoj večceličnih računskih modelov, ki bi omogočili povezovanje dinamike posameznih celic z nastankom kolektivne organizacije na različnih časovnih in prostorskih skalah.

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Unraveling Multiscale Dynamics and Collective Organization in Pancreatic Beta-Cell Networks

LANA KRALJ[1,3] and MARKO GOSAK[1,2,3]

[1] *Institute of Physiology, Faculty of Medicine, University of Maribor, Taborska ulica 8, SI-2000 Maribor, Slovenia*
[2] *Department of Physics, Faculty of Natural Sciences and Mathematics, University of Maribor, Koroška cesta 160, SI-2000 Maribor, Slovenia*
[3] *Alma Mater Europaea, Slovenska ulica 17, SI-2000 Maribor, Slovenia*
lana.kralj1@um.si, marko.gosak@um.si

Pancreatic β-cells within the islets of Langerhans represent a paradigmatic example of a complex multicellular system in which coordinated activity emerges from the interplay of processes operating across multiple temporal and spatial scales. Their dynamics are characterized by slow metabolic oscillations that are closely coupled to faster electrical activity and oscillations in intracellular Ca^{2+} concentration. Coordinated activity among interconnected cells gives rise to collective patterns that play an important role in pulsatile insulin secretion [1,2]. In the first part of the talk, we will focus on the multiscale organization of pancreatic β-cell dynamics, which can be investigated by combining experimental calcium imaging with advanced time-frequency analysis. Confocal calcium imaging in acute pancreatic tissue slices enables simultaneous monitoring of large populations of β-cells within their native multicellular environment [3]. The resulting calcium signals can then be examined using wavelet analysis, providing insights into the temporal evolution of distinct oscillatory components and interactions between slow and fast oscillations [4]. At the multicellular level, we will explore functional connectivity among β-cells and the organization of functional networks across different temporal scales. We will show that fast oscillatory components are predominantly associated with locally organized functional networks, whereas slow components reflect more spatially extended connectivity [4,5]. Particular attention will also be given to methodological aspects of functional network reconstruction, examining how signal preprocessing, the choice of analytical parameters, and methods for defining functional connections influence network topology and its physiological interpretation [5]. While experimental and analytical approaches provide valuable insights into the multiscale dynamics and functional organization of pancreatic β-cells, a mechanistic understanding of the processes underlying collective behavior remains incomplete. Addressing this challenge will require the development of multicellular computational models capable of linking individual cellular dynamics to emergent collective organization across multiple temporal and spatial scales.

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Complex Systems and AI: From Foundational Research to Technical Breakthroughs

KLAUS MAINZER

Senior Excellence Faculty, Technical University of Munich (TUM)
Carl Friedrich von Weizsäcker Center, University of Tübingen
President of European Academy of Sciences and Arts

Permanently increasing sets of data (Big Data) in machine learning are not sufficient to provide reliable and accurate artificial intelligence. What is needed are AI-methods based and regulated by models of logics, mathematics, and physical sciences. Recently, millenium problems of mathematics were claimed to be solved by statistical machine learning. But actually, big data and statistical learning algorithms are not sufficient to cheque these proofs seriously. That is also true for physical sciences and the safety relevant systems of engineering sciences (e.g., bridges, transport, traffic) which need the extension of statistical learning with solutions of natural laws and physical equations. Therefore, since many years, my research is dedicated to the foundations of computational complexity theory in computer science and dynamical complex systems in physical sciences. However, digitalisation requires a huge amount of energy. Therefore, classic digitalisation and artificial intelligence are examined together with future technologies such as neuromorphic computing (based on the energy-saving mode of natural brains with analog computing) as well as quantum computers and quantum technology. Digital and analogue technologies must then be integrated in a 'hybrid' IT and AI so that this innovation portfolio, together with the energy issue, is also geared towards a sustainable future of mankind.

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Invariantne algebraične krivulje polinomskih sistemov NDE

IVAN MASTEV

CAMTP - Center za uporabno matematiko in teoretično fiziko
Univerza v Mariboru, Mladinska 3, SI-2000 Maribor, Slovenija
ivan.mastev@student.um.si

Predstavil bom algebraični postopek za iskanje invariantnih algebraičnih krivulj določene stopnje za ravninske polinomske sisteme navadnih diferencialnih enačb s parametri. Iz pogoja invariantnosti dobimo polinomske enačbe v parametrih sistema ter neznanih koeficientih krivulje in njenega kofaktorja. S saturacijo odstranimo trivialne družine konstantnih polinomov, z algebraično eliminacijo pa dobimo pogoje za parametre. Ustrezne krivulje in njihove kofaktorje nato določimo z algoritmi računske algebre. Za preizkus našega algoritmičnega pristopa uporabimo postopek na sistemih, katerih invariantne algebraične krivulje so bile določene v predhodnih delih [1, 2], ter dobimo enake krivulje in pogoje za parametre. Poiščemo tudi invariantne krivulje nekaterih splošnejših polinomskih sistemov.

Soavtorja dela sta Bo Huang in Valerij Romanovskij.

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Invariant algebraic curves of polynomial systems of ODEs

IVAN MASTEV

CAMTP - Center for Applied Mathematics and Theoretical Physics
University of Maribor, Mladinska 3, SI-2000 Maribor, Slovenia
ivan.mastev@student.um.si

I describe an algebraic procedure for finding invariant algebraic curves up to a prescribed degree in planar polynomial differential systems with parameters. The invariance condition gives polynomial equations in the system parameters and the unknown coefficients of the curve and its cofactor. Saturation removes the trivial families of constant polynomials, and algebraic elimination gives conditions on the parameters. The corresponding curves and cofactors are then determined using algorithms of computational algebra. To test our algorithmic approach we apply the procedure and computational routines to systems whose invariant algebraic curves were determined in previous papers [1, 2] and obtain the same curves and parameter conditions. We also find invariant curves of some more general polynomial systems.

This is joint work with Bo Huang and Valery Romanovski.

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Perturbirani kvantni biljardi na hiperbolični ravnini

MATIC OREL

CAMTP - Center za uporabno matematiko in teoretično fiziko
Univerza v Mariboru, Mladinska 3, SI-2000 Maribor, Slovenija

Obravnavamo nastanek generičnega kvantnega kaosa v trikotnih biljardih na hiperbolični ravnini, ko z majhnimi geometrijskimi perturbacijami porušimo aritmetično strukturo. Izhodišče je Schmitov aritmetični trikotnik, katerega kvantni spekter kljub izrazito kaotičnemu klasičnemu gibanju kaže Poissonu podobno statistiko. S spremembami kotov, ki ohranjajo ploščino, preučujemo prehod k statistiki Gaussovega ortogonalnega ansambla (GOE). Z metodo robnih integralskih enačb in Beynovo konturno metodo izračunamo dolge zaporedne spektre z do 782 000 lastnimi vrednostmi. Analiza razmikov med sosednjimi nivoji, razmerij razmikov in dolgodosežnih spektralnih korelacij pokaže, da se z manjšanjem perturbacije prehod pomika k višjim valovnim številom. Struktura pri majhnih razmikih in rep porazdelitve pri velikih razmikih se spreminjata na različnih skalah, kar omejuje uporabnost Brodyjeve porazdelitve z enim samim parametrom. Za podrobnejšo analizo prehoda uporabimo posplošeno gama porazdelitev, pri najšibkejši perturbaciji pa tudi model šibko sklopljenih GOE blokov. Lokalizacijo lastnih stanj v faznem prostoru preučimo s Poincaré–Husimijevimi funkcijami in entropijsko mero lokalizacije. Empirične porazdelitve te mere v spektralnih oknih dobro opišemo z beta porazdelitvami. Po doseženem GOE režimu njihovi standardni odkloni sledijo skupnemu potenčnemu zakonu $\sigma_{\mathrm{Beta}} \sim k^{-\eta}$, z $\eta \simeq 0.38$, podobno kot v naši predhodni raziskavi evklidskega biljarda s simetrijo C_3 [7]. Rezultati povezujejo izgubo aritmetične strukture s semiklasičnim pojemanjem fluktuacij lokalizacije lastnih stanj.

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Perturbed quantum billiards on the hyperbolic plane

MATIC OREL

CAMTP - Center for Applied Mathematics and Theoretical Physics
University of Maribor, Mladinska 3, SI-2000 Maribor, Slovenia

We study the emergence of generic quantum chaos in hyperbolic triangle billiards when arithmeticity is broken by small geometric perturbations. The starting point is Schmit's arithmetic triangle, whose quantum spectrum exhibits Poisson-like statistics despite strongly chaotic classical motion. Area-preserving changes of its angles allow us to follow the transition towards the Gaussian orthogonal ensemble (GOE). Using a boundary-integral formulation and Beyn's contour method, we compute long consecutive spectra containing up to 782 000 eigenvalues. Nearest-neighbour spacings, gap ratios and long-range spectral correlations reveal a crossover that shifts to higher wavenumbers as the perturbation decreases. The small-spacing structure and the large-spacing tail evolve on different scales, limiting the applicability of one-parameter Brody fits; generalized-gamma fits and, for the weakest perturbation, a weakly coupled block-GOE model serve as complementary diagnostics. We also analyse the phase-space structure of the eigenstates through Poincaré–Husimi functions and an entropy-based localization measure. Its empirical distributions in spectral windows are well described by Beta distributions. After the GOE regime is reached, their standard deviations collapse onto a common power law, $\sigma_{\mathrm{Beta}} \sim k^{-\eta}$, with $\eta \simeq 0.38$, consistent with the decay observed in our previous study of a C_3-symmetric Euclidean billiard [7]. The results connect the loss of arithmetic structure with the semiclassical suppression of eigenstate localization fluctuations.

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Relativistic Poincaré-Invariant Theory of Baryon Electroweak Form Factors

WILLIBALD PLESSAS

Theoretical Physics, Institute of Physics
University of Graz, Universitätsplatz 5, A-8010 Graz, Austria
willibald.plessas@uni-graz.at

For any model of hadrons it is not only mandatory to describe their masses and excitation spectra [1] but also their structures as observed in experiment, notably the electromagnetic and axial form factors [2,3]. This is especially true for constituent-quark models, a long-standing effective approach to low-energy quantum chromodynamics (QCD). Contrary to frequent claims in the literature (and some folklore traceable even in contemporary textbooks) about the feasibility of the non-relativistic constituent-quark models there is no way of describing the hadron structures except for a relativistically invariant quantum theory [4].

I shall demonstrate, how in a relativistic quantum theory, formulated on a Hilbert space with finite degrees of freedom, covariant observables for electroweak form factors can be calculated. From the results, it will become evident that Poincaré invariance must be observed in order to reproduce the experimental data even in the low-energy regime. Neglecting relativistic effects constitutes an obsolete approach.

As the theory requires a Hilbert space in order to perform all transformations of the Poincaré algebra it can strictly be applied only to ground and excited hadron states but not to genuine resonances coupling to decay channels with two or more decay products. While the problems with resonances as complex poles and residua that are not square-integrable can be circumvented in a non-relativistic theory [5], there is no immediate solution for relativistic resonances in decay.

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Japonska jedrska energetika: 15 let po Fukušimi

ANITA PRAPOTNIK BRDNIK[a,b]

[a] *Fakulteta za gradbeništvo, prometno inženirstvo in arhitekturo, SI-2000 Maribor, Slovenija*

[b] *Fakulteta za energetiko*
Koroška cesta 62a, SI-3320 Velenje, Slovenija

anita.prapotnik@um.si

Avgusta in septembra 2026 sem se v Tokiu udeležila mednarodne šole za upravljanje jedrske energetike (Japan–IAEA Nuclear Energy Management School), organizirane v sodelovanju z Mednarodno agencijo za atomsko energijo (IAEA). V okviru šole smo se udeležili tudi štiridnevne strokovne ekskurzije, med katero smo obiskali več jedrskih objektov na Japonskem. Med drugim smo si ogledali delujočo jedrsko elektrarno Onagava (Onagawa), ki je bila ob potresu in cunamiju 11. marca 2011 bližje epicentru potresa kot Fukušima, vendar se je izognila katastrofalnim posledicam. Obiskali smo tudi jedrsko elektrarno Fukušima Daiiči (Fukushima Daiichi), kjer poteka dolgotrajen in tehnično zahteven proces razgradnje, ter raziskovalne objekte Japonske agencije za atomsko energijo (JAEA), namenjene raziskavam jedrskih reaktorjev, jedrske varnosti in izrabljenega jedrskega goriva. V okviru predavanj so nam predstavili tudi razvoj japonske jedrske energetike in energetsko strategijo države, ki po nesreči v Fukušimi ponovno povečuje pomen jedrske energije. Na letošnjem božičnem simpoziju bom predstavila svoje vtise z obiskov ter podala pregled stanja japonske jedrske energetike pred nesrečo leta 2011, posledic nesreče, trenutnega stanja jedrskih elektrarn in načrtov za prihodnost.

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Nuclear Energy in Japan: 15 Years After Fukushima

ANITA PRAPOTNIK BRDNIK[a,b]

[a] *Faculty of Civil Engineering, Transport Engineering and Architecture, SI-2000 Maribor, Slovenia*

[b] *Faculty of Energy Technology*
Koroška cesta 62a, SI-3320 Velenje, Slovenia

anita.prapotnik@um.si

In August and September 2026, I had the opportunity to attend the Japan–IAEA Nuclear Energy Management School in Tokyo, organized in cooperation with the International Atomic Energy Agency (IAEA). The programme included a four-day technical field trip, during which we visited several nuclear facilities in Japan. Among them was the operating Onagawa Nuclear Power Plant, which was located closer to the epicentre of the earthquake on 11 March 2011 than Fukushima Daiichi, yet managed to avoid a major nuclear accident. We also visited the Fukushima Daiichi Nuclear Power Plant, where a long and technically challenging decommissioning process is underway, as well as research facilities of the Japan Atomic Energy Agency (JAEA), dedicated to nuclear reactor research, nuclear safety and spent nuclear fuel. The lectures also provided an overview of the development of nuclear energy in Japan and the country's energy strategy, which once again places increasing emphasis on nuclear power following the Fukushima accident. At this year's Christmas Symposium, I would like to share my impressions from these visits and present an overview of nuclear power in Japan before the 2011 accident, its consequences, the current status of Japanese nuclear power plants and the country's plans for the future.

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Kvantni kaos v odsotnosti klasične korespondence

TOMAŽ PROSEN

Fakulteta za matematiko in fiziko, oddelek za fiziko
Univerza v Ljubljani, Jadranska 19, SI-1000 Ljubljana, Slovenija
tomaz.prosen@fmf.uni-lj.si • chaos.fmf.uni-lj.si

Klasični kaos je matematično dobro opredeljen pojem, njegov natančni pomen v kvantnem svetu pa ostaja nejasen. V zadnjem desetletju so raziskave kvantnega kaosa napredovale predvsem v dveh dopolnjujočih se smereh. Prva išče kvantne ustreznice klasične Ljapunovove nestabilnosti s pomočjo časovno neurejenih korelatorjev, zlasti v mnogodelčnih sistemih in kvantnih teorijah polja v režimu velikega N. Druga kvantni kaos opredeljuje z univerzalnostjo teorije naključnih matrik – karakterizirane na primer s spektralno statistiko. S tem dobimo abstraktnejši in širše uporaben pojem kvantnega kaosa, ki se ne opira na korespondenco s klasično fiziko niti na obstoj majhnega ali velikega parametra.

Predstavil bom razred modelov, ki ponazarjajo drugi pristop: dualno unitarne sisteme in njihove perturbacije. Za te modele je mogoče izpeljati eksaktne – in pogosto tudi matematično rigorozne – rezultate za spektralno statistiko, rast operatorjev in druge mere dinamične kompleksnosti. Zato zagotavljajo dobro obvladljiv teoretični okvir za raziskovanje pomena kvantnega kaosa v odsotnosti klasičnega analoga.

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Quantum Chaos Without a Classical Correspondence

TOMAŽ PROSEN

Faculty of Mathematics and Physics, Department of Physics
University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia
tomaz.prosen@fmf.uni-lj.si • chaos.fmf.uni-lj.si

Classical chaos is a mathematically well-defined concept, but its precise meaning in the quantum realm remains elusive. Over the past decade, research on quantum chaos has advanced mainly along two complementary directions. The first seeks quantum analogues of classical Lyapunov instability through out-of-time-ordered correlators, particularly in many-body systems and quantum field theories in a large-N regime. The second characterizes quantum chaos through random-matrix universality—for example, in spectral statistics. This provides a more abstract and broadly applicable notion of quantum chaos that does not rely on a classical correspondence or on the existence of a small or large parameter.

I will discuss a class of models exemplifying the second approach: dual-unitary systems and their perturbations. For these models, one can derive exact—and often rigorous—results for spectral statistics, operator growth, and other measures of dynamical complexity. They therefore provide a controlled setting in which to explore what quantum chaos means in the absence of a classical counterpart.

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Towards "Riemann Dynamics"

KLAUS RICHTER

Institute for Theoretical Physics
University of Regensburg, 93050 Regensburg, Germany
klaus.richter@ur.de

As known for a long time [1] various striking facts hint towards a deep connection between quantum chaos and number theory, more precisely, the zeros of the Riemann zeta function. For example, the Riemann zeros, interpreted as energy eigenvalues, obey GUE random matrix statistics. Moreover, comparing counting functions of the positions of the Riemann zeros and formulae for semiclassical quantum eigenvalues suggests that the Riemann zeros are eigenvalues of a hermitean quantum mechanical operator, semiclassically related to a classical dynamical system with specific features. These features are provided by analogy, e.g., the dynamics should be "chaotic" and have periodic orbits whose periods are multiples of logarithms of prime numbers.

Starting from a recently proposed class of quantum hamiltonians [2,3] that provide the Riemann zeros as eigenvalues I will consider its semiclassical limit. There are clear indications that the corresponding "classical" Riemann dynamics is not chaotic in the usual sense but rather equivalent to dynamics on peculiar quantum graphs (for an earlier attempt see [4]). Massive (destructive) interference reduces the number of contributing periodic orbits, labeled by integers, to those with periods given by the logarithms of primes.

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Kaos in kvantizacija tridelčnega modela Fermi-Pasta-Ulam-Tsingou: gostota stanj, spektralna statistika in fenomenologija kvantnih lastnih stanj

MARKO ROBNIK [1]

CAMTP - Center za uporabno matematiko in teoretično fiziko
Univerza v Mariboru, Mladinska 3, SI-2000 Maribor, Slovenia
Marko.Robnik@guest.um.si • www.camtp.uni-mb.si

Proučujemo klasično dinamiko mešanega tipa v tridelčnem modelu Fermi-Pasta-Ulam-Tsingou (FPUT) [1-3] v povezavi s pridruženim kvantnim sistemom in predstavimo nove rezultate o vidikih kvantnega kaosa v tem sistemu. Najprej [4] izpeljemo za splošen N-delčni sistem FPUT transformacijo v predstavitev z normalnimi načini. Nato se omejimo na tridelčni FPUT primer in analitično izpeljemo semiklasično gostoto energije stanj, z uporabo pravila Thomasa-Fermi-ja. Rezultat se ujema z numerično energijsko gostoto dobljeno z metodo podprostora Krylova, kakor tudi z energijsko gostoto z uporabo kvantne tipičnosti. Proučimo statistiko energijskih nivojev (porazdelitev razmikov med sosednjimi nivoji), v vseh dinamičnih režimih, ki nas zanimajo: skoraj povsem regularne, povsem kaotične in režime mešanega tipa. Zelo jasno potrdimo, ustrezno, Poissonovo statistiko, statistiko Gausovega ortogonalnega ansambla, ter statistiko Berry-Robnik-Brody (BRB) v mešanem režimu. Ugotovimo, da porazdelitev BRB razmikov med sosednjimi nivoji popolnoma ustreza numeričnim podatkom. Odčitani Berry-Robnikov parameter se ujema s klasično vrednostjo z natančnostjo boljšo od enega procenta. Proučimo vlogo lokalizacije kaotičnih stanj, ter njene pojavnosti, v odnosu do strukture klasičnega faznega prostora (Poincaré jevi diagrami ter diagrami SALI - manjši indeks poravnave; Smaller ALignment Index).

Nato [5] temeljito proučujemo fenomenologijo kvantnih lastnih stanj. Uporabljajoč razne Husimijeve funkcije, se naša obravnava osredotoči na model tipa α, ki je kanonično ekvivalenten znanemu hamiltonijanu Henona-Heilesa, in je neintegrabilen sistem mešanega tipa, ter na splošen sistem pri energiji sedla, kjer je sistem povsem kaotičen. Na osnovi Husimijeve kvantne presečne ploskve vidimo v mešanem režimu, da delež mešanih lastnih stanj v energijski lupini $[E - \delta E/2, E + \delta E/2]$ ob $\delta E << E$ pojema kot potenčni zakon kot funkcija Planckove konstante $\hbar$ [6-10]. Ob definiciji lokalizacijskih mer v smislu Rényi-Wehrl-ove entropije vidimo, da je tako v mešanem režimu kakor tudi v povsem kaotičnem režimu ujemanje porazdelitve lokalizacijskih mer z beta prazdelitvijo boljše in stopnja lokalizacije manjša, ko se kontrolni indeks $\alpha_L = t_H/t_T$ med Heisenbergovim časom t_H in klasičnim transportnim časom t_T povečuje. Ta prehod v odnosu do α_L ter potenčni zakon pojemanja deleža mešanih stanj, skupaj podajajo dodatno evidenco Principa enakomerne semiklasične kondenzacije (Principle of Uniform Semiclassical Condensation, PUSC) [11-12] v semiklasični limiti. Nadalje, pokažemo, da je v povsem kaotičnem splošnem sistemu maksimalno loalizirano stanje pod vplivom stabilne in nestabilne mnogoterosti sedla (hiperbolične fiksne točke), medtem ko se maksimalno delokalizirana stanja izogibajo teh točk, in se razširjajo čez preostali prostor, v medsebojnem komplementu.

[1] skupni projekt s Hua Yan-nom, CAMTP

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Chaos and quantization of the three-particle Fermi-Pasta-Ulam-Tsingou model: Density of states, spectral statistics and phenomenology of quantum eigenstates

MARKO ROBNIK [2]

CAMTP - Center for Applied Mathematics and Theoretical Physics
University of Maribor, Mladinska 3, SI-2000 Maribor, Slovenia
Marko.Robnik@um.si • www.camtp.uni-mb.si

We study the mixed-type classical dynamics of the three-particle Fermi-Pasta-Ulam-Tsingou (FPUT) model [1-3] in relationship with its quantum counterpart and present new results on aspects of quantum chaos in this system. First [4] we derive for the general N-particle FPUT system the transformation to the normal mode representation. Then we specialize to the three-particle FPUT case and derive analytically the semiclassical energy density of states, using the Thomas-Fermi rule. The result agrees with the numerical energy density from the Krylov subspace method, as well as with the energy density obtained by the method of quantum typicality. We study the energy level statistics (level spacing and spacing ratios), in all classical dynamical regimes of interest: the almost entirely regular, the entirely chaotic, and the mixed-type regimes. We clearly confirm, correspondingly, the Poissonian statistics, the Gaussian orthogonal ensemble statistics, and the Berry-Robnik-Brody (BRB) statistics in the mixed-type regime. It is found that the BRB level spacing distribution perfectly fits the numerical data. The extracted quantum Berry-Robnik parameter is found to agree with the classical value within better than one percent. We discuss the role of localization of chaotic eigenstates, and its appearances, in relation to the classical phase space structure (Poincaré and smaller alignment index plots).

Then [5] we undertake a thorough investigation into the phenomenology of quantum eigenstates. Employing different Husimi functions, our study focuses on both the α-type, which is canonically equivalent to the celebrated Hénon-Heiles Hamiltonian, a nonintegrable and mixed-type system, and the general case at the saddle energy where the system is fully chaotic. Based on Husimi quantum surface of sections, we find that in the mixed-type system, the fraction of mixed eigenstates in an energy shell $[E - \delta E/2, E + \delta E/2]$ with $\delta E << E$ obeys a power-law decay with respect to the decreasing Planck constant $\hbar$ [6-10]. Defining the localization measures in terms of the Rényi-Wehrl entropy, in both the mixed-type and fully chaotic systems, we find a better fit with the beta distribution and a lesser degree of localization, in the distribution of localization measures of chaotic eigenstates, as the controlling ratio $\alpha_L = t_H/t_T$ between the Heisenberg time t_H and the classical transport time t_T increases. This transition with respect to α_L and the power-law decay of the mixed states, together provide supporting evidence for the Principle of Uniform Semiclassical Condensation (PUSC) [11-12] in the semiclassical limit. Moreover, we find that in the general case which is fully chaotic, the maximally localized state, is influenced by the stable and unstable manifold of the saddles (hyperbolic fixed points), while the maximally extended state notably avoids these points, extending across the remaining space, complementing each other.

[2] joint work with Hua Yan, CAMTP

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Superintegrabilnost in perturbacije polinomskih hamiltonskih sistemov

VALERY ROMANOVSKI[1,2,3]

[1] *CAMTP -Center za uporabno matematiko in teoretično fiziko Univerza v Mariboru, Mladinska 3, SI-2000 Maribor, Slovenia*
[2] *Fakulteta za elektrotehniko, računalništvo in informatiko*
[3] *Fakulteta za naravoslovje in matematiko*
Univerza v Mariboru, SI-2000 Maribor, Slovenia
valerij.romanovskij@um.si

Raziskujemo superintegrabilnost polinomskih hamiltonskih sistemov z dvema prostostnima stopnjama in njeno izgubo pri perturbacijah. Naš pristop združuje metode računske algebre s teorijo Poincaré–Dulacovih normalnih form in Brunovimi pogoji za konvergenco normalizacijskih transformacij.

Obravnavamo družino štiridimenzionalnih polinomskih Hamiltonovih sistemov, katerih linearni deli imajo lastne vrednosti 1,-1,2,-2, kar ustreza dvema harmoničnima oscilatorjema v resonanci 1:2. Z uporabo algebraičnih pogojev, izpeljanih iz teorije normalnih form, določimo tri družine lokalno analitično superintegrabilnih sistemov in konstruiramo njihove prve integrale.

Nato raziskujemo perturbacije teh družin, pri katerih se izgubi eden ali oba dodatna prva integrala. Dinamiko dobljenih sistemov proučujemo s Poincaréjevimi preseki pri različnih energijskih nivojih. Numerični eksperimenti ponazarjajo prehod od superintegrabilnega gibanja k Liouvillovo integrabilni dinamiki in pri splošnejših perturbacijah k mešani dinamiki, za katero so značilni invariantni torusi, resonančne strukture in kaotično obnašanje.

Rezultati kažejo, kako lahko teorijo normalnih form in računsko algebro združimo z numeričnimi metodami pri raziskovanju povezav med integrabilnostjo, superintegrabilnostjo in pojavom kompleksne dinamike v hamiltonskih sistemih.

Raziskava je skupno delo z Ivanom Mastevom.

Superintegrability and perturbations of polynomial Hamiltonian systems

VALERY ROMANOVSKI[1,2,3]

[1]*CAMTP - Center for Applied Mathematics and Theoretical Physics*
University of Maribor, Mladinska 3, SI-2000 Maribor, Slovenia
[2]*Faculty of Electrical Engineering and Computer Science*
[3]*Faculty of Natural Science and Mathematics*
University of Maribor, SI-2000 Maribor, Slovenia
valerij.romanovskij@um.si

We study superintegrability and its loss under perturbations in polynomial Hamiltonian systems with two degrees of freedom. Our approach combines methods of computational algebra with Poincaré–Dulac normal form theory and Bruno's conditions for the convergence of normalizing transformations.

We consider a family of four-dimensional polynomial Hamiltonian systems whose linear parts have eigenvalues $1, -1, 2, -2$, corresponding to two harmonic oscillators in $1 : 2$ resonance. Using algebraic conditions derived from normal form theory, we identify three families of locally analytically superintegrable systems and construct their additional first integrals.

We then investigate perturbations of these families that destroy one or both additional first integrals. The resulting dynamics is studied using Poincaré sections at different energy levels. The numerical experiments illustrate the transition from superintegrable motion to Liouville-integrable dynamics and, under more general perturbations, to mixed dynamics exhibiting invariant tori, resonant structures, and chaotic behavior.

The results demonstrate how normal form theory and computational algebra can be combined with numerical methods to investigate the relationship between integrability, superintegrability, and the emergence of complex dynamics in Hamiltonian systems.

This is a joint work with Ivan Mastev.

Atomic Bright Solitons in Optical Lattices

LUCA SALASNICH

Dipartimento di Fisica e Astronomia "Galileo Galilei"
University of Padova, Italy

Bright solitons are self-localized nonlinear wave packets arising from the balance between dispersion and self-focusing nonlinearity [1]. They occur in a variety of physical systems, including nonlinear optical media and Bose-Einstein condensates. In ultracold atomic gases with attractive interactions, bright matter-wave solitons were experimentally realized more than two decades ago [2,3], and in quasi-one-dimensional geometries they may represent the ground-state configuration of the condensate.

I will discuss their description within the Gross-Pitaevskii equation and effective one-dimensional models obtained by dimensional reduction of a transversely confined condensate. In particular, the nonpolynomial Schrödinger equation accurately describes quasi-one-dimensional bright solitons and the onset of collapse at sufficiently strong attraction [4,5]. I will then consider bright solitons in a periodic optical lattice, where the competition between attraction, transverse confinement and the lattice gives rise to single-site and multi-site localized states with well-defined stability regions [6]. These predictions have recently been investigated experimentally with ultracold cesium atoms in an optical accordion lattice [7]. I will briefly compare the theoretical stability diagram with the observed localization regimes.

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Latent and Hidden Symmetries: Fundamentals and First Applications

PETER SCHMELCHER

ZOQ - Center for Optical Quantum Technologies
University of Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany
peter.schmelcher@uni-hamburg.de

Symmetries represent a cornerstone for a broad range of physical theories. Knowing the underlying symmetries allows to make prediction for a physical system without even performing concrete analytical or numerical investigations of the specific setup. But what about a system that has no obvious e.g. geometrical symmetries ? We introduce so-called latent symmetries which belong to a setup with no symmetries but shows in a corresponding isospectrally reduced Hamiltonian a hidden symmetry. Recent graph theoretical results provide a unique connection between the original and the reduced system, such that the impact of those latent symmetries become manifest in the original setup. We describe the isospectral reduction techniques in graph theory and their applications to a class of different physical problems. We show that flat bands can be generated and controllably designed from a latent symmetry of the lattice unit cell. The developed framework offers perspectives to analyze and design flat band structures in a variety of physical systems, including cold atoms, optical and acoustic wave setups. Latent symmetries possess a range of different applications such as hidden (higher order) topological properties, optimal (quantum) state transfer, and in particular the explanation and design of spectral degeneracies which otherwise would be considered as accidental.

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Cross-relaxation in NMR: Fifty years ago and today

HANS-JÜRGEN STÖCKMANN

Fachbereich Physik
Philipps-Universität Marburg, Renthof 5, D-35032 Marburg, Germany
stoeckmann@physik.uni-marburg.de

Nuclear magnetic resonance (NMR) is a well-established method to study in particular the local surroundings of nuclei in solids and molecules. In an NMR experiment a sample containing a nuclear spin ensemble is spin-polarized, and a radio frequency (rf) field is applied. Whenever the frequency of the rf field is in resonance with transitions between sub-states in the spin system, dips in the nuclear polarization are found yielding information on local magnetic fields and electric field gradients. Similar dips may occur if the polarization is studied in dependence of the applied magnetic field without applying a rf field. In this case the dips are caused by energy-conserving mutual spin-flips between the polarized spin-ensemble and neighboring unpolarized spins caused by the nuclear dipole-dipole interaction. Thereby polarization is transferred from the polarized to the unpolarized ensemble, a phenomenon called cross-relaxation.

In standard NMR the spin polarization results from a Boltzmann distribution of the occupation numbers of the spin sub-states in a strong magnetic field, and the polarization is detected via induction. In a nuclear variant, β radiation detected NMR, practiced in the last three decades of the past century, polarized β active nuclei are produced in a nuclear reaction, and the polarization is detected via the β decay asymmetry. The speaker participated in β NMR cross-relaxation studies fifty years ago [1], and became involved again very recently in a standard NMR experiment on the same topic [2]. It will be explained how this could happen, an why there is still an interest in the topic after fifty years of research.

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Coherence-incoherence patterns in networks of excitable systems

ECKEHARD SCHÖLL

Institut für Physik und Astronomie
Technische Universität Berlin, Hardenbergstr. 36, D-10623 Berlin, Germany
schoell@physik.tu-berlin.de • www.tu.berlin/en/physik/agschoell

As a dynamical paradigm, excitability permeates local and collective activity in a broad variety of living systems, from neurons and cardiomyocytes to pancreatic beta-cells, cancer cells, and the emerging field of network physiology. Here we outline the state-of-the-art in the field of coherence-incoherence patterns in coupled excitable systems, covering theoretical advances and surveying experimental evidence on how they support or mediate transitions between different physiological and pathological states in living systems [1]. While observing patterns like chimeras and solitary states has primarily been associated with self-sustained oscillators, here we highlight how local excitability impacts generic mechanisms of onset and promotes new types of patterns inherent to excitable systems. We illustrate the importance of the constructive role of noise in pattern formation, like coherence resonance, and present alternatives to the classical scenario where chimeras emerge along desynchronization from complete coherence to incoherence. We evaluate physiological evidence relating chimera-like with phenomena like epileptic seizures, ventricular fibrillation, unihemispheric sleep, or cognition.

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Gutzwiller periodic-orbit sums beyond the Heisenberg time

AKIRA SHUDO

Department of Physics, Tokyo Metropolitan University
Minami-Osawa, Hachioji, Tokyo 192-0397, Japan

We investigate the long-time validity of the Gutzwiller trace formula for a smoothly perturbed cat map. By the Faure–Tsujii theorem, the unresummed periodic-orbit sum agrees with the trace of a canonical N-dimensional natural quantization up to an exponentially small error. Removing a classically determined common phase preserves accuracy far beyond the Heisenberg time T_H. Breakdown occurs through level-dependent dephasing rather than non-unitary amplification. Numerically, the first stationary-phase correction describes the phase shifts, whose variance follows the Feingold–Peres formula, yielding the breakdown scale $n_* \propto T_H \hbar^{-1/2} \propto \hbar^{-3/2}$.

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Numerical investigation of effective stability and chaos in the J_2–Solar radiation pressure problem

HARIS SKOKOS

Nonlinear Dynamics and Chaos Group
Department of Mathematics and Applied Mathematics
University of Cape Town, South Africa
and
Max Planck Institute for the Physics of Complex Systems
Dresden, Germany
haris.skokos@gmail.com • *haris.skokos@uct.ac.za*
http://math_research.uct.ac.za/∼hskokos/

The rapid increase in satellite activity in recent years has transformed the Low Earth Orbit (LEO) region into a congested and collision-prone environment. In this talk, following [1], we investigate the regular and chaotic dynamics of satellites under the combined effects of Earth's oblateness (J_2) and solar radiation pressure (SRP) [2-4], with particular emphasis on natural reentry pathways. Using the Smaller Alignment Index (SALI) method [5,6] to distinguish between regular and chaotic orbits, we explore how the phase space structure and reentry dynamics depend on the satellite's area-to-mass ratio. We find that, for sufficiently large area-to-mass ratios, reentry lifetimes follow a power law distribution. Remarkably, despite the presence of chaotic regions in phase space, trajectories satisfying the 25-year reentry criterion remain predominantly regular until atmospheric decay. These results provide insight into the dynamical mechanisms underlying natural de-orbiting and their potential relevance to passive space debris mitigation.

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Simulacija sistema smučarske letalnice in letalca

BOŽIDAR ŠARLER, KRIŠTOF KOVAČIČ, BOR ZUPAN

Laboratorij za dinamiko fluidov in termodinamiko
Fakulteta za strojništvo, Univerza v Ljubljani, Aškerčeva ulica 6, SI-1000 Ljubljana, Slovenia
bozidar.sarler@fs.uni-lj.si • https://web.fs.uni-lj.si/lfdt/LFDT_ en.php

V prispevku predstavimo koncept projektiranja smučarske letalnice, ki zagotavlja varne polete. Varno letenje naj bi bilo smiselno oddaljeno od hrbtišča letalnice, sile na letalca ob doskoku pa naj bi bile čim manjše. Za odgovor, ali letalnica zadostuje tem zahtevam, smo razvili računalniški model aerodinamike skakalca [1], ki na podlagi Reynoldsovo povprečenih Navier-Stokesovh enačb računa sile vzgona in upora med tremi fazami (vzlet, letenje, pristanek) leta. Aerodinamični model je bil zgrajen na podlagi različnih konfiguracije skakalca in smuči med letom. Validacija modela je bila narejena na podlagi primerjave izračunanih krivulj njihovega leta z eksperimentalno merjenimi ter na podlagi primerjave izračunanih sil pri doskoku z meritvami teh sil. Na opisani način smo sodelovali pri modifikaciji Planiške letalnice [2], na kateri je bil dosežen trenutno veljaven svetovni rekord 254,5 m, da bo omogočala varne polete do 270 m. Modifikacijo je potrdila FIS.

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Simulation of ski-flying hill and ski flyer system

BOŽIDAR ŠARLER, KRIŠTOF KOVAČIČ, BOR ZUPAN

Laboratory for Fluid Dynamics and Thermodynamics
Faculty of Mechanical Engineering, University of Ljubljana, Aškerčeva ulica 6,
SI-1000 Ljubljana, Slovenia
bozidar.sarler@fs.uni-lj.si • https://web.fs.uni-lj.si/lfdt/LFDT_ en.php

In this paper, we present a concept for designing a ski-flying hill to ensure safe flights. Safe flying should not be reasonably far from the ski flying hill's profile, and the forces on the ski flyer during landing should be as small as possible. To determine whether the ski flying hill meets these requirements, we have developed a computational model of the ski jumper's aerodynamics [1], which, based on the Reynolds-averaged Navier-Stokes equations, calculates the lift and drag forces across the three phases of flight (takeoff, flight, and landing). The aerodynamic model was built based on several configurations of the ski jumper and skis during flight. The model was validated by comparing the calculated flight curves with experimentally measured curves and the calculated landing forces with measured forces. In this manner, we contributed to modifying the Planica ski flying hill [2], which currently holds the world record of 254.5 m, to enable safe flights up to 270 m. The modification was approved by FIS.

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Od akiralnih dimerov do fotonskih vijačnic: raznolike kiralne strukture v tekočih kristalih

NATAŠA VAUPOTIČ

Fakulteta za naravoslovje in matematiko
Univerza v Mariboru, Koroška 160, SI-2000 Maribor, Slovenija
Institut Jožef Stefan
Jamova 39, SI-1000 Ljubljana, Slovenia
natasa.vaupotic@um.si • fnm.um.si

Spontani nastanek kiralnih struktur iz akiralnih gradnikov predstavlja enega najzanimivejših pojavov samoorganizacije v mehki snovi. Tekoči kristali iz akiralnih molekul lahko tvorijo tvorijo presenetljivo raznolike kiralne smektične strukture, od nanovijačnic do fotonskih nadstruktur, ki selektivno odbijajo vidno svetlobo.

V predavanju bom predstavila metodo resonančnega sipanja mehkih rentgenskih žarkov (RSoXS), ki omogoča neposreden vpogled v orientacijsko ureditev molekul v smektičnih tekočekristalnih fazah. V nasprotju s konvencionalnim rentgenskim sipanjem, s katerim zaznamo prostorsko modulacijo gostote, RSoXS razkriva tudi orientacijsko organizacijo molekul in s tem bistveno dopolnjuje strukturne informacije o sistemu. Kombinacija eksperimentalnih rezultatov in teoretičnega modeliranja sipalnega odziva omogoča zanesljivo določitev kompleksnih mezoskopskih struktur [1].

Na primeru akiralnih tekočekristalnih dimerov bom pokazala, kako smo z modeliranjem in primerjavo s sipalnimi eksperimenti najprej določili večnivojsko kiralno organizacijo smektičnih faz ter pojasnili mehanizme, ki vodijo do nastanka hierarhičnih vijačnih struktur [2]. Takšno razumevanje je omogočilo napoved struktur s periodami v območju valovnih dolžin vidne svetlobe in vodilo do odkritja akiralnih tekočih kristalov, ki izkazujejo fotonsko prepovedano območje [3]. Predstavila bom tudi novejše rezultate, ki razkrivajo še širšo paleto vijačnih struktur in kažejo na omejitve prvotnih modelskih predpostavk [4].

Predavanje bo osvetlilo, kako lahko tesno prepletanje eksperimenta in teorije ne le pojasni obstoječa opažanja, temveč tudi odpira nova vprašanja ter vodi do napovedi in odkrivanja doslej neznanih oblik samoorganizacije v mehki snovi.

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From Achiral Dimers to Photonic Helices: Diverse Chiral Structures in Liquid Crystals

NATAŠA VAUPOTIČ

Faculty of Natural Sciences and Mathematics
University of Maribor, Koroška 160, SI-2000 Maribor, Slovenia
Jozef Stefan Institute
Jamova 39, SI-1000 Ljubljana, Slovenia
natasa.vaupotic@um.si • fnm.um.si

The spontaneous emergence of chirality from achiral building blocks is one of the most fascinating examples of self-organization in soft matter. In liquid-crystalline dimers, molecules that are individually achiral can assemble into a remarkable variety of chiral smectic structures, ranging from short-pitch helices to photonic superstructures capable of selectively reflecting visible light.

In this lecture, I will show how resonant soft X-ray scattering (RSoXS) provides direct access to the orientational organization of molecules in layered liquid-crystalline phases, revealing structural details that remain inaccessible to conventional X-ray diffraction, which is primarily sensitive to electron-density variations. Combining RSoXS experiments with theoretical modelling of molecular arrangements and their expected scattering response enables an unambiguous identification of complex mesoscale structures [1].

Starting from the first determination of multi-level chiral smectic organization in achiral liquid-crystal dimers [2], I will discuss how modelling revealed the driving tendencies underlying these hierarchical helical phases and led to the prediction of structures with periodicities comparable to wavelengths of visible light. This understanding subsequently guided the discovery of materials exhibiting photonic band-gap behaviour in completely achiral systems [3]. I will then present recent results demonstrating an even broader spectrum of helical organizations, including structures beyond the assumptions of earlier models [4].

The lecture will illustrate how theoretical modelling can serve not only as a tool for interpreting experiments, but also as a guide for identifying promising directions in the search for new soft-matter structures and functionalities.

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Transport onkraj difuzije

LEV VIDMAR

Odsek za teoretično fiziko, Institut Jožef Stefan
Fakulteta za matematiko in fiziko, Univerza v Ljubljani
lev.vidmar@ijs.si • web-f1.ijs.si/user/lev.vidmar/

Kvantna termalizacija pomeni, da se vsi relevantni relaksacijski procesi zaključijo na časovnih skalah, krajših od obratne vrednosti povprečne razlike med energijskimi nivoji, ki jo označujemo kot Heisenbergov čas. Ko pa najdaljša relaksacijska časovna skala postane primerljiva s Heisenbergovim časom, sistem vstopi v kvantno kritični režim.

V tem režimu lahko transport izkazuje nekonvencionalno obnašanje, od superdifuzije do subdifuzije. Predstavil bom izpeljavo dinamičnega eksponenta z v kritični točki in pokazal, da je za enodelčne sisteme enak razmerju med dimenzijo mreže d_l in spektralno fraktalno dimenzijo d_s, torej $z = d_l/d_s$ [1]. Nato bom izpeljal sorodno, a vendar drugačno skalirno zvezo za verjetnost preživetja v kritični točki ter razpravljal o možnih povezavah med sistemi brez interakcij in sistemi z interakcijami [2,3].

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Transport beyond diffusion

LEV VIDMAR

Department of Theoretical Physics, Jožef Stefan Institute
Faculty of Mathematics and Physics, University of Ljubljana
lev.vidmar@ijs.si • web-f1.ijs.si/user/lev.vidmar/

Quantum thermalization implies that all relevant relaxation processes are completed on timescales shorter than the inverse mean level spacing, known as the Heisenberg time. However, when the slowest relaxation timescale becomes comparable to the Heisenberg time, the system enters a quantum critical regime.

In this regime, transport can exhibit unconventional behavior, ranging from superdiffusion to subdiffusion. I will present a derivation of the dynamical exponent z at the critical point, showing that, for short-range single-particle systems, it is given by the ratio of the lattice dimensionality d_l to the spectral fractal dimension d_s, i.e., $z = d_l/d_s$ [1]. I will then derive a related, yet distinct, scaling relation for the survival probability at criticality and discuss possible connections between noninteracting and interacting systems [2,3].

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Dynamical aspects of spin-boson systems with two-photon interactions

DAVID VILLASEÑOR

CAMTP - Center for Applied Mathematics and Theoretical Physics
University of Maribor, Mladinska 3, SI-2000 Maribor, Slovenia
Institute of Particle and Nuclear Physics
Faculty of Mathematics and Physics, Charles University
V Holešovičkách 2, 18000 Prague, Czechia
d.v.pcf.cu@gmail.com

Spin-boson systems with two-photon interactions display fascinating behaviors, such as spectral collapse [1] and the disappearance of the superradiant phase in the semiclassical limit [2]. In this talk, I will explore the localization of eigenstates and the dynamical localization of coherent states within the mixed phase space of the two-photon Dicke model [3]. I will utilize two distinct localization measures based on Husimi functions to highlight fundamental differences: one measure is defined in the classical energy subspace, while the other is defined in the atomic subspace of the system. Finally, I will examine the evolution of the survival probability for coherent states, which may reveal signatures of quantum chaos, as the correlation hole. Overall, these results [4] offer a comprehensive characterization of the dynamical features in spin-boson systems with two-photon interactions, which may be valuable for advancing quantum applications.

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Impact of quantum coherence and excited-state quantum phase transition on nonequilibrium thermodynamics in quenched Lipkin model

QIAN WANG

Department of Physics, Zhejiang Normal University, Jinhua 321004, China
CAMTP - Center for Applied Mathematics and Theoretical Physics
University of Maribor, Mladinska 3, SI-2000 Maribor, Slovenia
qwang@zjnu.edu.cn

We study the nonequlibrium thermodynamics of the quenched Lipkin model by means of the quantum work statistics. Using the Kirkwood-Dirac quasiprobability (KDQ) distribution, we investigate in detail the effects of quantum coherence and the excited-state quantum phase transition (ESQPT) on the statistical properties of quantum work for different coherent initial states. We show that the presence of coherence in the initial state leads to a work distribution with negative and/or complex values, which reflect non-classical features and are independent of the occurrence of the ESQPT. Moreover, although the statistical moments of the work distribution are strongly affected by the coherence of the initial states, they are insensitive to whether the underlying ESQPT occurs. However, by focusing on the work distribution entropy, we uncover how quantum coherence and the ESQPT shape the complexity of the work KDQ distribution. Our findings contribute to the understanding of the roles played by quantum coherence and the ESQPT in nonequilibrium thermodynamics and may have practical applications.

Quantum Walks and Topology

SANDRO WIMBERGER

Dipartimento di SMFI, Università di Parma
INFN, Sezione di Milano Bicocca, Gruppo Collegato di Parma
sandromarcel.wimberger@unipr.it • https://sites.google.com/view/cdyn

Quantum walks in momentum space are reported with a rubidium spinor Bose-Einstein condensate by applying a periodic kicking potential as a walk operator and a resonant microwave pulse as a coin toss operator [1]. The generated quantum walks are stable for up to about ten steps and their coherence properties can be tested by artificial noise [2]. Experimental limitations induced by phase noise, quasimomentum distributions, symmetries, and the ac-Stark shift are considered [1-4]. Topological phase transitions occur in a double-kicked version of our walker [3, 5, 6]. Motivated by experimental constraints, we also analyze the effects of open and periodic boundary conditions in contrast to the idealized case of infinite momentum space [6]. As a bulk probe for topological invariants, we focus on the mean chiral displacement and show that it exhibits sensitivity to boundary conditions.

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Weyl law for flat-band Dirac billiards

HONG-YA XU

Lanzhou Center for Theoretical Physics
Lanzhou University, Lanzhou, China
xuhongya@lzu.edu.cn

Weyl's law characterizes the mean density of states in confined wave systems and underpins a broad range of applications across physics. It links spectral counting to geometry, with perimeter corrections subleading to the area term in two dimensions. This area-perimeter hierarchy becomes non-uniform in flat-band Dirac billiards for three-component Dirac-Weyl waves. We derive a Weyl formula with an anomalous perimeter term set by flat-band-assisted boundary spinor matching. This term can dominate any finite spectral window without counting flat-band states. It arises from reflection phase shifts, edge modes, and grazing singularities, which determine an anomalous perimeter coefficient encoding boundary-projected spinor geometry. This coefficient therefore makes spectral counting a probe of flat-band quantum geometry. We validate the formula against circular and chaotic billiard spectra computed with a regularized boundary-integral method developed for confined flat-band Dirac systems.

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Structured eigenstates in quantum maps: stickiness and crystalline order

HUA YAN

CAMTP - Center for Applied Mathematics and Theoretical Physics
University of Maribor, Mladinska 3, SI-2000 Maribor, Slovenia

Quantum maps with sharply divided, non-KAM mixed phase spaces support two forms of structured eigenstates: sticky states near regular–chaotic boundaries and crystalline states predominantly supported in the chaotic component. Classical trajectories exhibit prolonged trapping near boundaries formed by marginally unstable periodic orbits or quasiperiodic trajectories, giving rise to algebraic recurrence statistics. The quantum counterpart appears as sticky eigenstates with enhanced Husimi weight near these boundaries. For marginally unstable periodic-orbit boundaries, their fraction decreases approximately as $\hbar^{1/2}$, a scaling associated with classical stickiness and algebraic recurrence.

Crystalline eigenstates exhibit ordered Husimi patterns with strongly suppressed weight inside the regular island. These states can share resonant eigenphase clusters with regular states, and resolving the island weight within these clusters separates their different phase-space contributions. Their occurrence displays a pronounced arithmetic dependence on the effective Planck constant. Invariant classical lattices account for part of this structure but do not generally reproduce the quantum zero arrangements, which include staggered patterns and repeating cells containing multiple zeros. Together, these results reveal complementary roles of classical recurrence, arithmetic structure, and quantum interference in shaping eigenstates beyond their regular–chaotic classification.

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Ograjena topološka aktivna mehka snov

SLOBODAN ŽUMER

Fakulteta za matematiko in fiziko, Univerza v Ljubljani, Slovenija
slobodan.zumer@fmf.uni-lj.si • softmatter.fmf.uni-lj.si

Topološka aktivna mehka snov je tekočina, za katero je značilna orientacijska urejenost njenih gradnikov – od anizotropnih koloidnih delcev, ki črpajo energijo iz okolice, do bioloških in živih sistemov, ki uporabljajo kemijsko energijo. Zaradi svoje mehkobe je taka snov le redko homogeno orientacijsko urejena. Frustracija, ki nastane zaradi nasprotujočih si vplivov anizotropne elastičnosti, kiralnosti, omejujočih geometrij, sidranja na površinah in zunanjih polj, vodi do nastanka stabilnih in metastabilnih točkastih defektov, defektnih linij ter solitonov v polju parametra orientacijske urejenosti. Dinamiko urejanja in defektov spremljajo tokovi, ki so posledica aktivnosti in zunanjih vplivov. Stacionarna dinamika defektov pri nizki aktivnosti se z naraščanjem aktivnosti razvije v kaotično tridimenzionalno gibanje – t. i. aktivno nematično turbulenco [1–4]. Dva primera bom ponazoril z numeričnim modeliranjem ograjene aktivne nematične snovi: v sferični votlini in v tanki sferični lupini, obdani s pasivnim nematikom, kjer pride do z aktivnostjo pogojenega plavanja [5].

Raziskave so potekale v sodelovanju s skupinama iz Ljubljane (Miha Ravnik, Simon Čopar, Žiga Kos, Nika Kralj in Andraž Gnidovec) in Pariza (Teresa Lopez Leon s študenti, ESPCI).

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Confined topological active soft matter

SLOBODAN ŽUMER

Faculty of Mathematics and Physics, University of Ljubljana, Slovenia
slobodan.zumer@fmf.uni-lj.si • softmatter.fmf.uni-lj.si

Topological active soft matter covers the fluids characterized by orientational ordering of constituent objects, ranging from anisotropic colloidal particles that receive energy from their surroundings to biological and living systems that convert chemical energy. Because of its softness, topological soft matter rarely exhibits homogeneous orientational order. The frustration due to the conflicting effects of anisotropic elasticity, chirality, confining geometries, surface anchoring, and external fields leads to stable and metastable point defects, defect lines, and solitons in the orientational order parameter field. The dynamics of ordering and defects are accompanied by flows caused by activity and external driving. The low-activity stationary dynamic defect structures with increasing activity evolve to chaotic 3D motions—active nematic turbulence [1-4]. We illustrate these processes by modeling active nematic confined to a spherical cavity and thin spherical shells dispersed in passive nematics, where spontaneous activity-driven swimming appears [5].

Studies were done in collaboration with groups from Ljubljana (Miha Ravnik, Simon Čopar, Žiga Kos, Nika Kralj, and Andraž Gnidovec) and ESPCI Paris (Teresa Lopez Leon with students).

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Svetlobni odmevi supernov

TOMAŽ ZWITTER

Fakulteta za matematiko in fiziko
Univerza v Ljubljani, Jadranska 19, 1000 Ljubljana
tomaz.zwitter@fmf.uni-lj.si • fiz.fmf.uni-lj.si/zwitter

Eksplozije lokalnih supernov omogočajo geometrijsko določitev njihovih oddaljenosti. Geometrija razloži tudi nadsvetlobno gibanje, ki ga ob teh dogodkih pogosto opazimo. Svetlobni odmevi omogočajo, da sodobne opazovalne tehnike uporabimo za študij svetlobe in razjasnitev fizike eksplozij supernov, ki smo jih odkrili pred stoletji. Enake geometrijske koncepte lahko uporabimo tudi za učinkovito iskanje elektromagnetnih signalov drugih civilizacij.

Te, na prvi pogled raznorodne teme, za razlago uporabljajo isti napredni, vendar lahko razumljivi koncept iz učbenika, ki je lansko leto izšel v založbi UL FMF. Njegova prilagojena in posodobljena verzija bo v teh dneh izšla pri založbi Springer.

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Light echoes of supernovae

TOMAŽ ZWITTER

Faculty of Mathematics and Physics
University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia
tomaz.zwitter@fmf.uni-lj.si • fiz.fmf.uni-lj.si/zwitter

Distances to local supernovae explosions can be determined geometrically. Geometry also explains the superluminal motion which is often observed soon after the explosion. Light echoes even allow the use of modern observing techniques to study the light and clarify the physics of supernova explosions that were first observed centuries ago. Finally, the same geometric concepts can be used to effectively search for electromagnetic signals from extraterrestrial civilisations.

These, seemingly disparate topics base their explanations on the same advanced, yet easy to understand concept from a textbook, which was published last year by UL FMF. Its modified and upgraded version is being published by Springer.

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