Transport and localization of Bose-Einstein condensates in disorder

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Outline

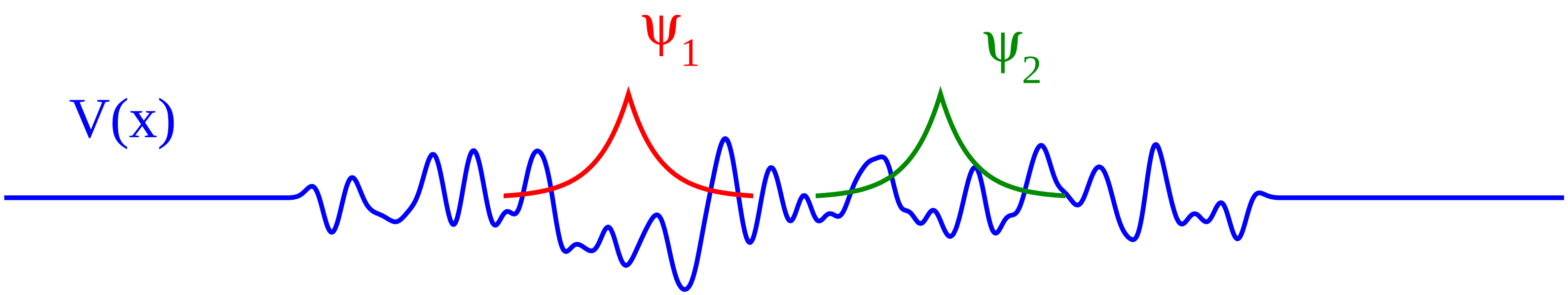
- Introduction: disorder with cold atoms
- Transport of Bose-Einstein condensates through disorder: solving the nonlinear scattering problem
- Transmission of condensates through 1D disorder potentials: Anderson localization with mean-field interaction?
- Transport through 2D disorder potentials: Coherent backscattering with interaction?
- Conclusion

Disorder with cold atoms

⟶ Anderson localization

P. W. Anderson, Phys. Rev. 109, 1492 (1958)

- exponential localization of eigenstates



- exponential decrease of the transmission with the length of the disordered region

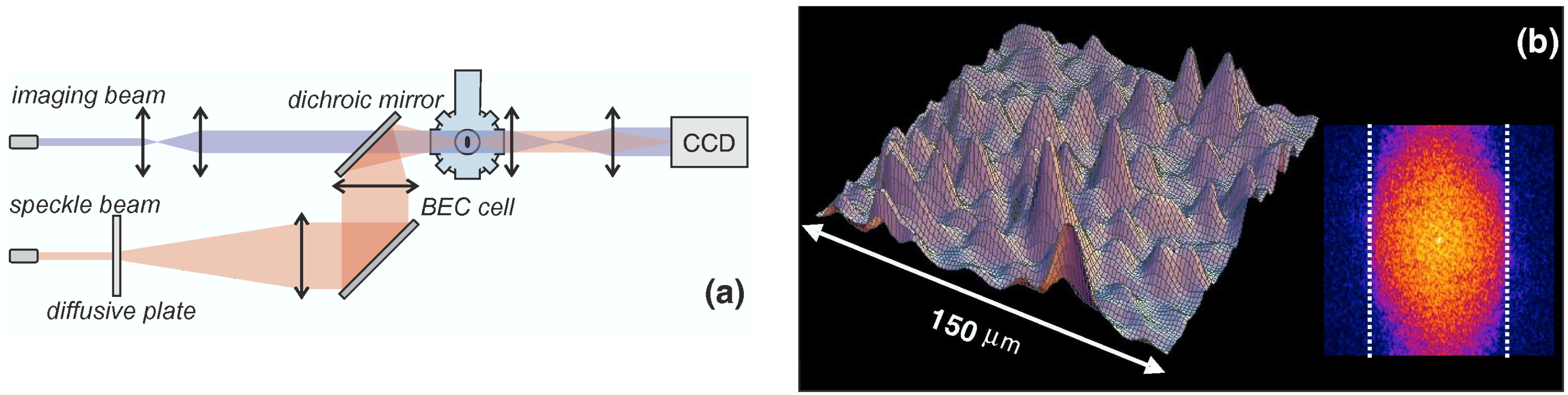
⟶ realization with Bose-Einstein condensates?

Disorder potentials for cold atoms

Experimental realization of disorder potentials with

- optical speckle fields:

  J. E. Lye *et al.*, PRL 95, 070401 (2005)



Disorder potentials for cold atoms

Experimental realization of disorder potentials with

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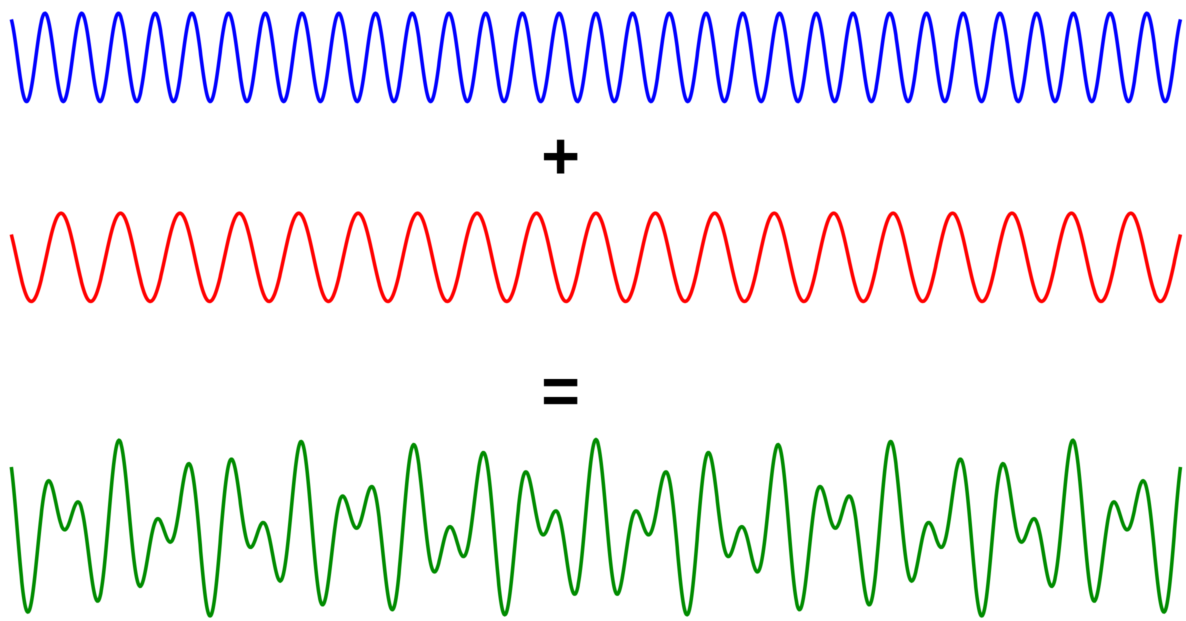
  J. E. Lye *et al.*, PRL 95, 070401 (2005)

- incommensurate optical lattices

  L. Fallani *et al.*, PRL 98, 130404 (2007)

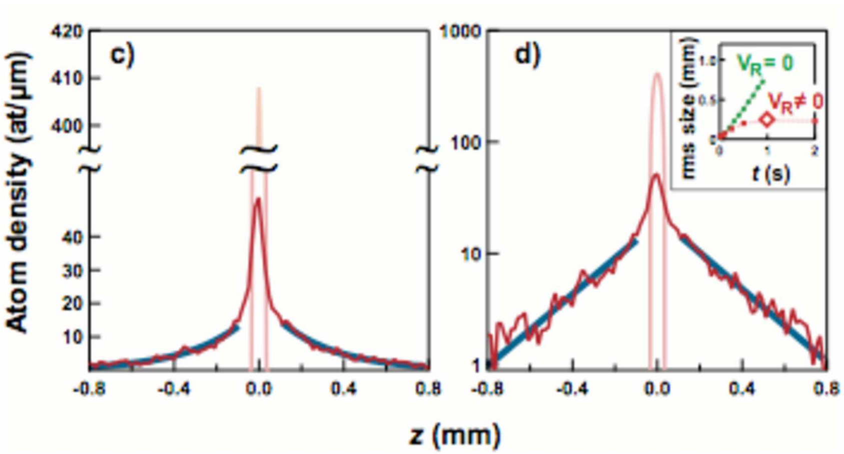


Expansion experiments in disorder potentials

$\longrightarrow$ exponential tails in the noninteracting regime,
as predicted by the theory of Anderson localization

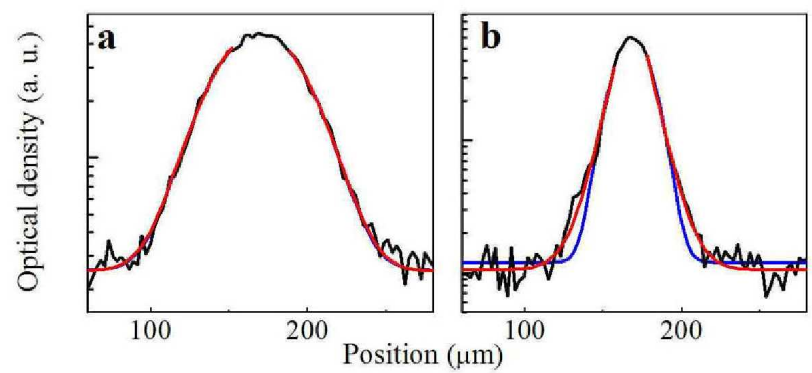
optical speckle fields:

J. Billy *et al.*, Nature 453, 891 (2008)



bichromatic optical lattices:

G. Roati *et al.*, Nature 453, 895 (2008)



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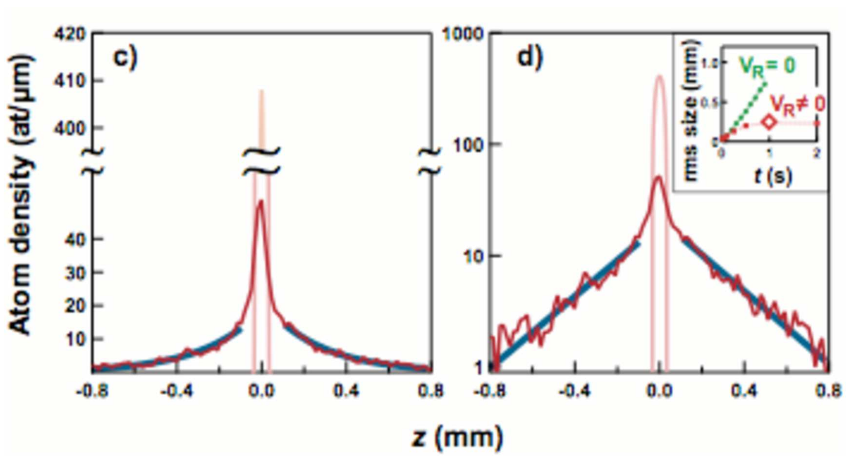
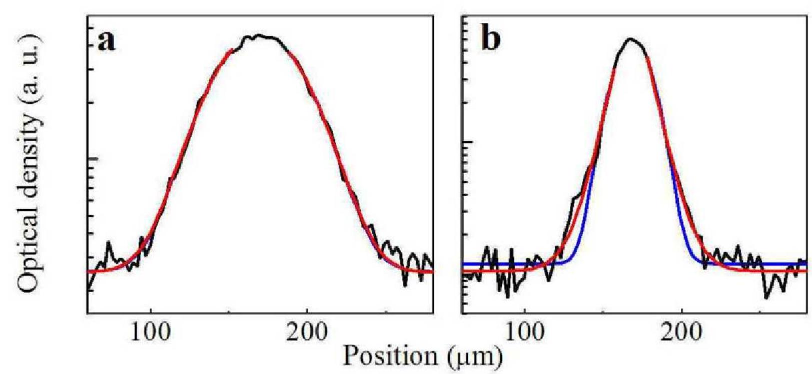


bichromatic optical lattices:

G. Roati *et al.*, Nature 453, 895 (2008)

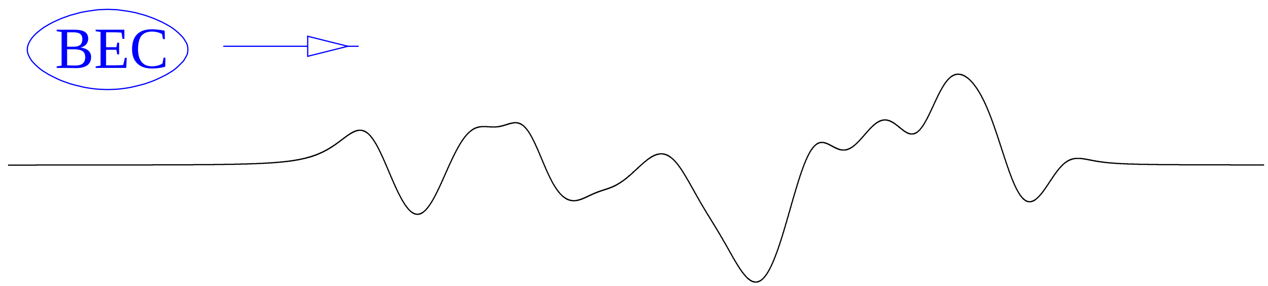


$\rightarrow$ transmission properties in disordered systems?

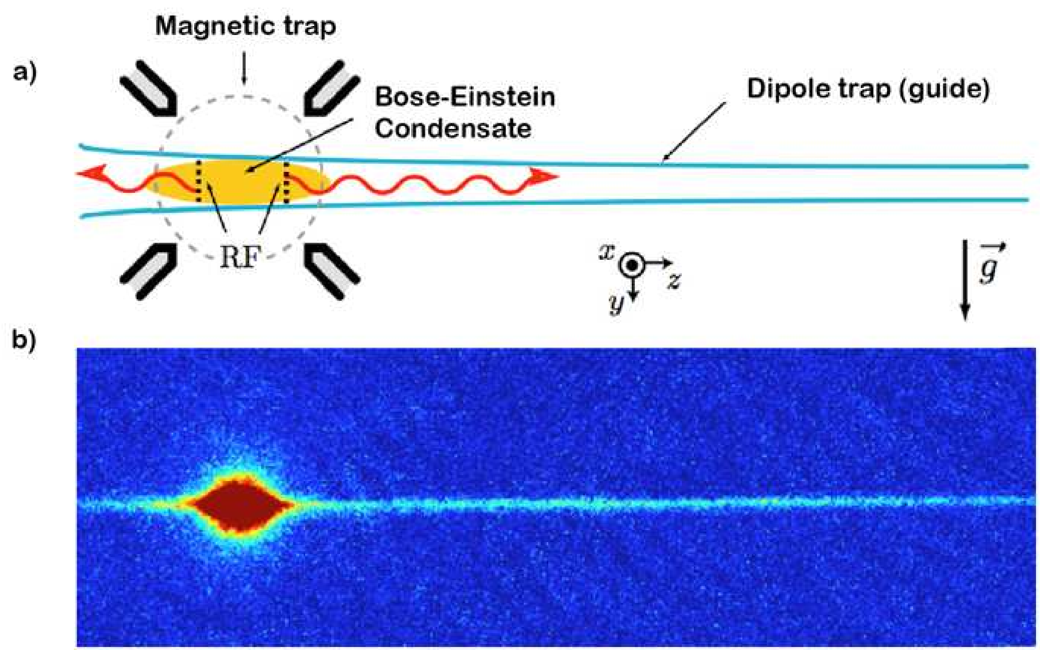
$\rightarrow$ interaction effects?

Transmission through disorder potentials

$\longrightarrow$ quasi-stationary transport process of the condensate:



Experimental realization: W. Guerin *et al.*, PRL 97, 200402 (2006)



Transmission through disorder potentials

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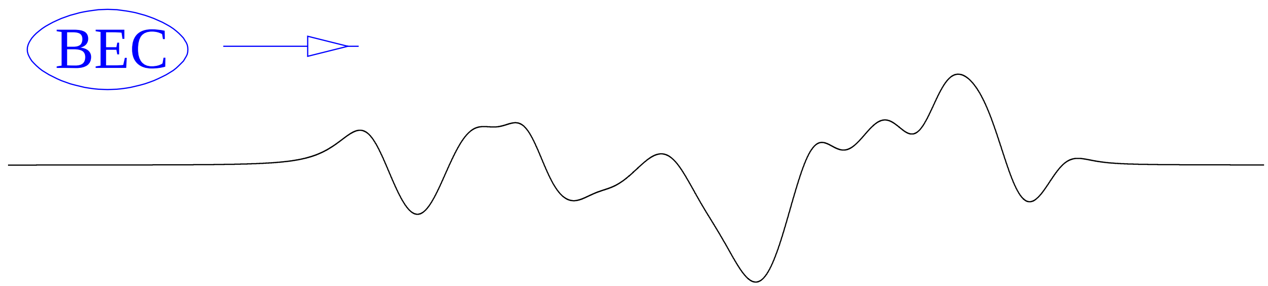


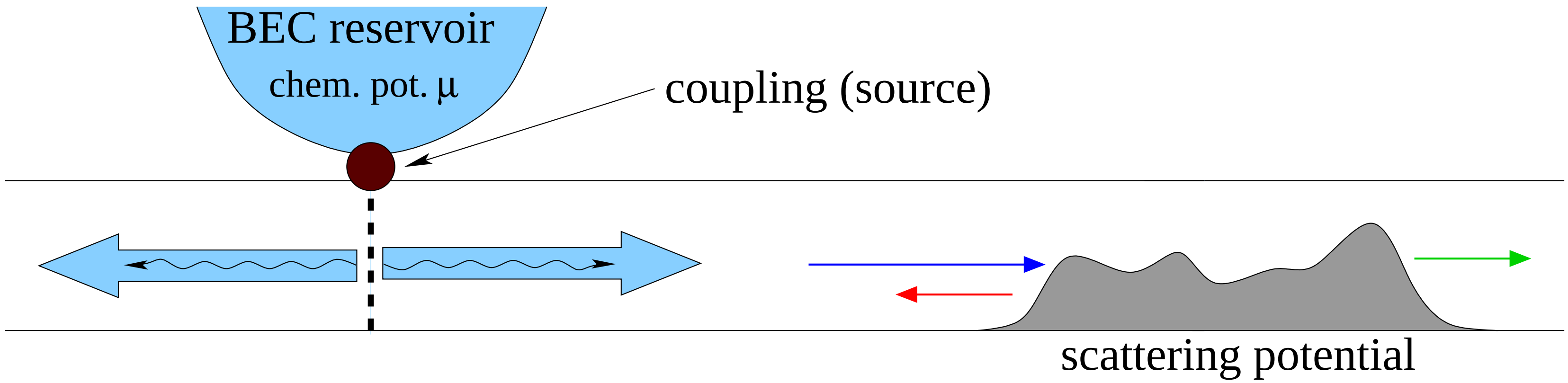
Connection with mesoscopic transport physics in solids / optics:

→ scaling theory of localization

→ conductance fluctuations

→ strong and weak localization in 2D disorder
Anderson transition in 3D

Transport theory of Bose-Einstein condensates



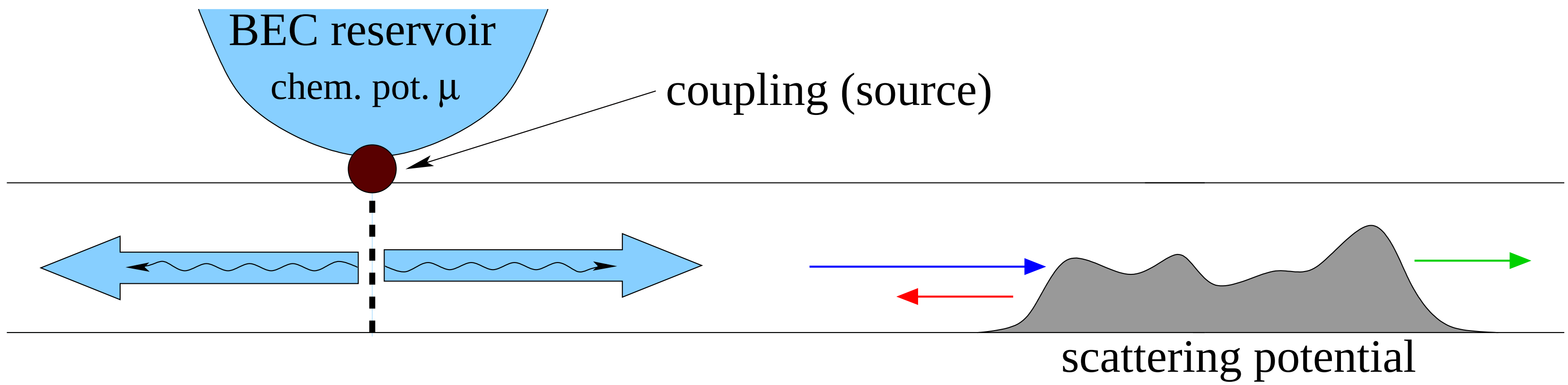
Mean-field description of a condensate in a waveguide (1D mean-field regime):

1D Gross-Pitaevskii equation

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) + 2a_s\hbar\omega_\perp|\psi(x,t)|^2\right)\psi(x,t)$$

with $a_s = s$-wave scattering length between the atoms,
$\omega_\perp =$ transverse confinement frequency of the waveguide

Transport theory of Bose-Einstein condensates



Numerical simulation of quasi-stationary scattering processes:

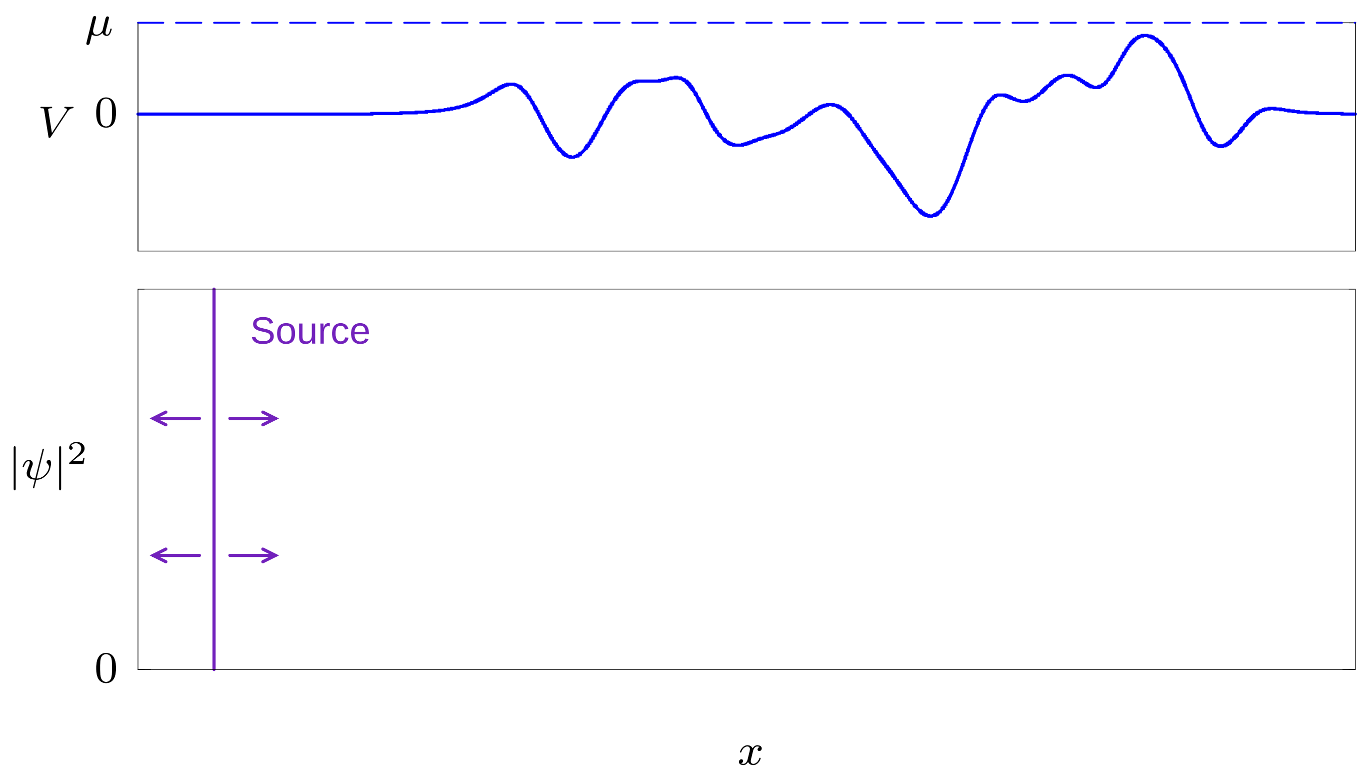
⟶ integrate Gross-Pitaevskii equation in presence of a source term

$$
\begin{aligned}
i\hbar\frac{\partial}{\partial t}\psi(x,t) &= \left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) + g|\psi(x,t)|^2\right)\psi(x,t) \\
&\quad + S_0\delta(x-x_0)\exp(-i\mu t/\hbar)
\end{aligned}
$$

T. Paul, K. Richter, and P.S., PRL 94, 020404 (2005)

Transport through 1D disorder potentials

No interaction between the atoms:



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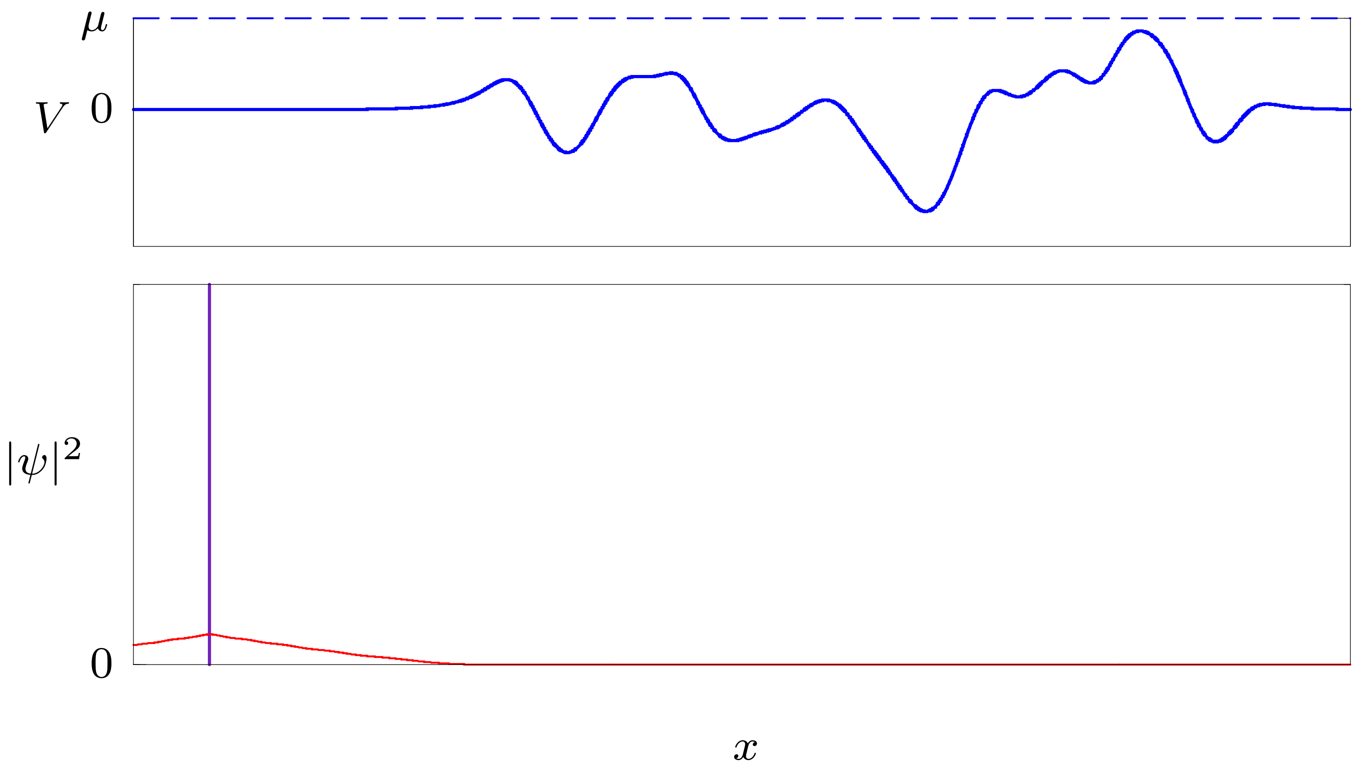

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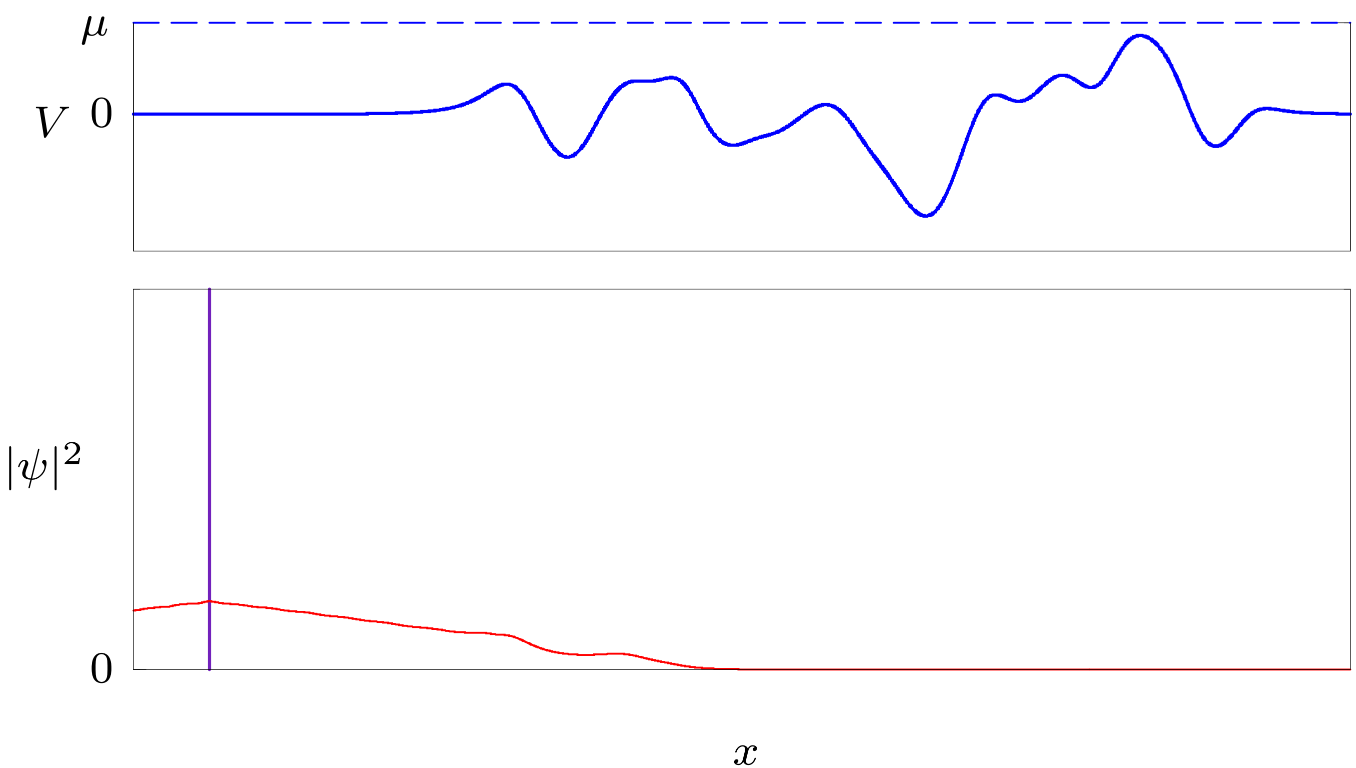

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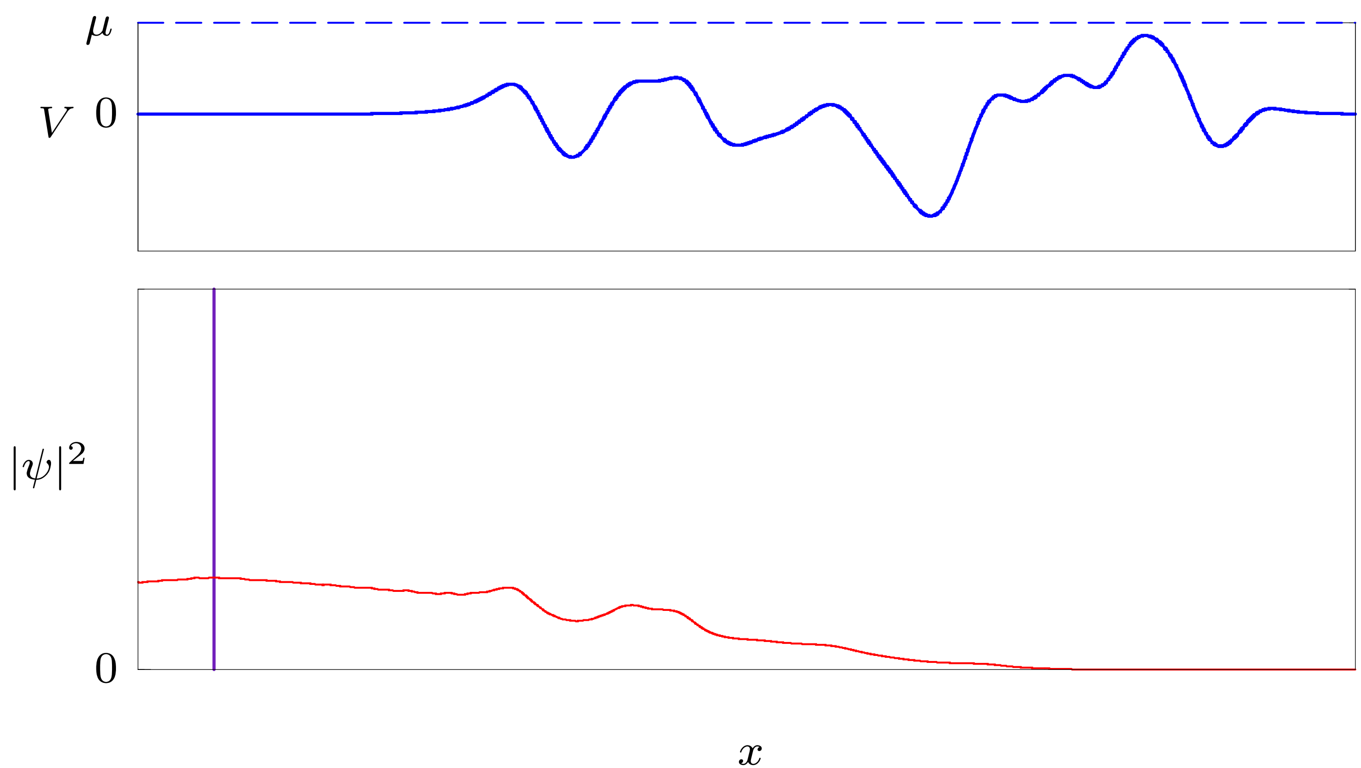

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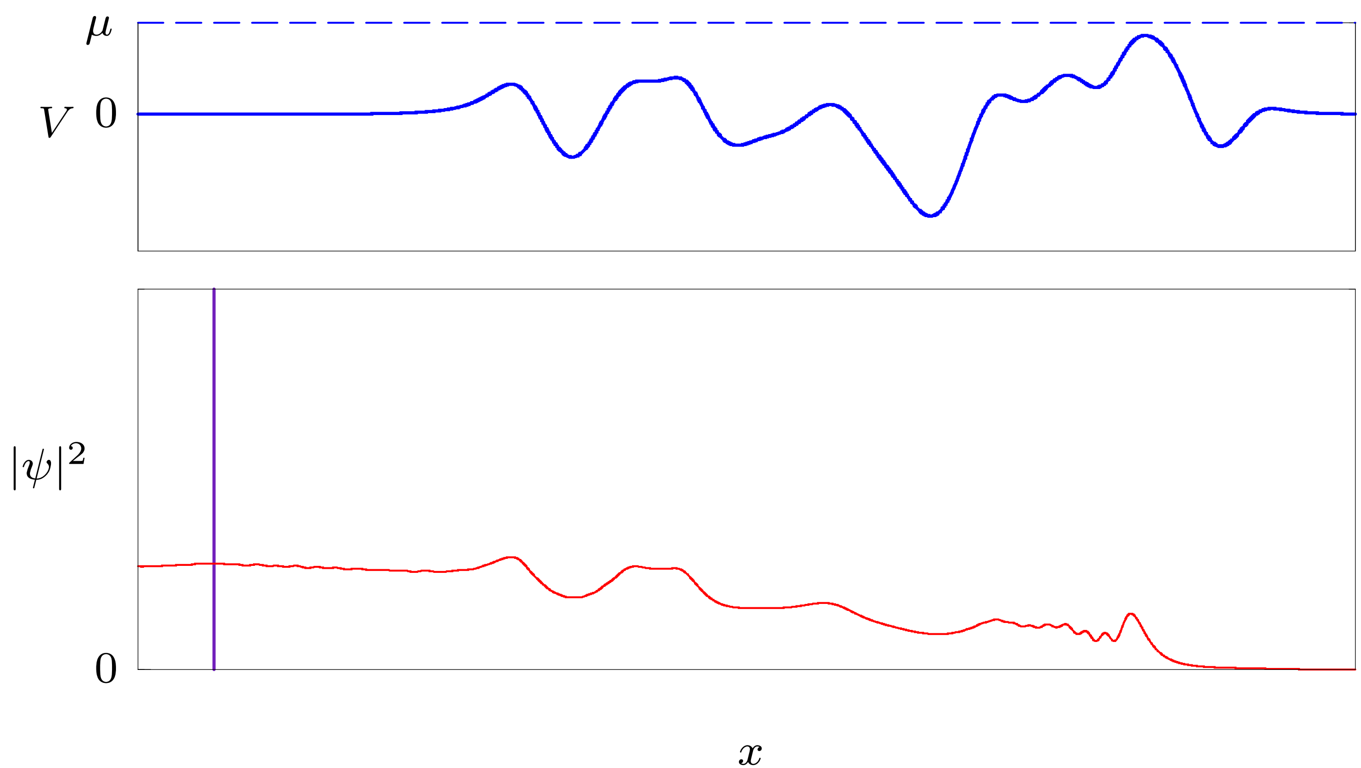

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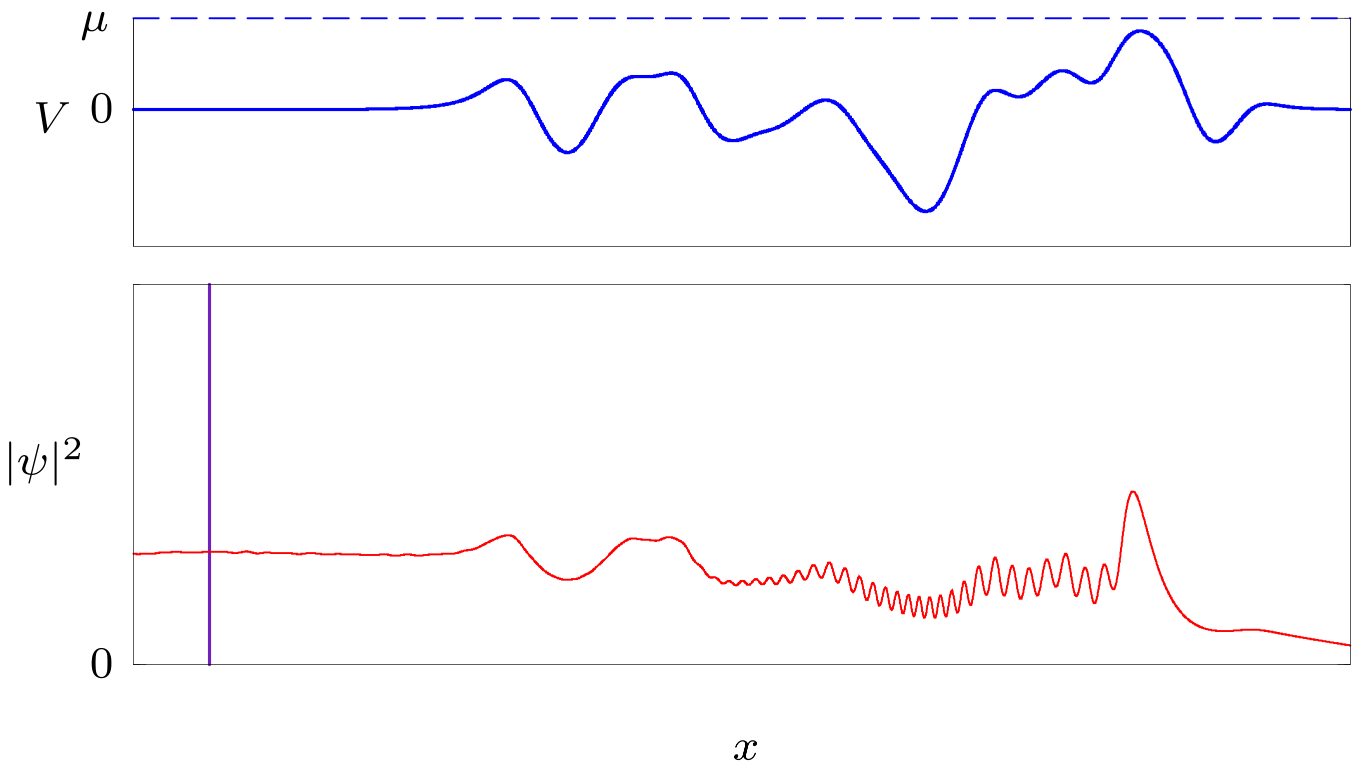

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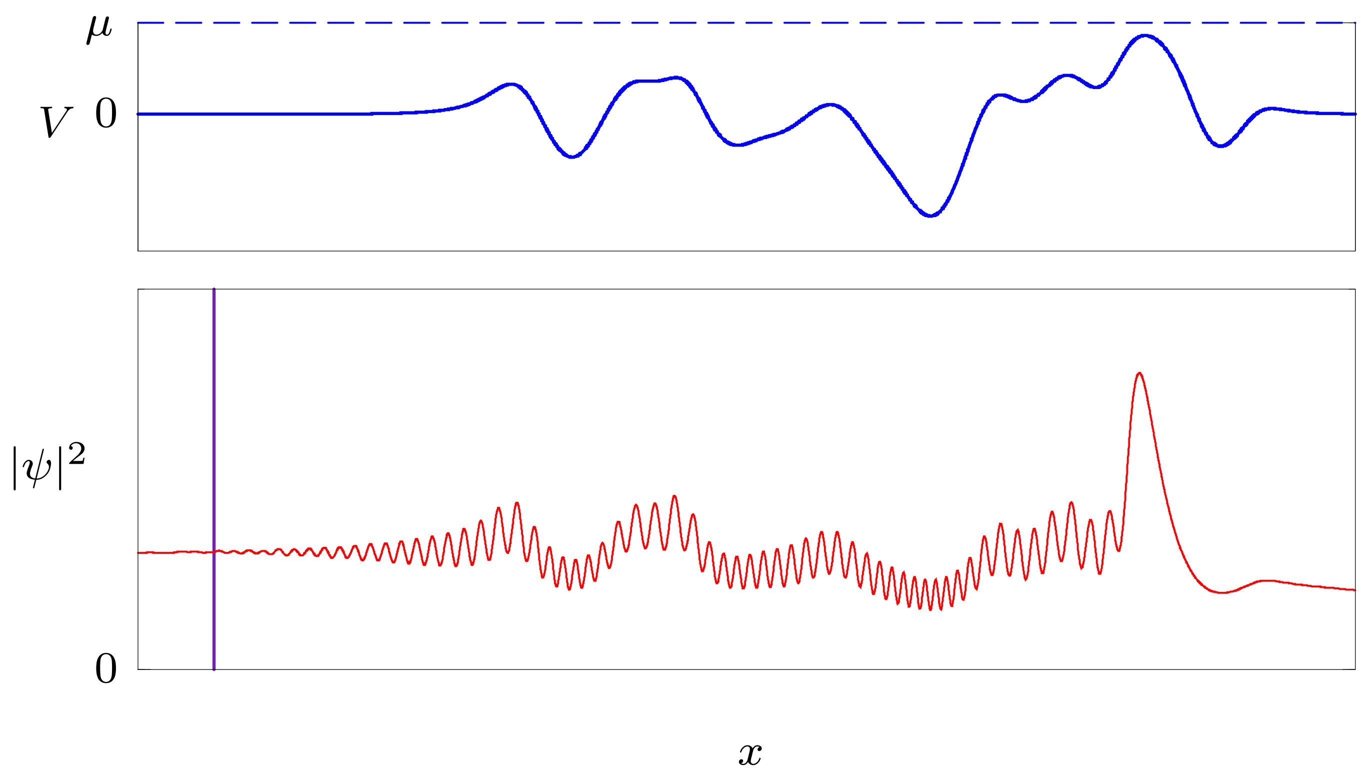

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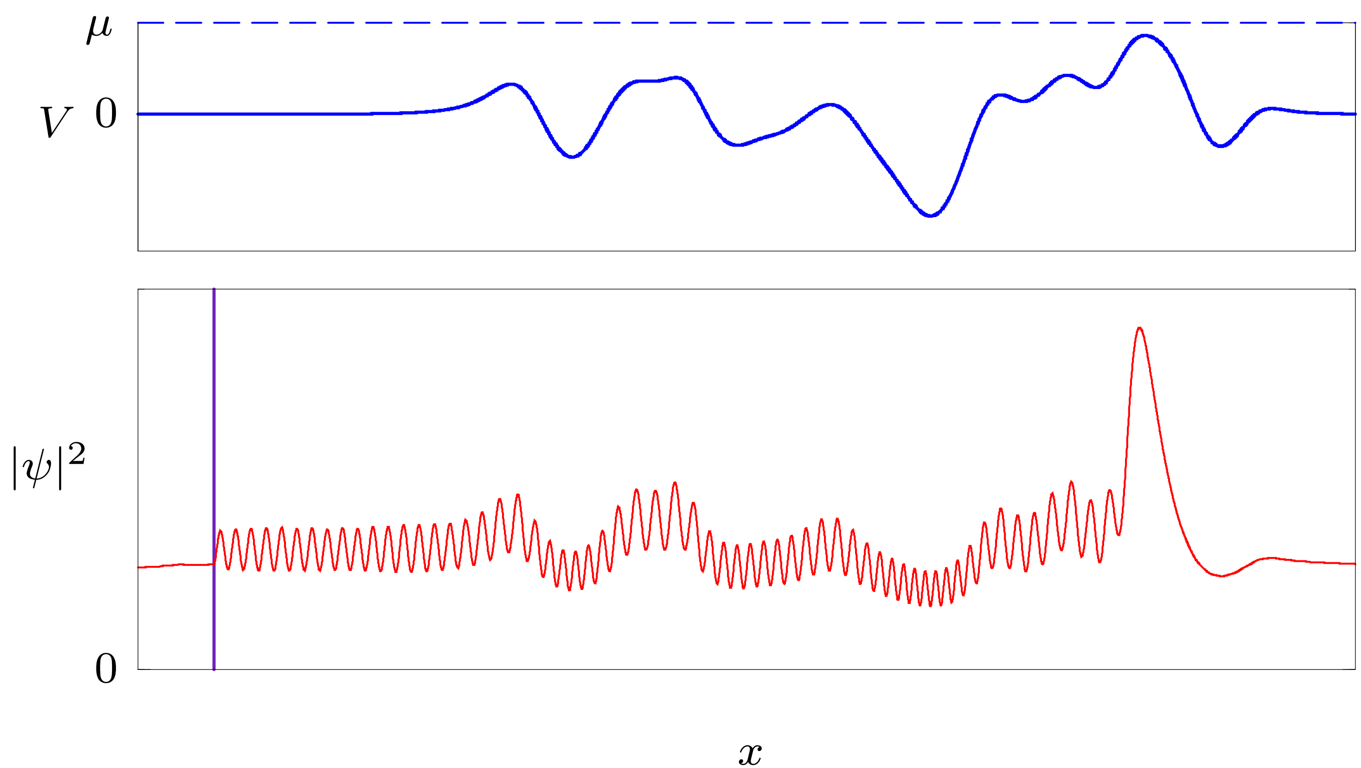

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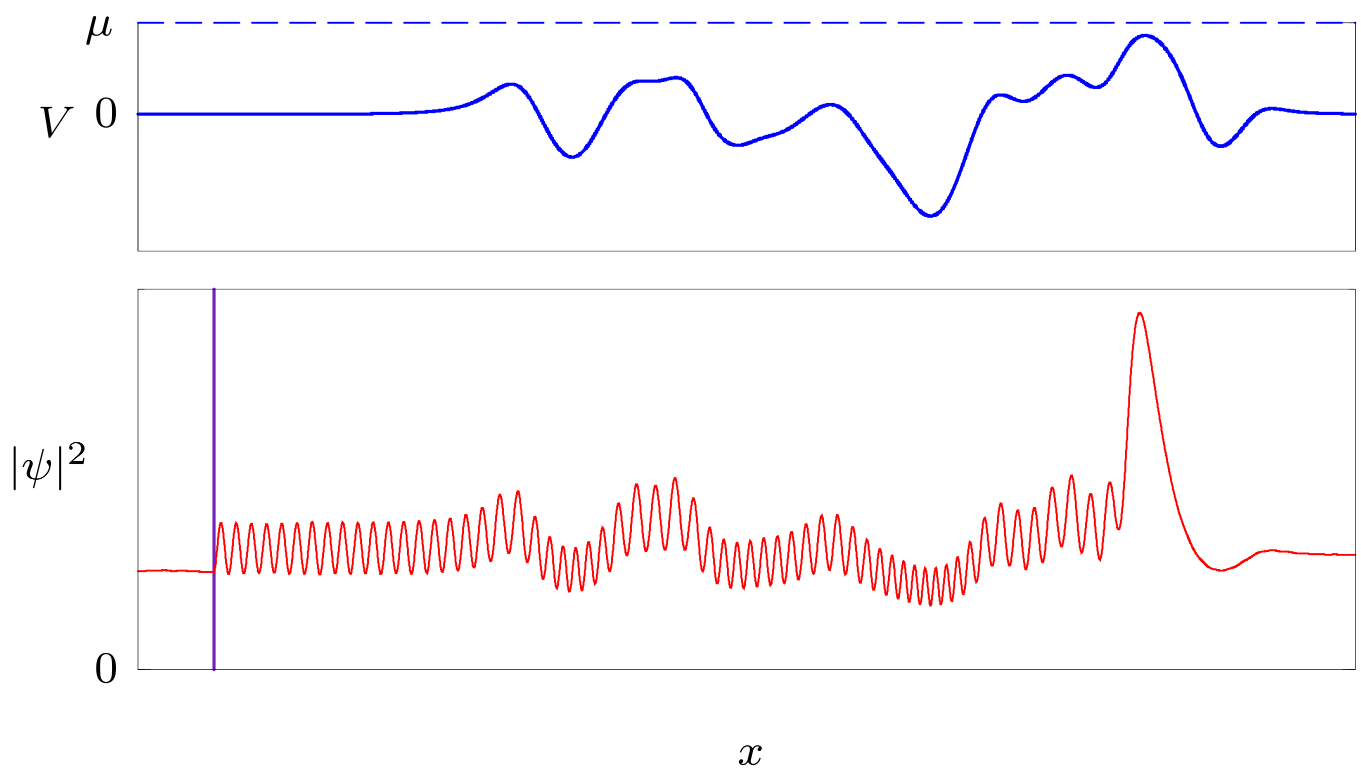
No interaction between the atoms:



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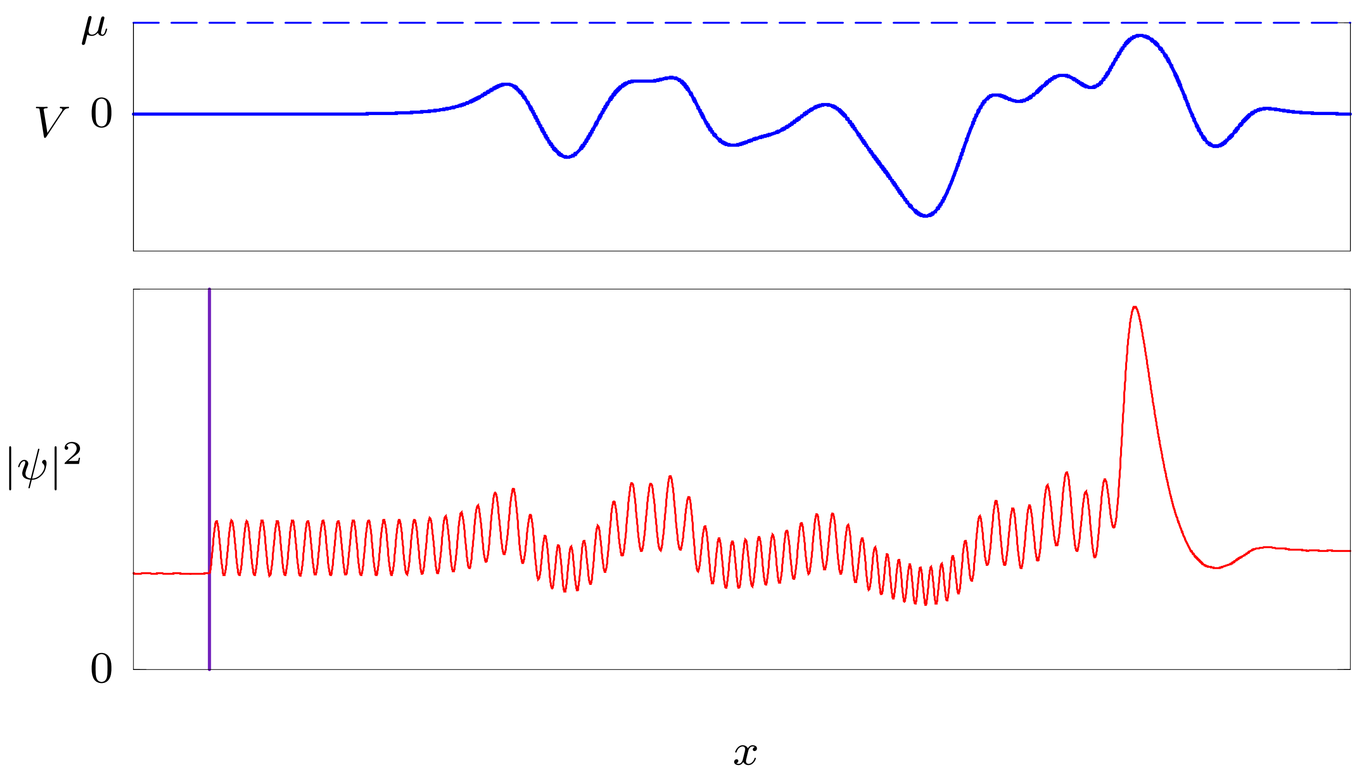


Transport through 1D disorder potentials

No interaction between the atoms:

$\longrightarrow$ exponential decrease of the average transmission with the length L of the disorder region

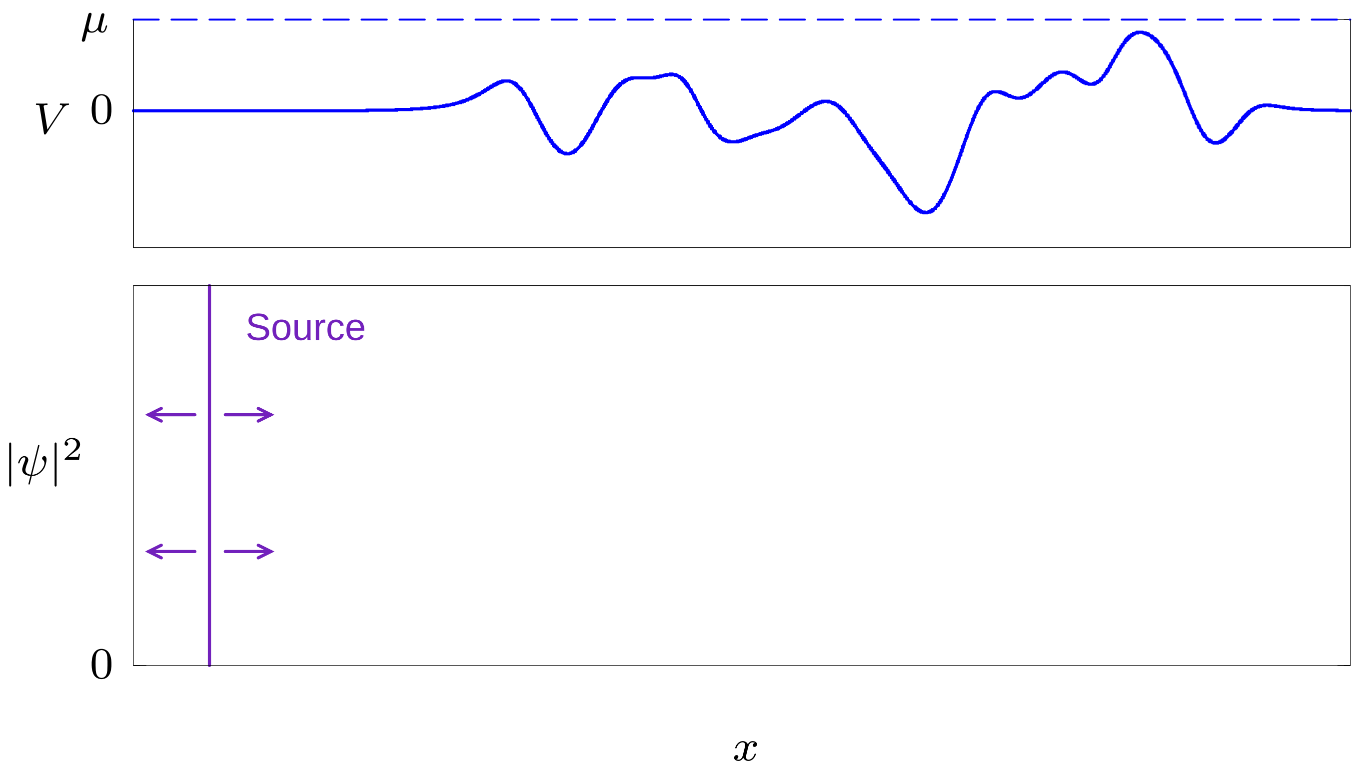
$\longrightarrow$ lognormal-type probability distribution for the transmission T at fixed length L:

$$P(\ln T) = \sqrt{\frac{L_{\rm loc}}{4\pi L}} \exp\left[-\frac{L_{\rm loc}}{4L}\left(\frac{L}{L_{\rm loc}} + \ln T\right)^2\right]$$

J.-L. Pichard, 1991

Transport through 1D disorder potentials

Finite interaction between the atoms: $g|\psi|^2 \simeq 0.1\mu$



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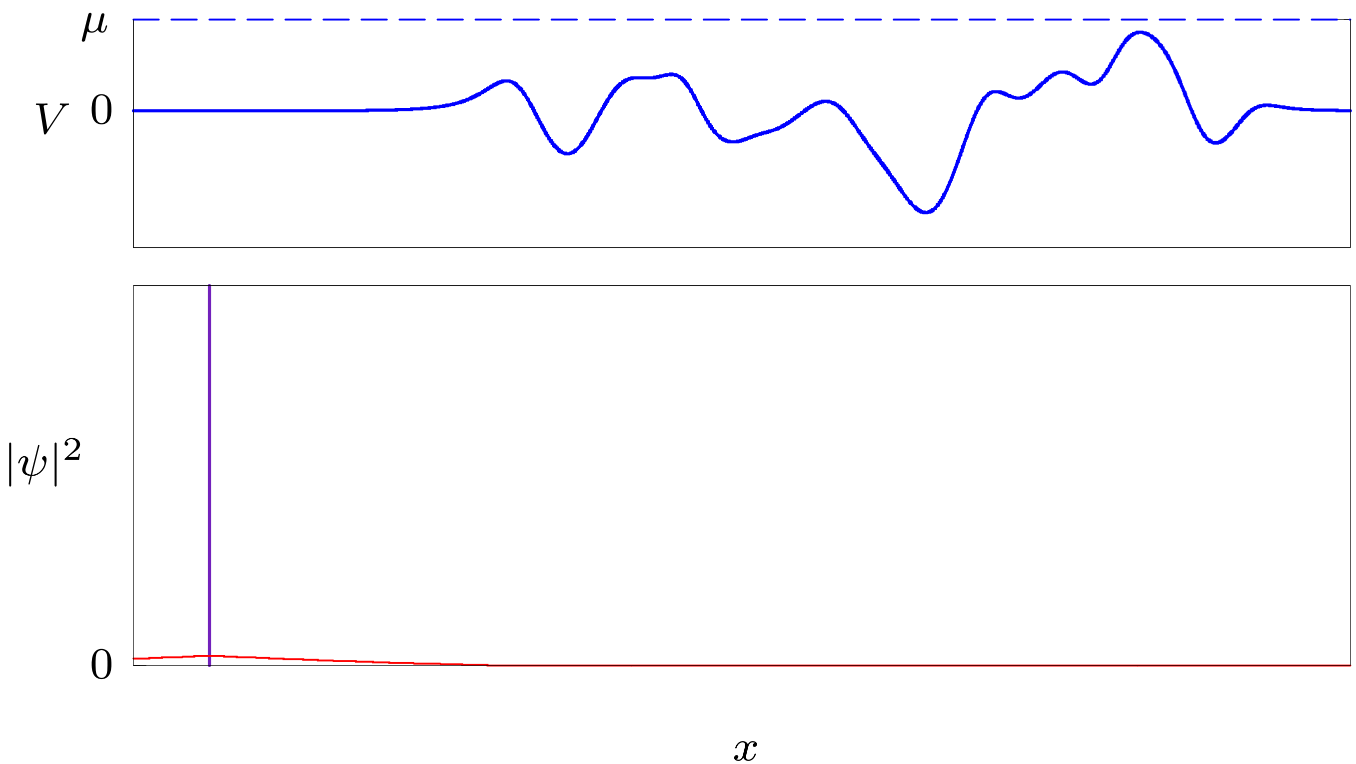

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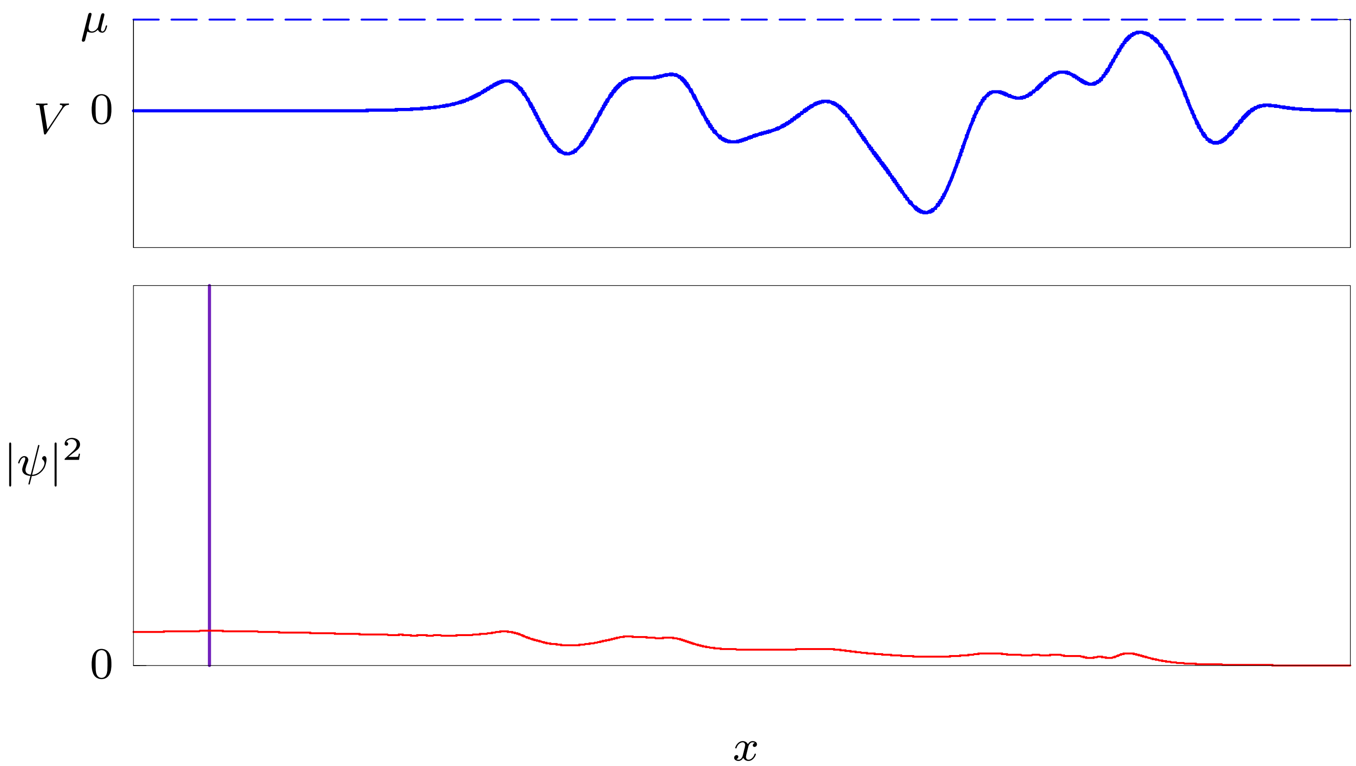

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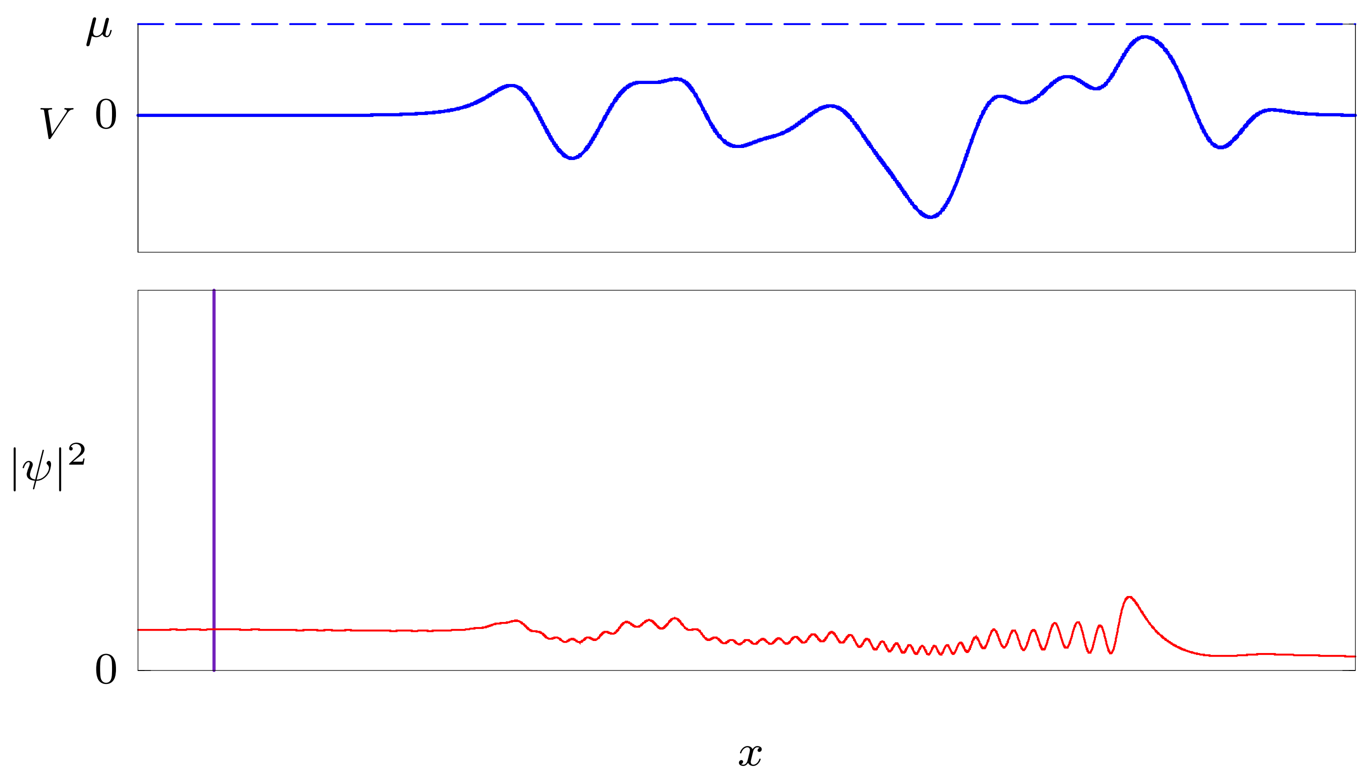

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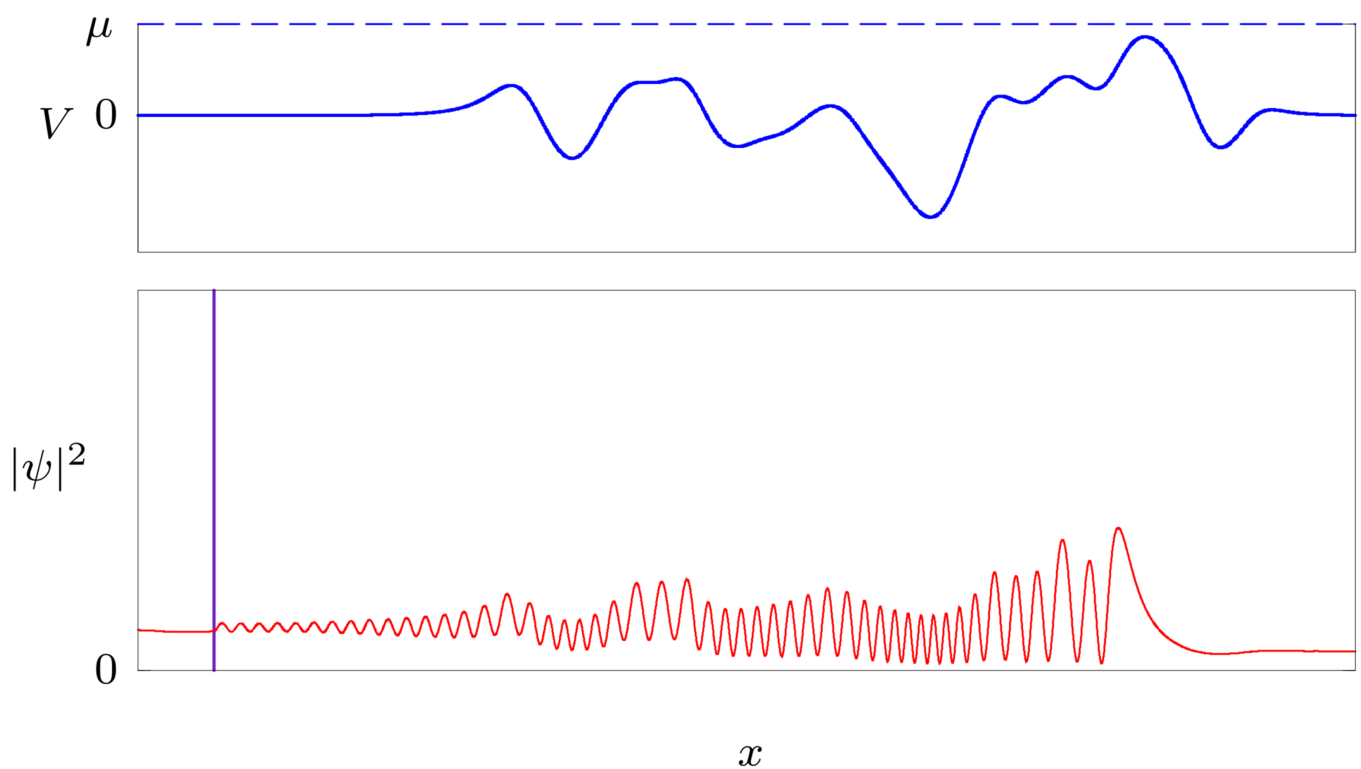

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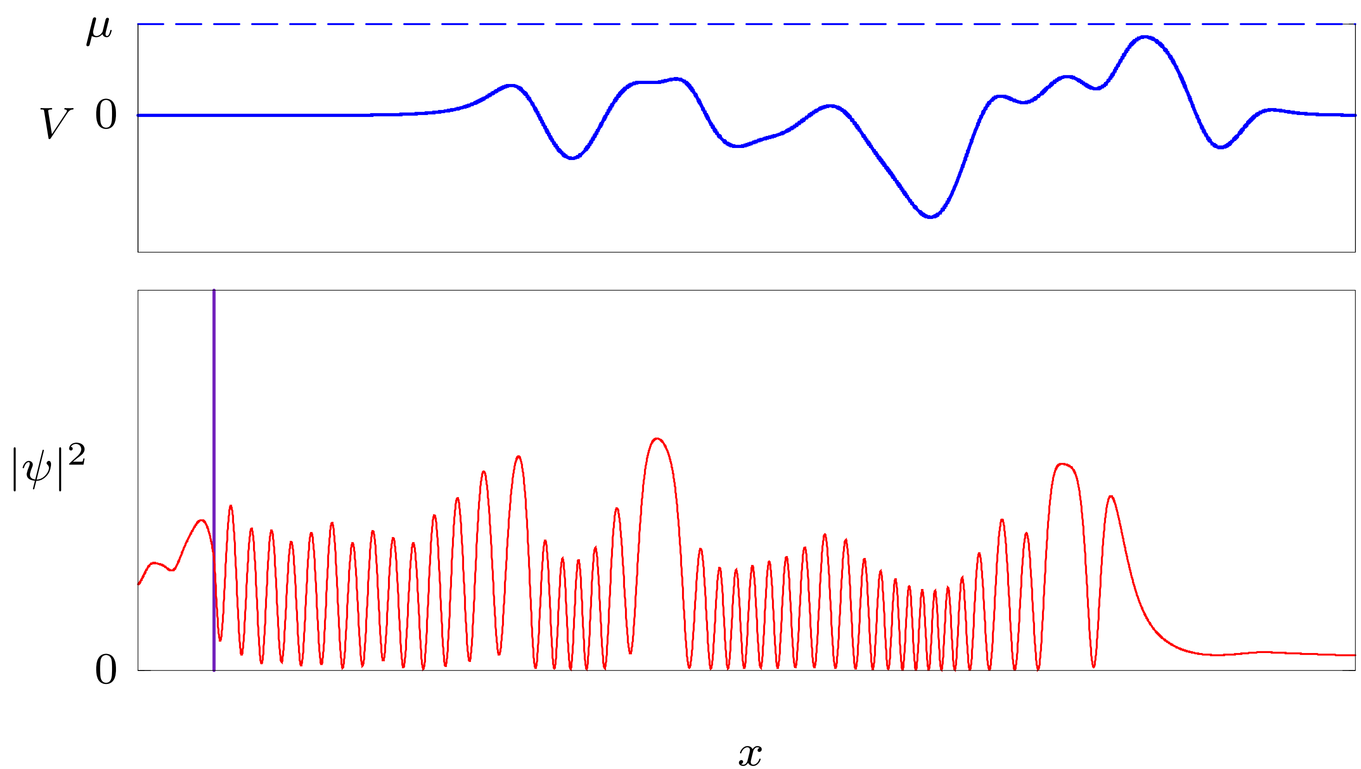

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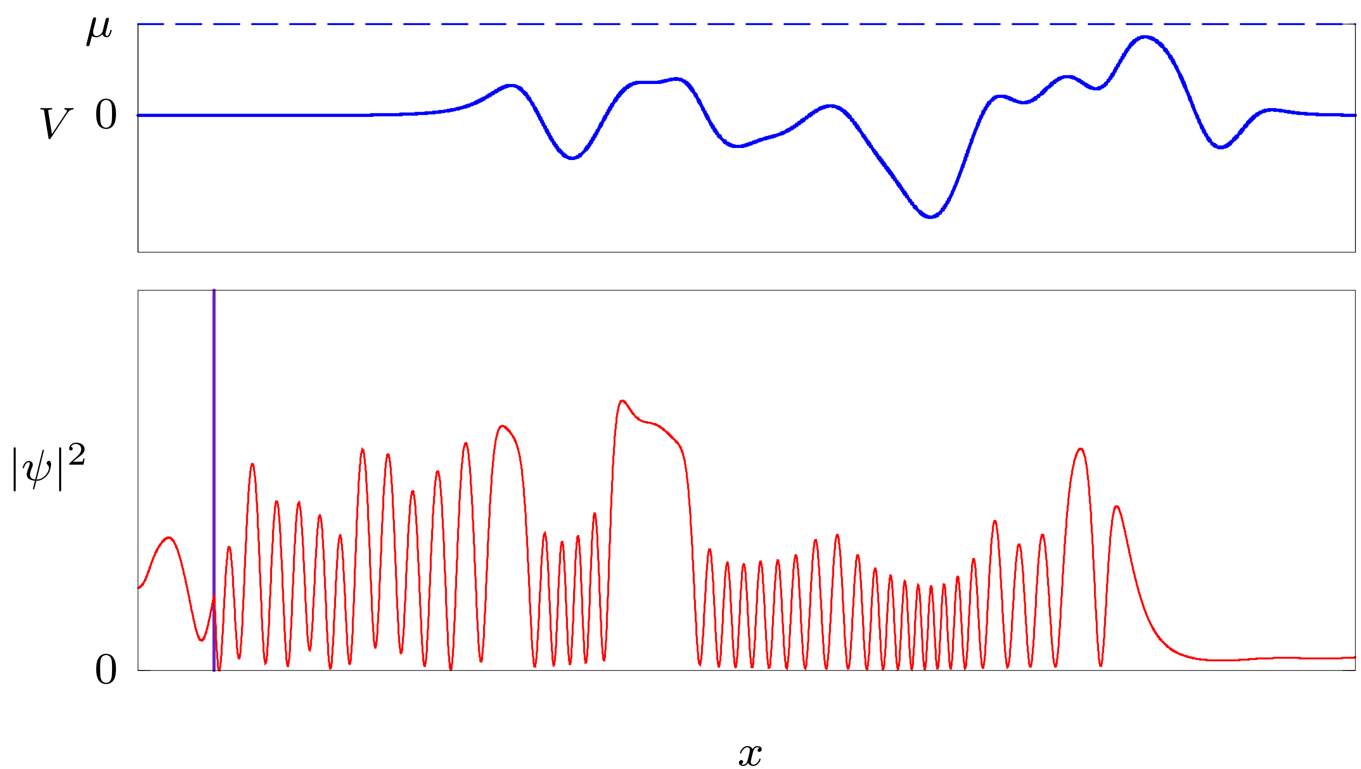

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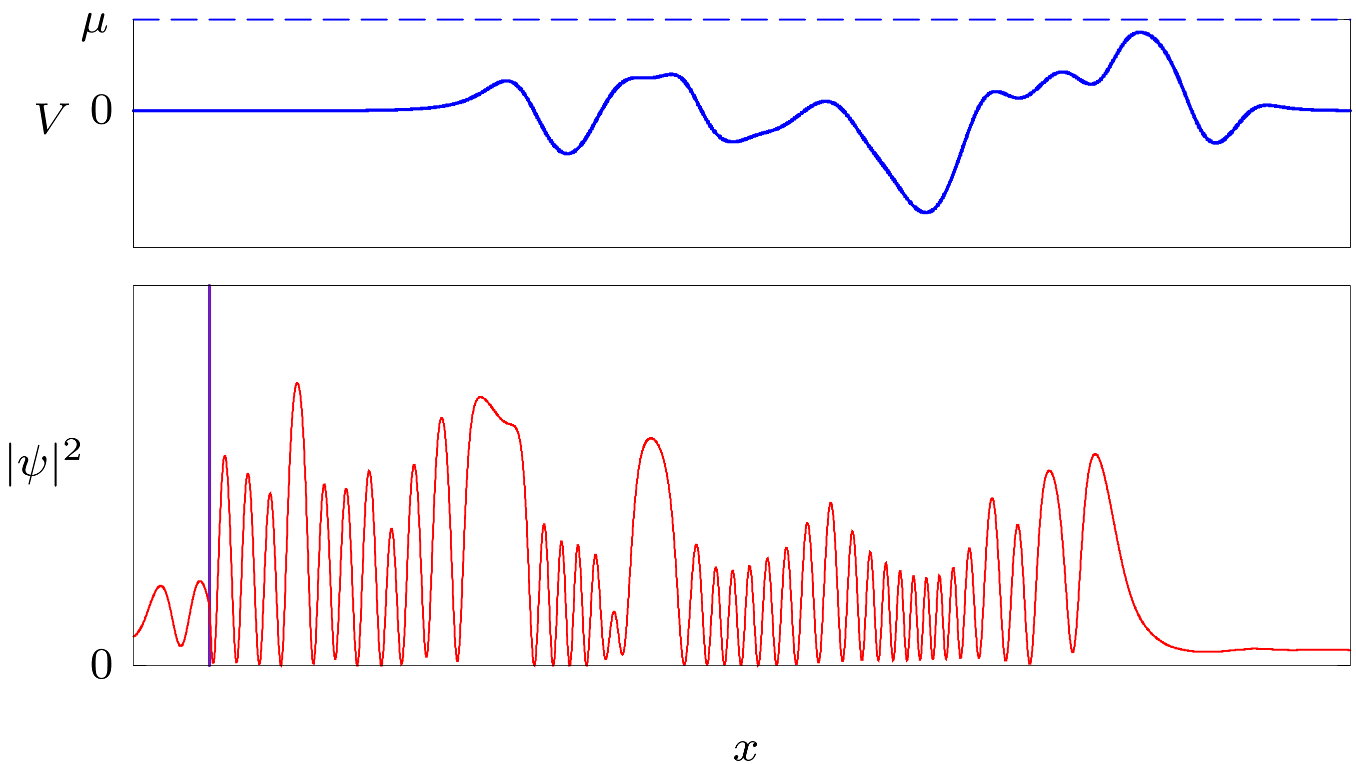

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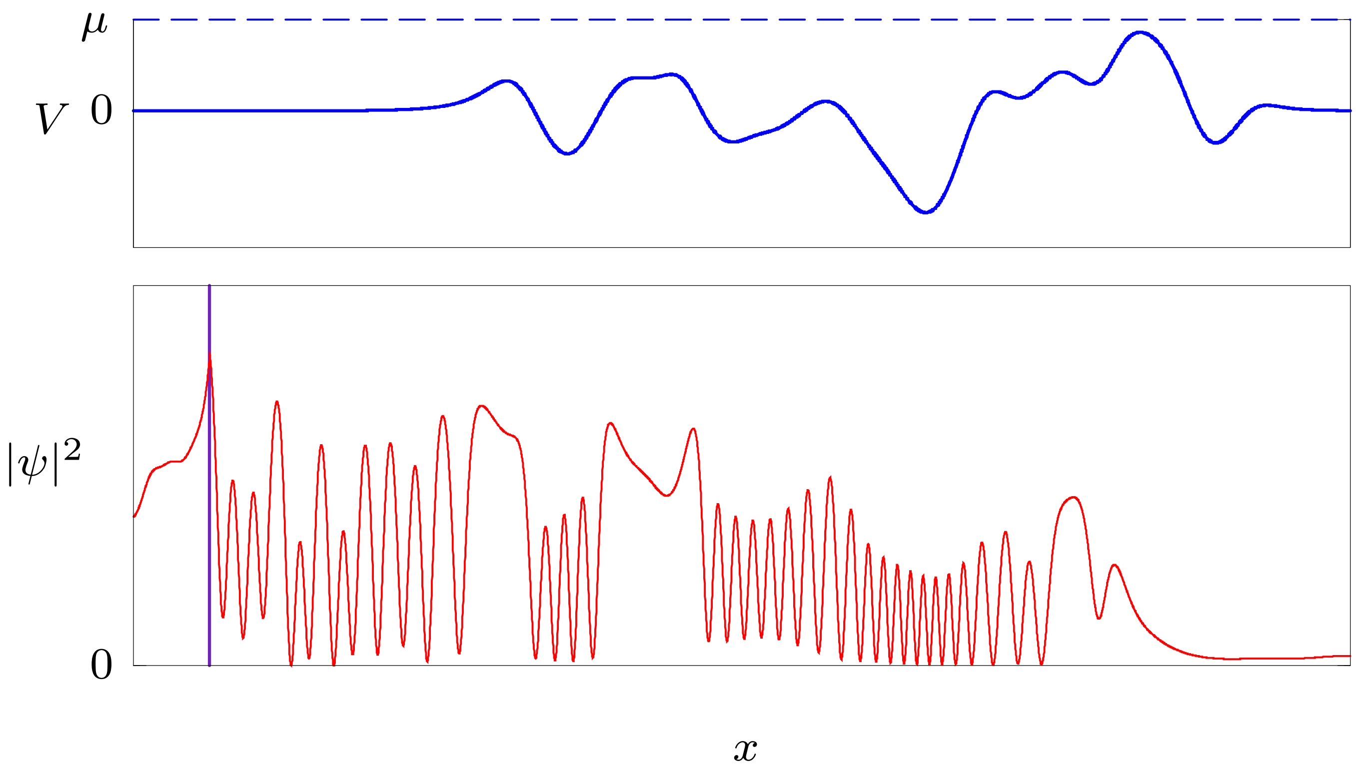

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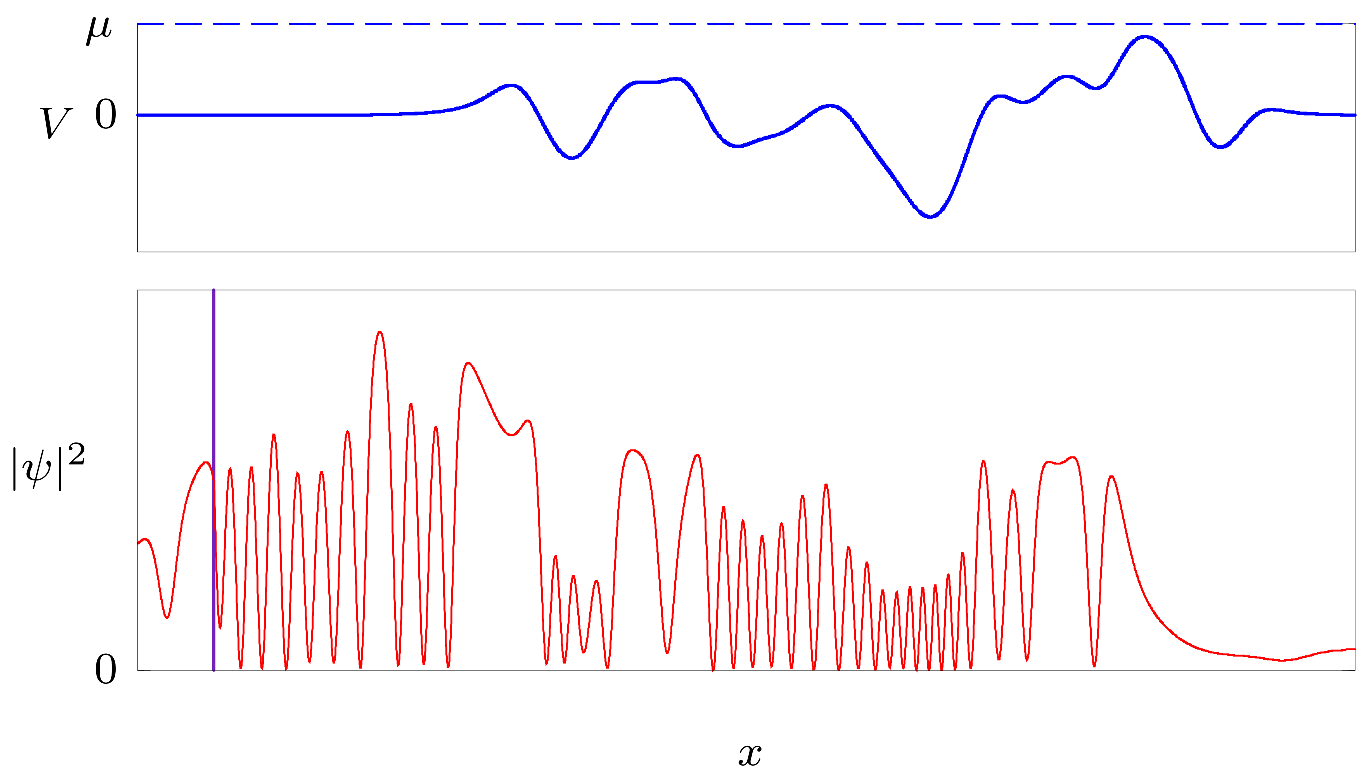


Transport through 1D disorder potentials

Finite interaction between the atoms: $g|\psi|^2 \simeq 0.1\mu$

$\longrightarrow$ permanently time-dependent scattering, except for very short disorder samples

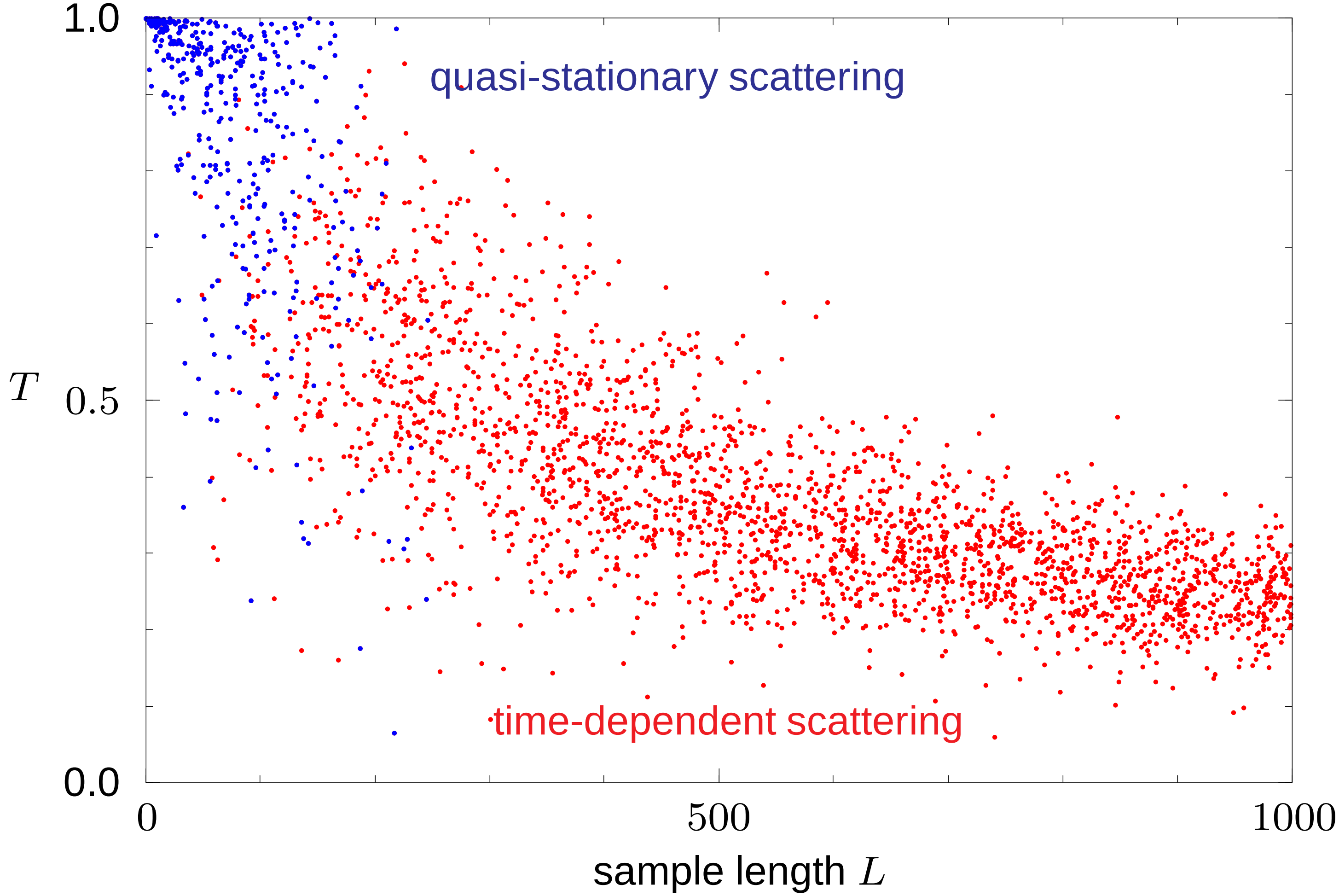
$\Longrightarrow$ compute time-averaged transmission: $\overline{T} = \frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t} T(t)dt$

$\longrightarrow$ algebraic (Ohm-like) decrease of the average transmission:

$$\overline{T} \simeq \frac{1}{1 + L/L_0}$$

T. Paul, P. Leboeuf, N. Pavloff, K. Richter, and P.S., PRA 72, 063621 (2005)

Transmission with finite interaction



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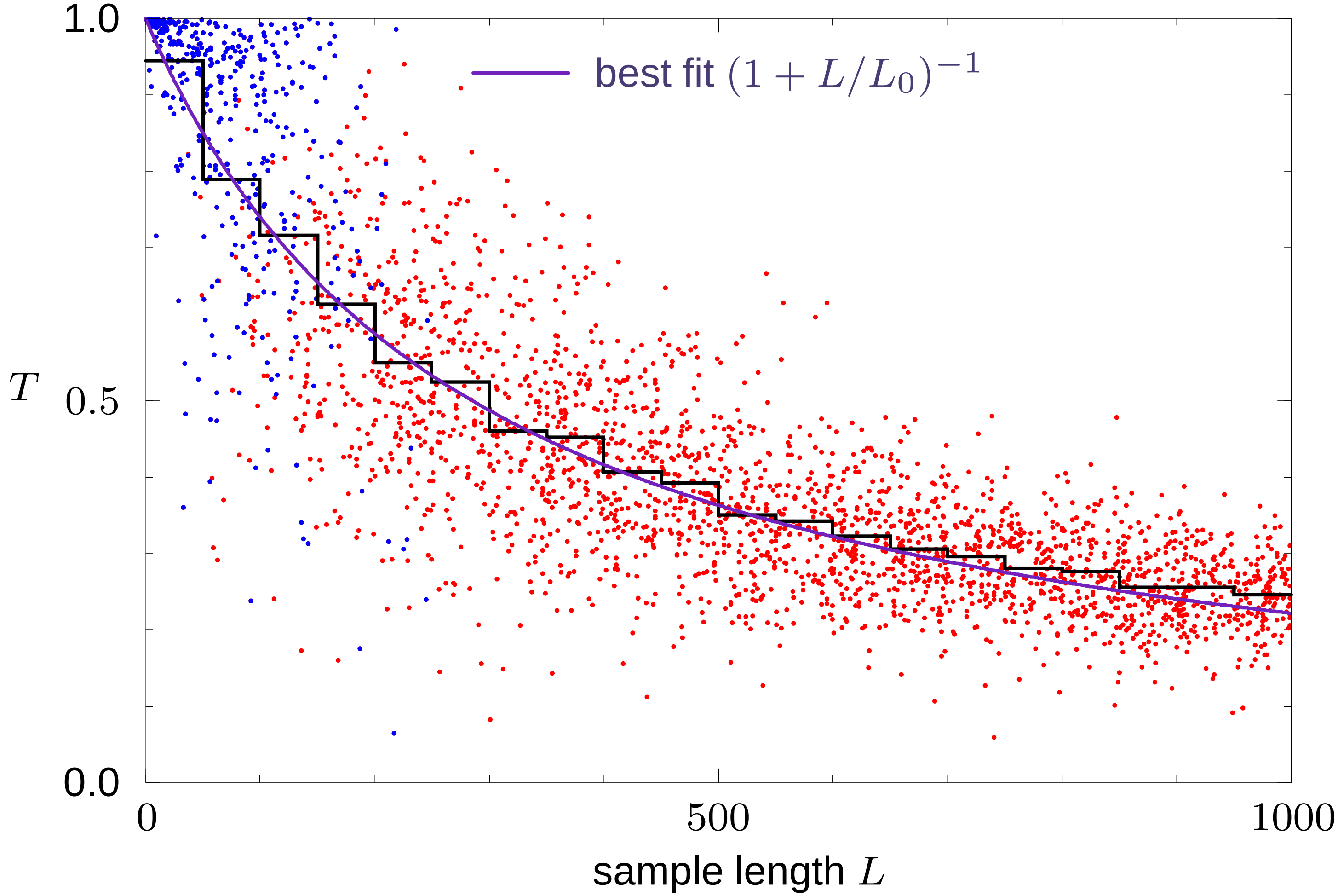
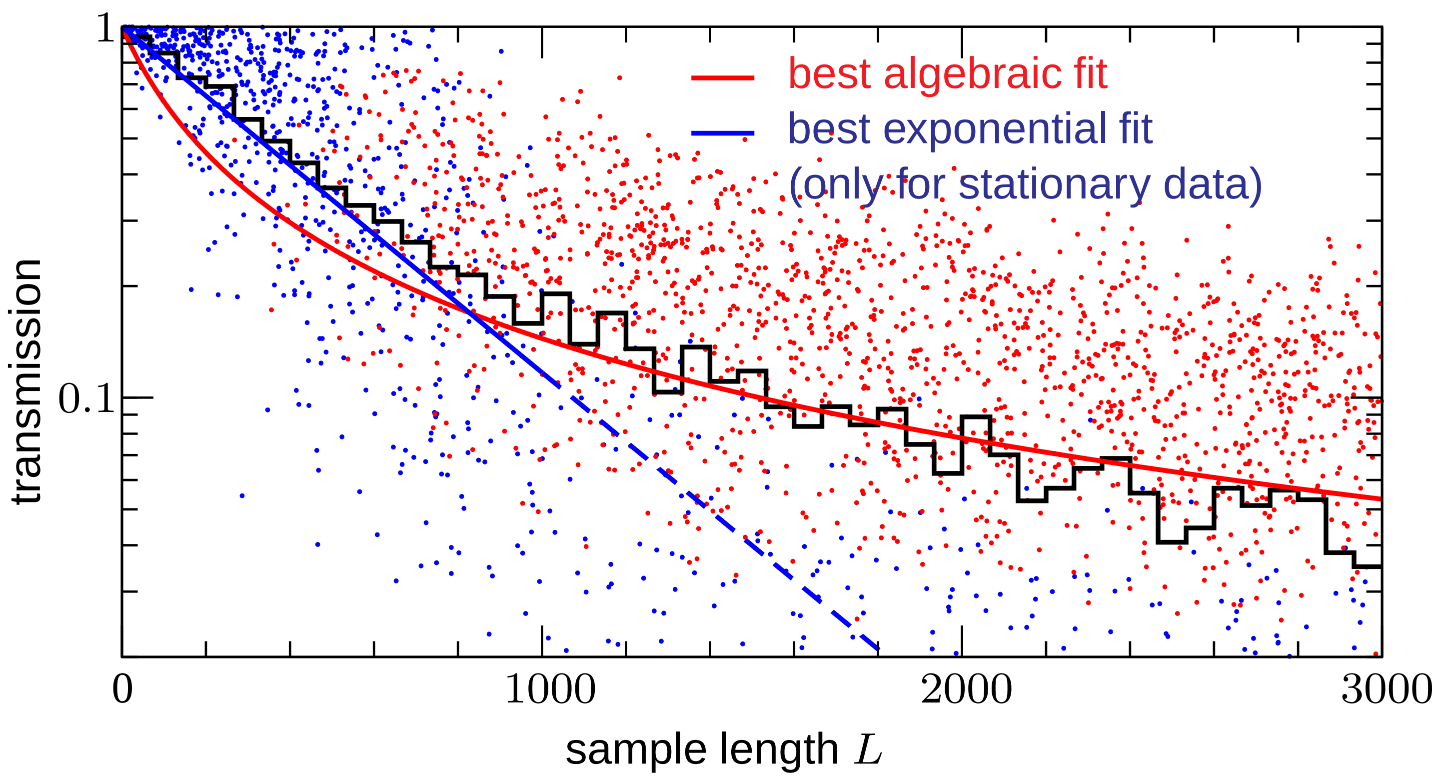


Crossover at weak interaction

$$g|\psi|^2 \simeq 0.01\mu$$



$\longrightarrow$ correlated with crossover from quasi-stationary to time-dependent scattering at $L = L^*$

T. Paul, P. Leboeuf, N. Pavloff, K. Richter, and P.S., PRA 72, 063621 (2005)

Crossover at weak interaction

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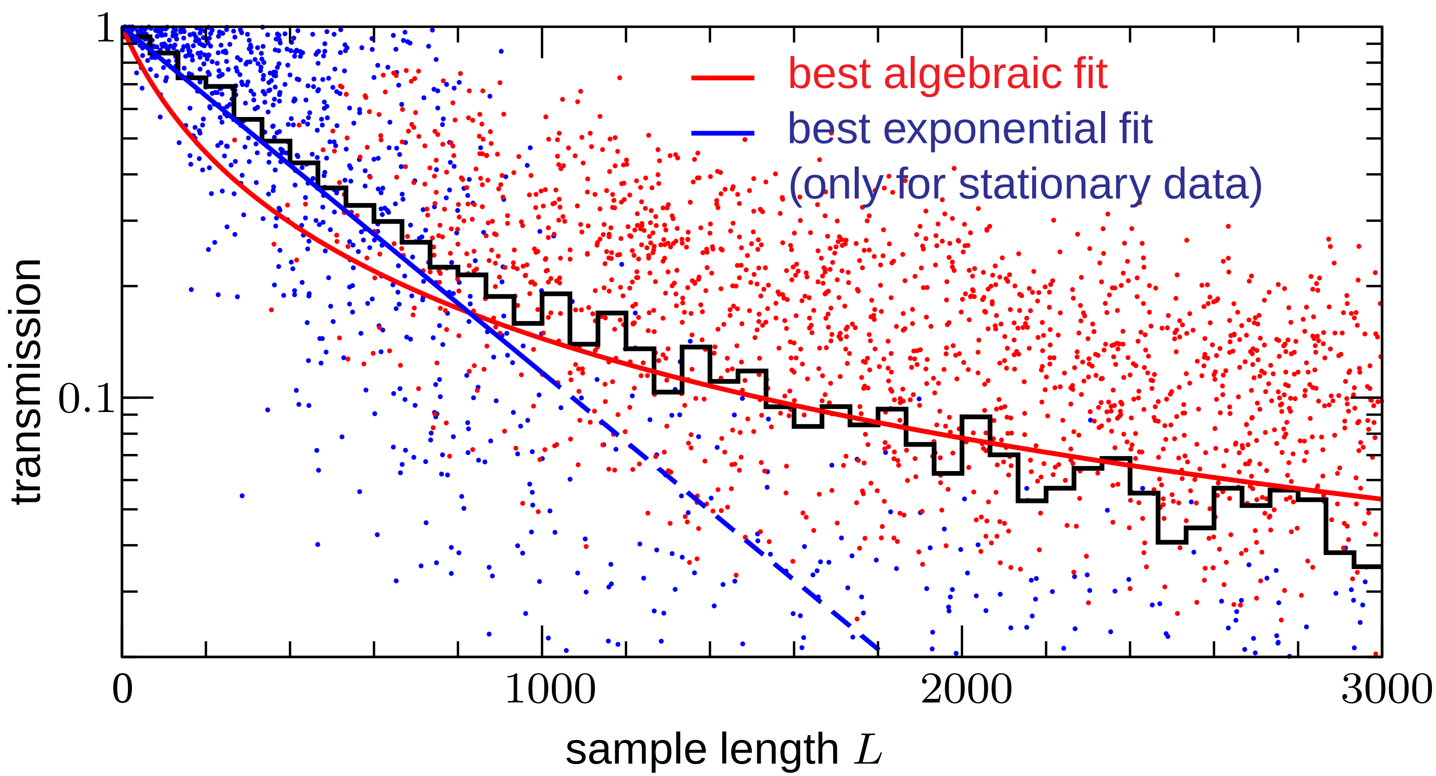
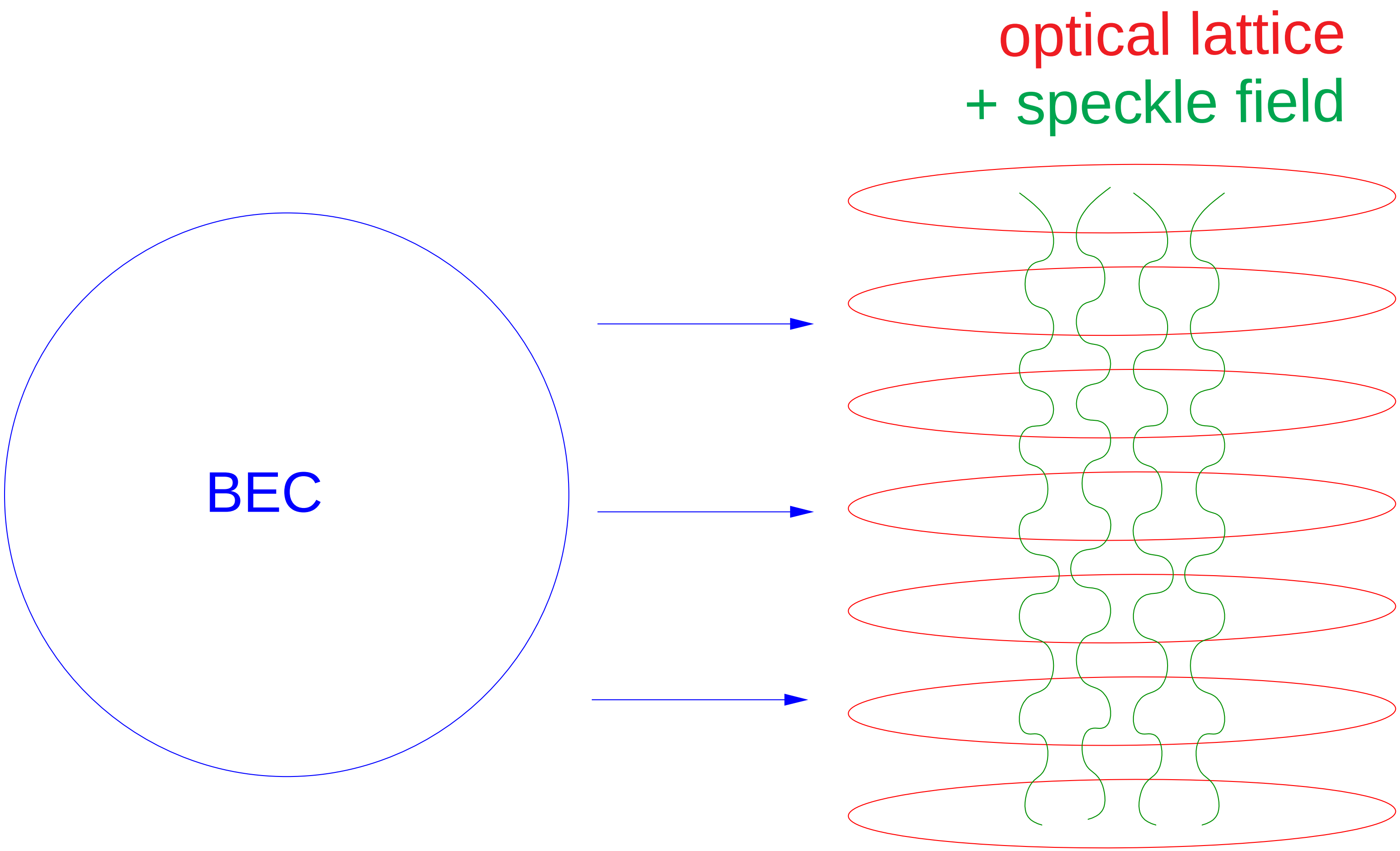


crossover length scale: $L^* \sim L_{\mathrm{loc}} \ln \left(\frac{\mu}{g|\psi|^2_{x>L}} \right)$

T. Paul, P.S., P. Leboeuf, and N. Pavloff, PRL 98, 210602 (2007)

Transport of condensates through 2D disorder

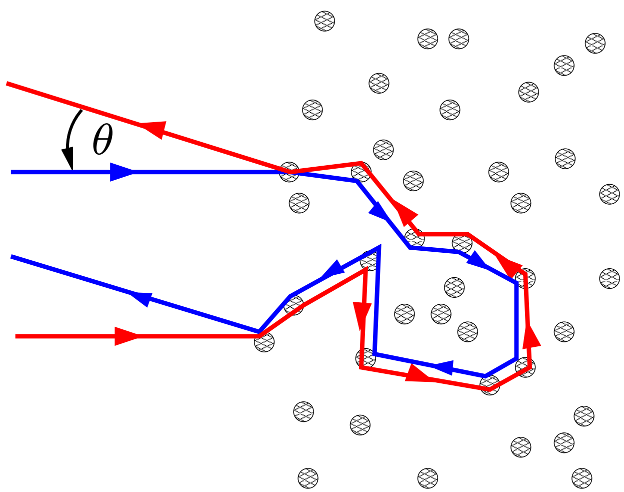
Possible experimental realization:

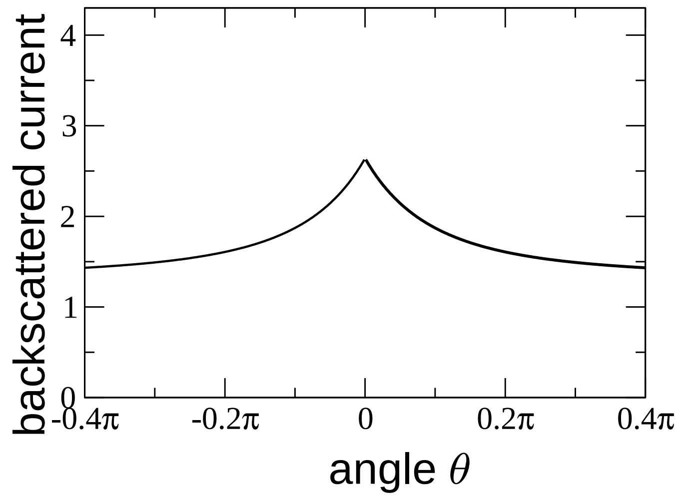


⟶ measure angle-resolved flux of backscattered atoms

Weak localization in two-dimensional disorder

- Constructive interference between reflected paths and their time-reversed counterparts





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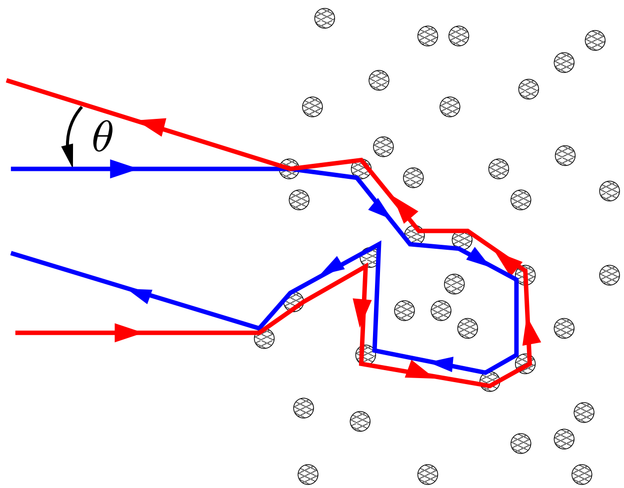
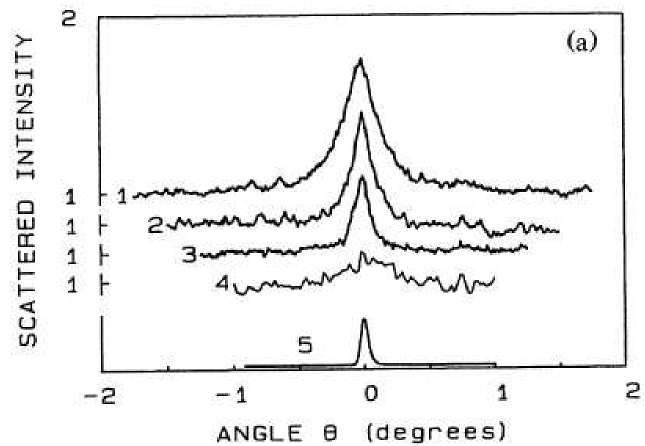




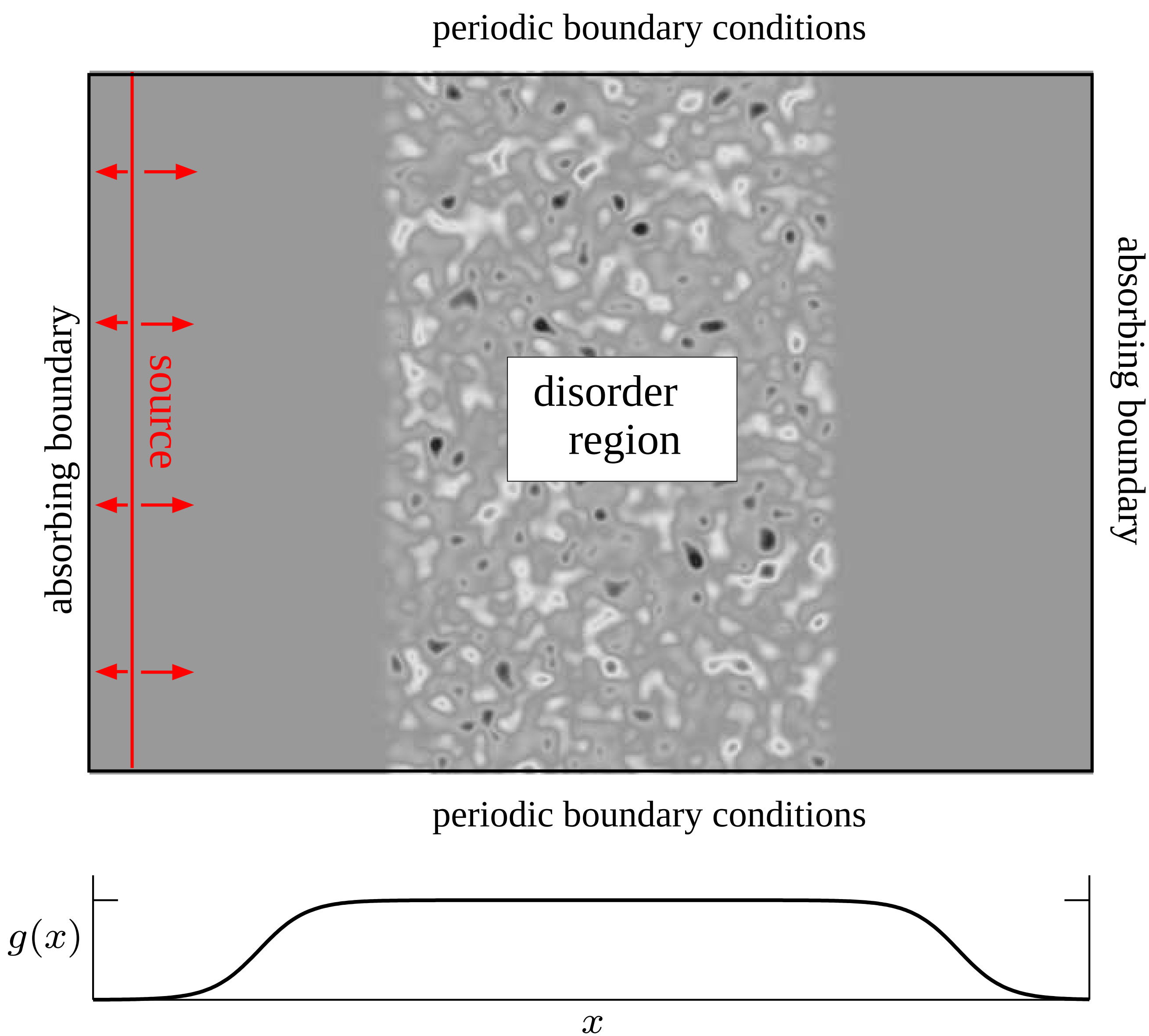
→ enhanced coherent backscattering of laser light from disordered media

M. P. Van Albada and A. Lagendijk, PRL 55, 2692 (1985)

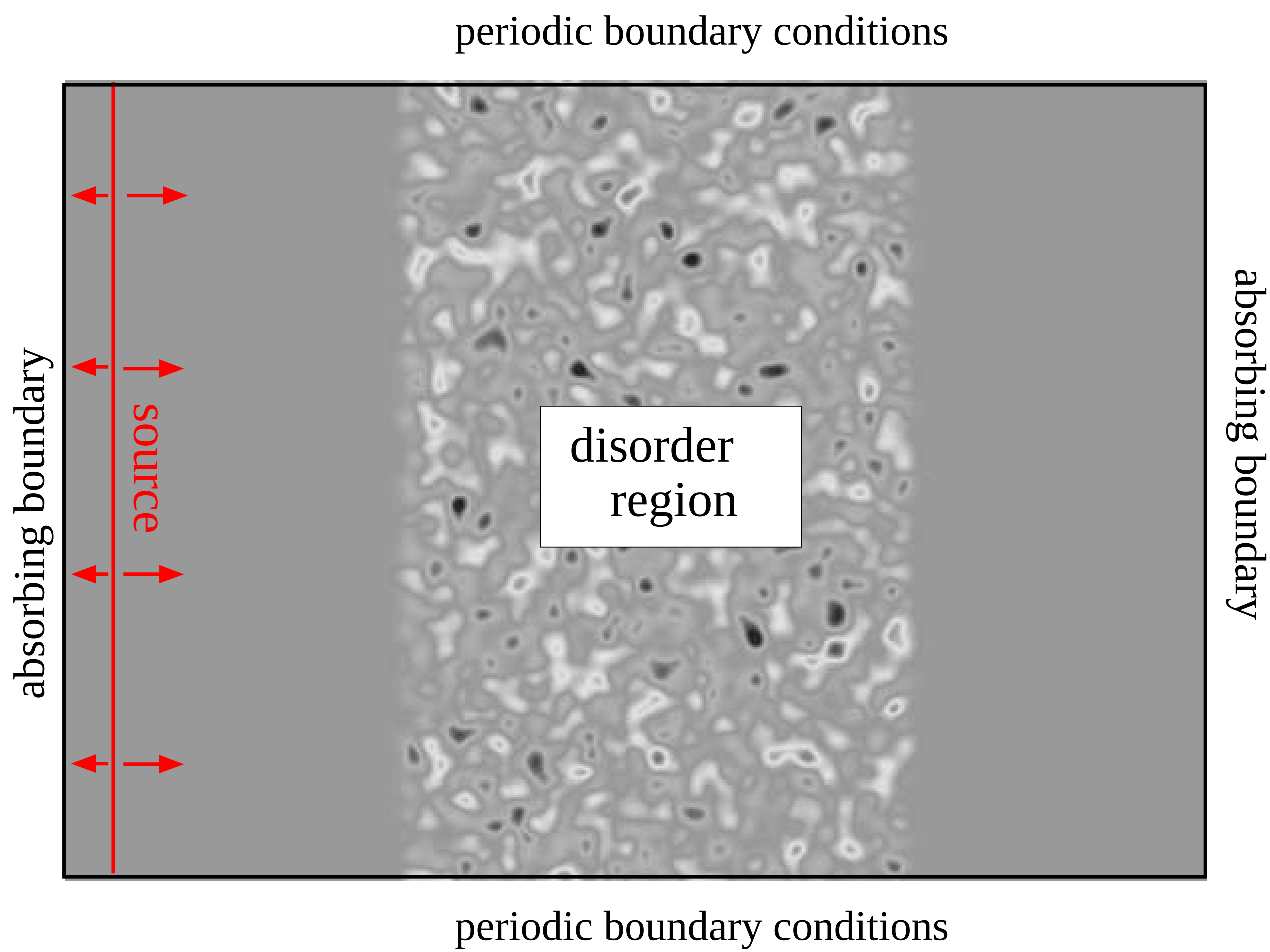
P.-E. Wolf and G. Maret, PRL 55, 2696 (1985)

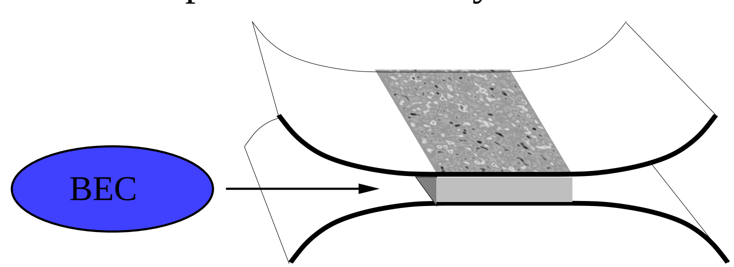
→ magnetoresistance in disordered 2D metals

Transport of condensates through 2D disorder

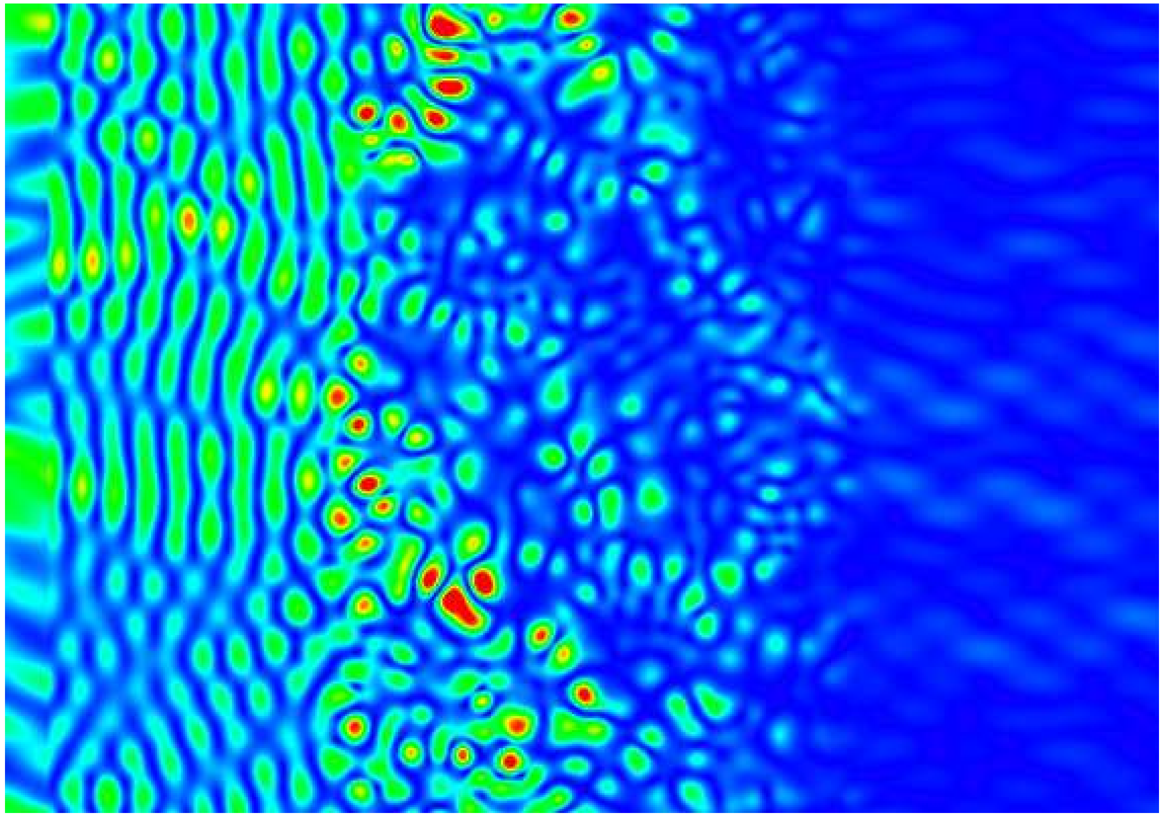


Transport of condensates through 2D disorder





Stationary scattering state of the condensate



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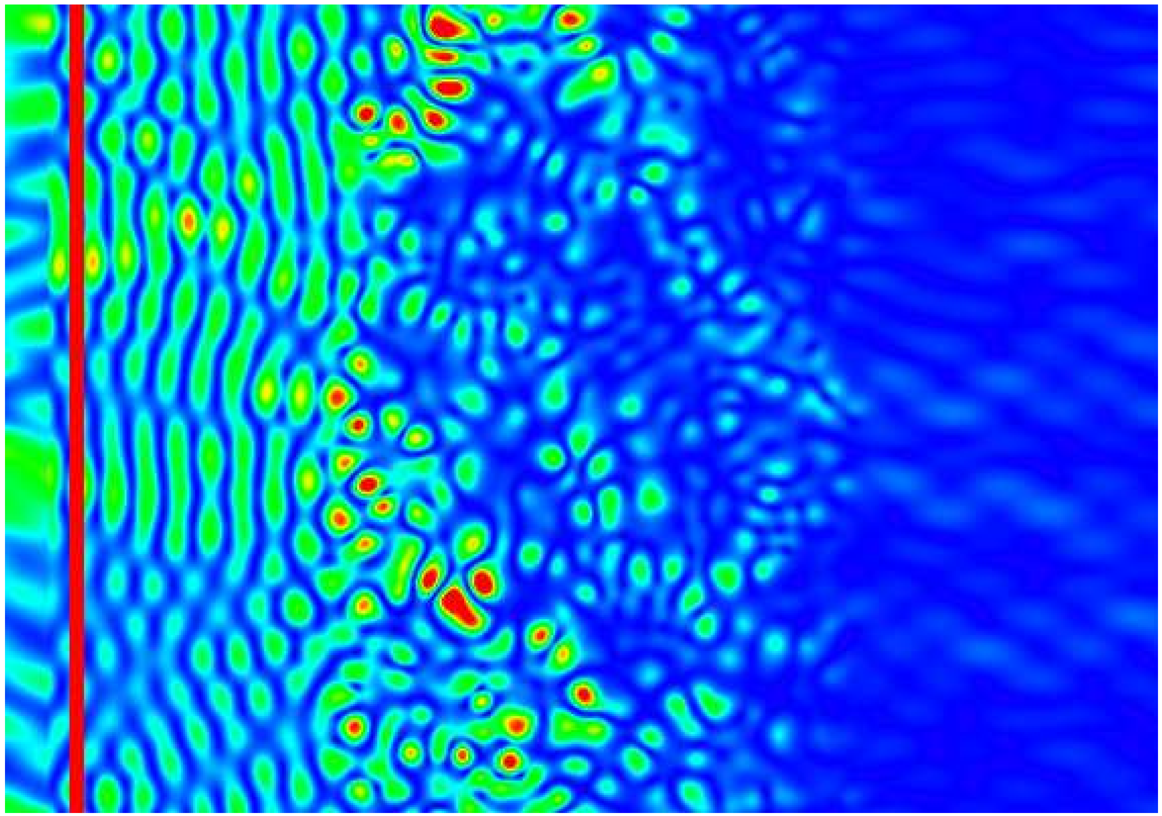
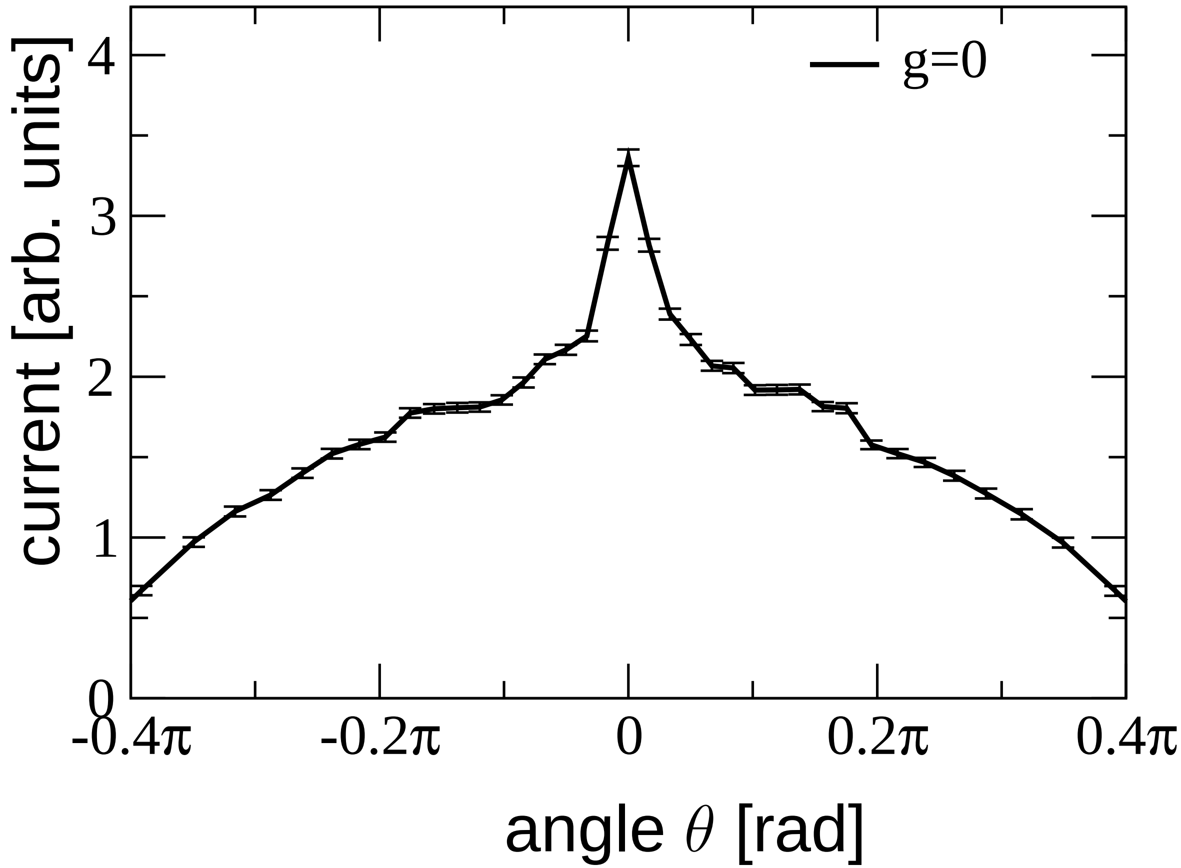


decomposition of reflected wave into transverse eigenmodes
$\rightarrow$ angle-resolved backscattered current (time-of-flight image)

Coherent backscattering of the condensate



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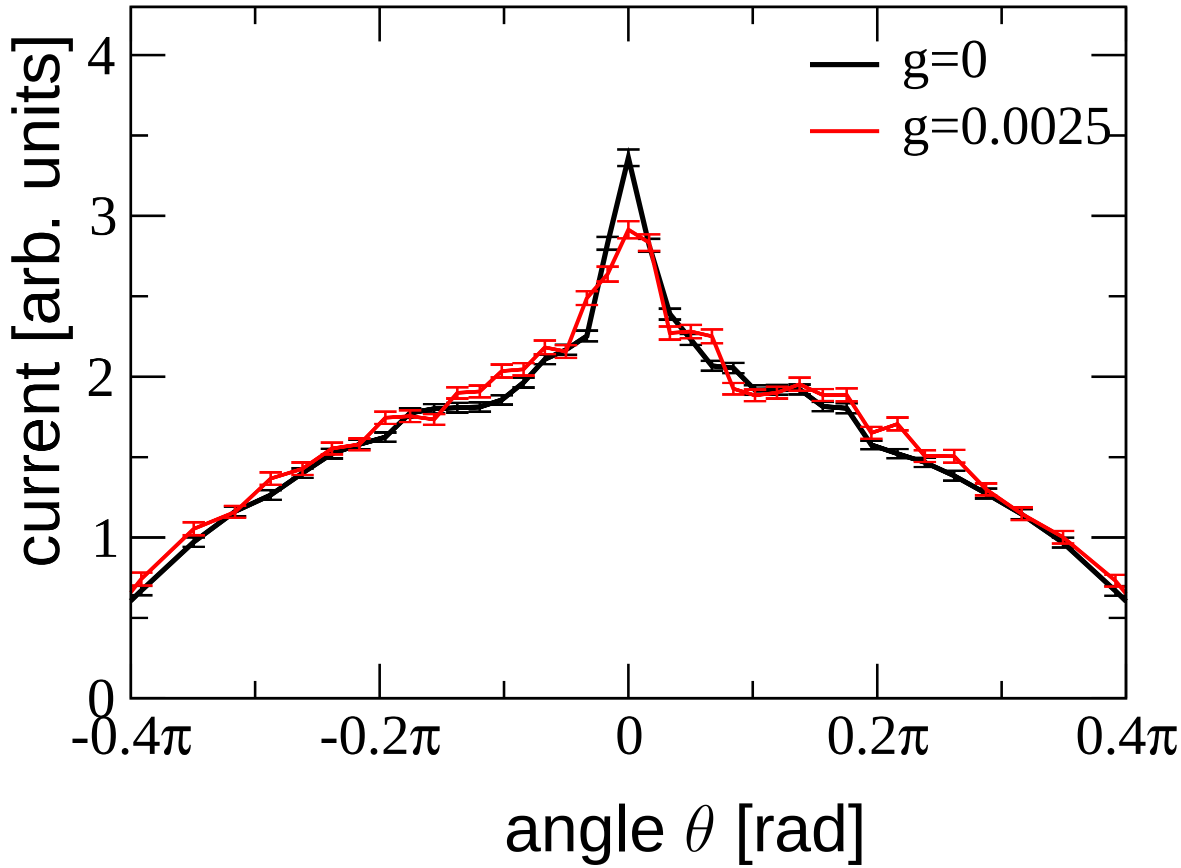

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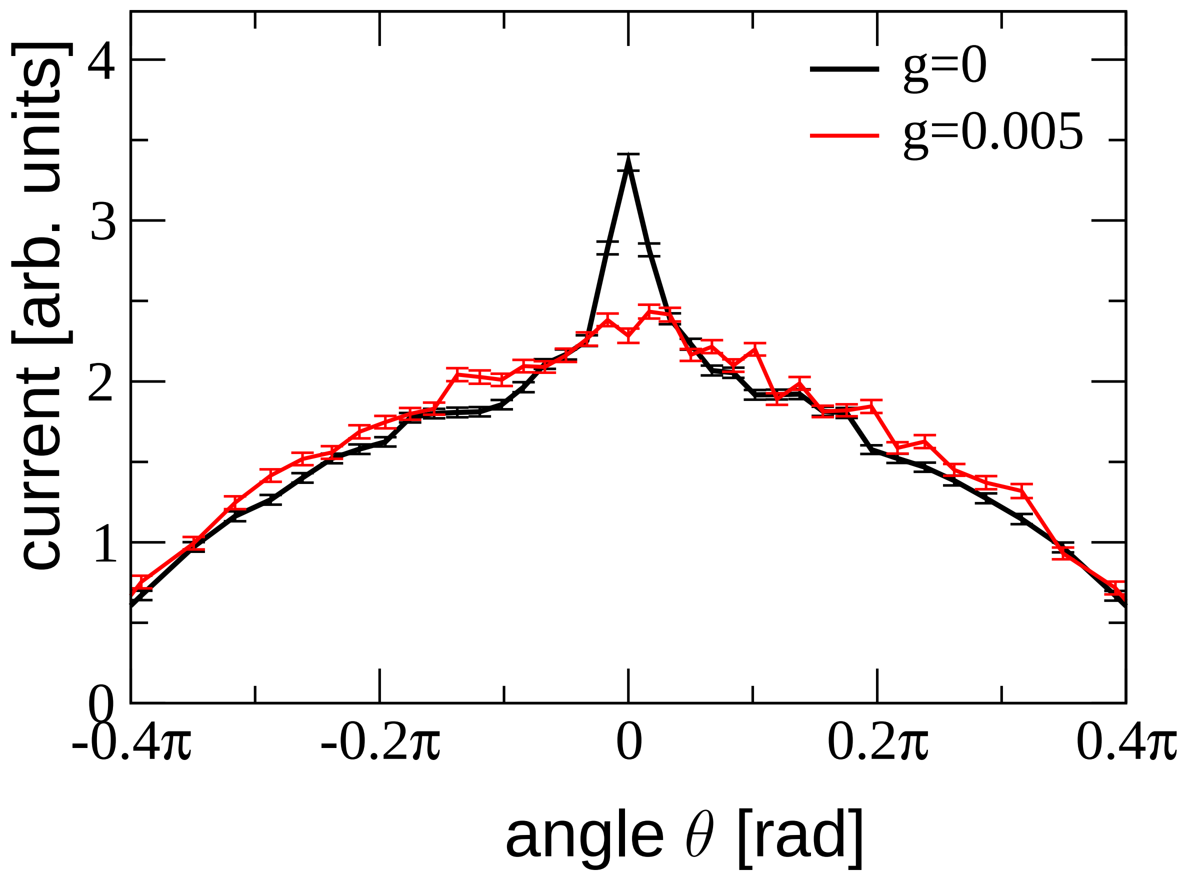

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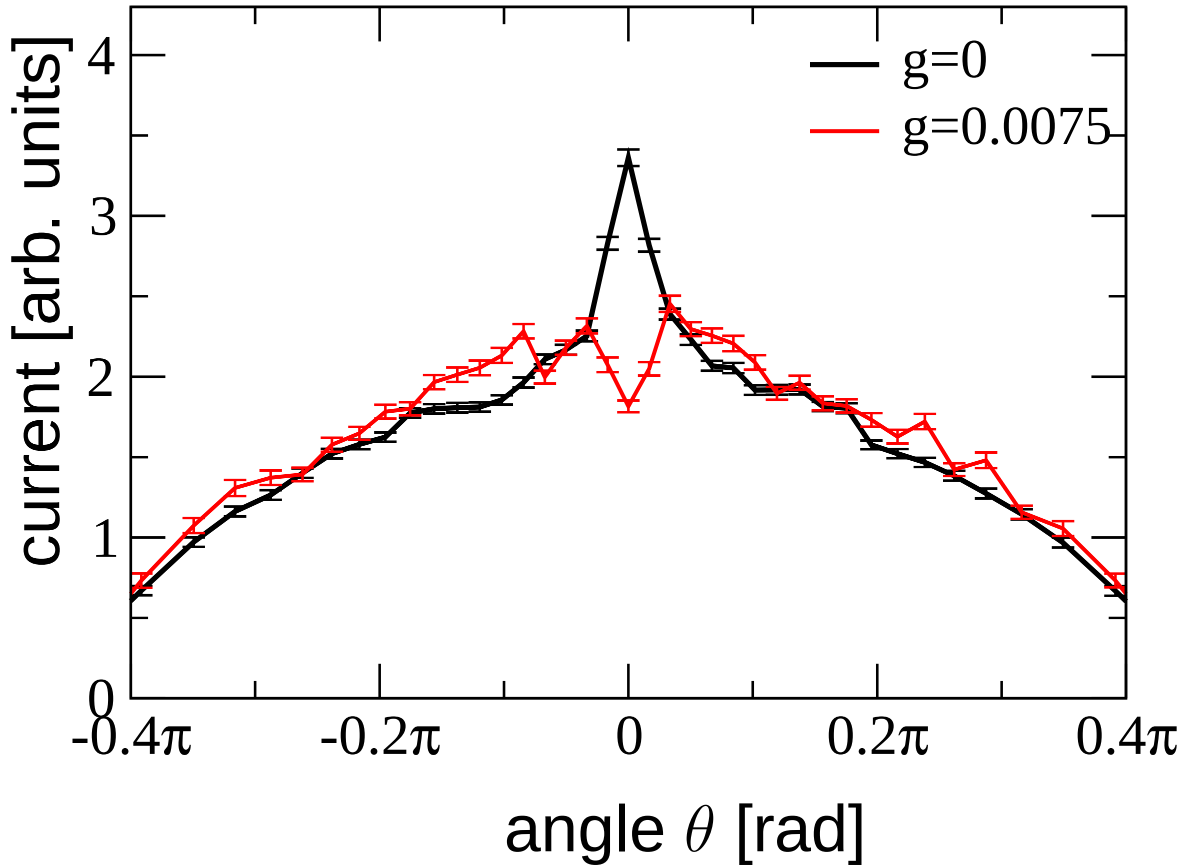

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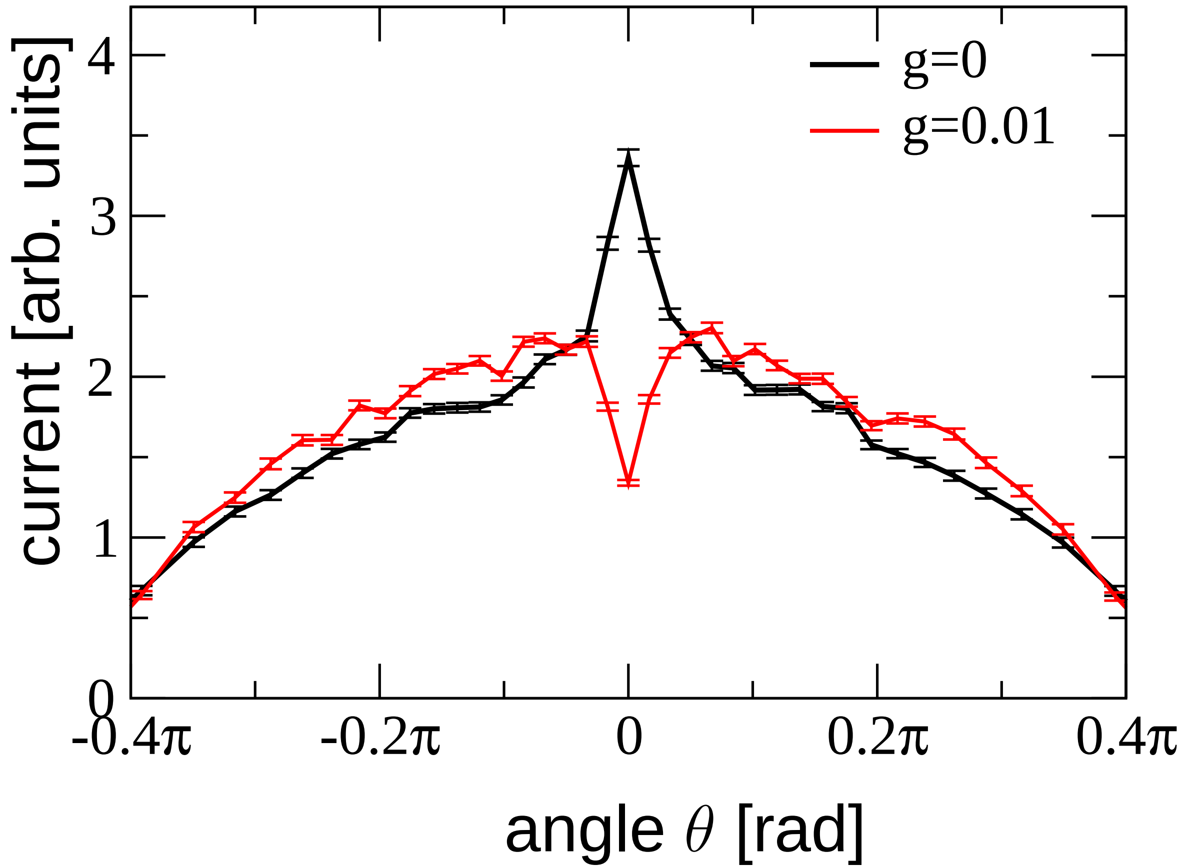
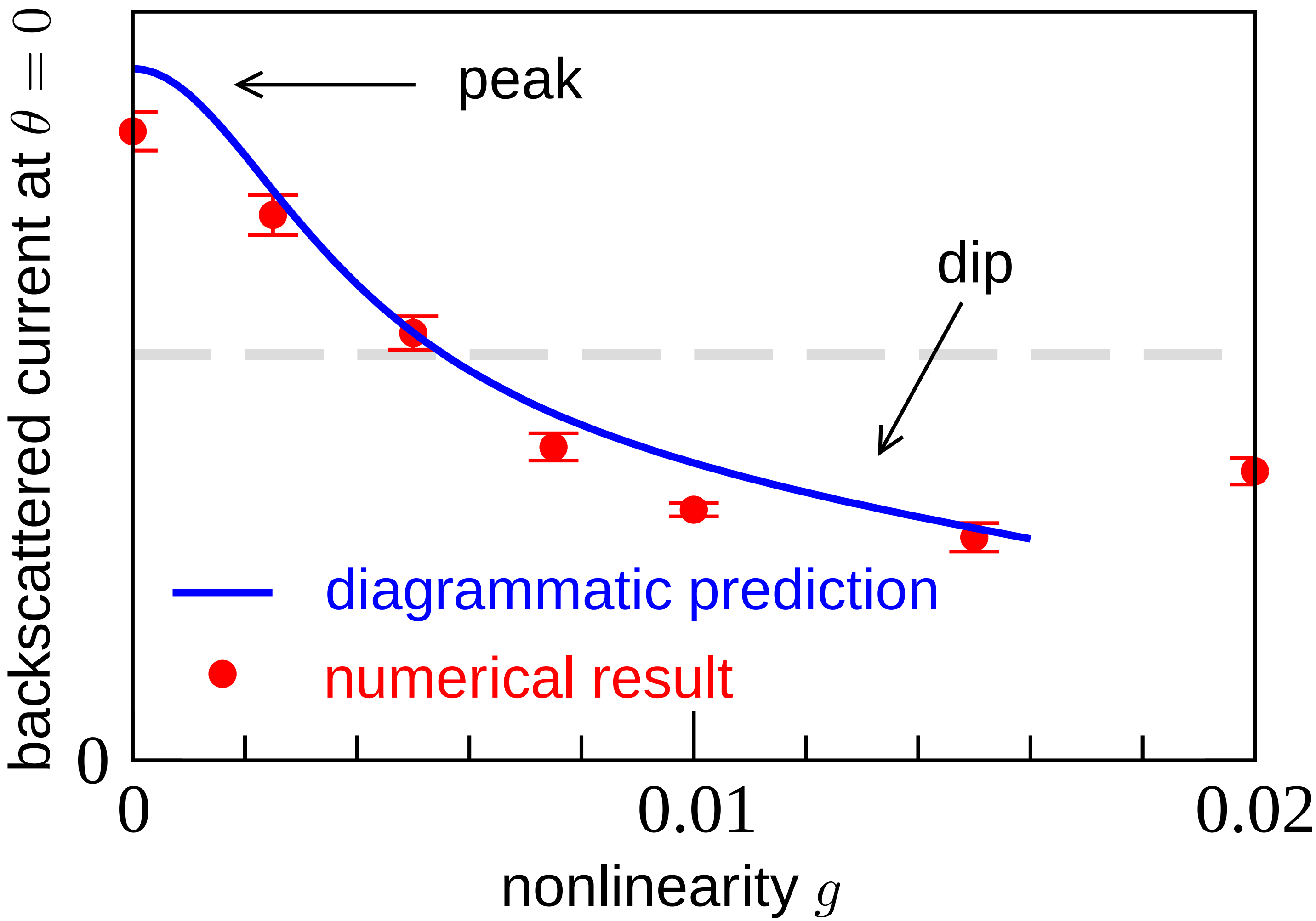


$\longrightarrow$ inverted cone in presence of finite interaction:
crossover from constructive to destructive interference

Comparison with analytical diagrammatic theory

→ based on: T. Wellens and B. Grémaud, PRL 100, 033902 (2008)



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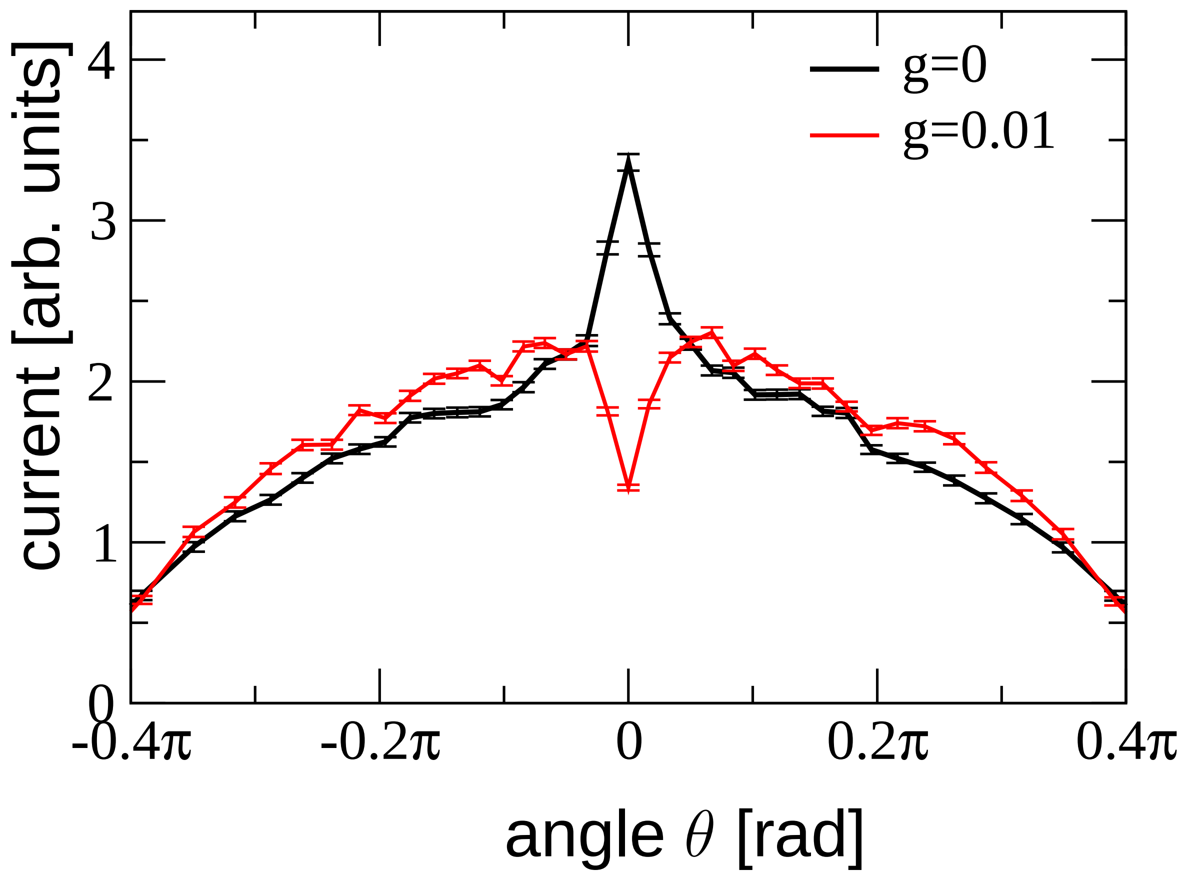

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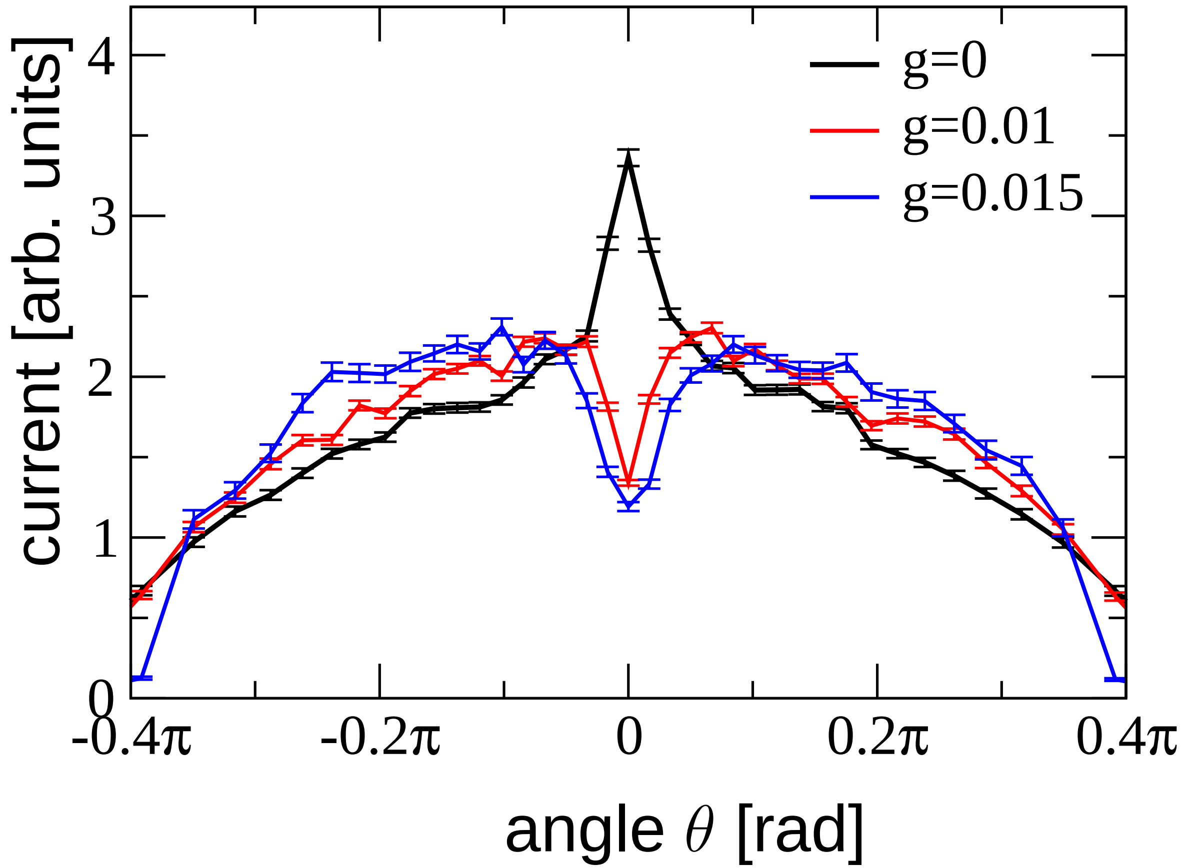

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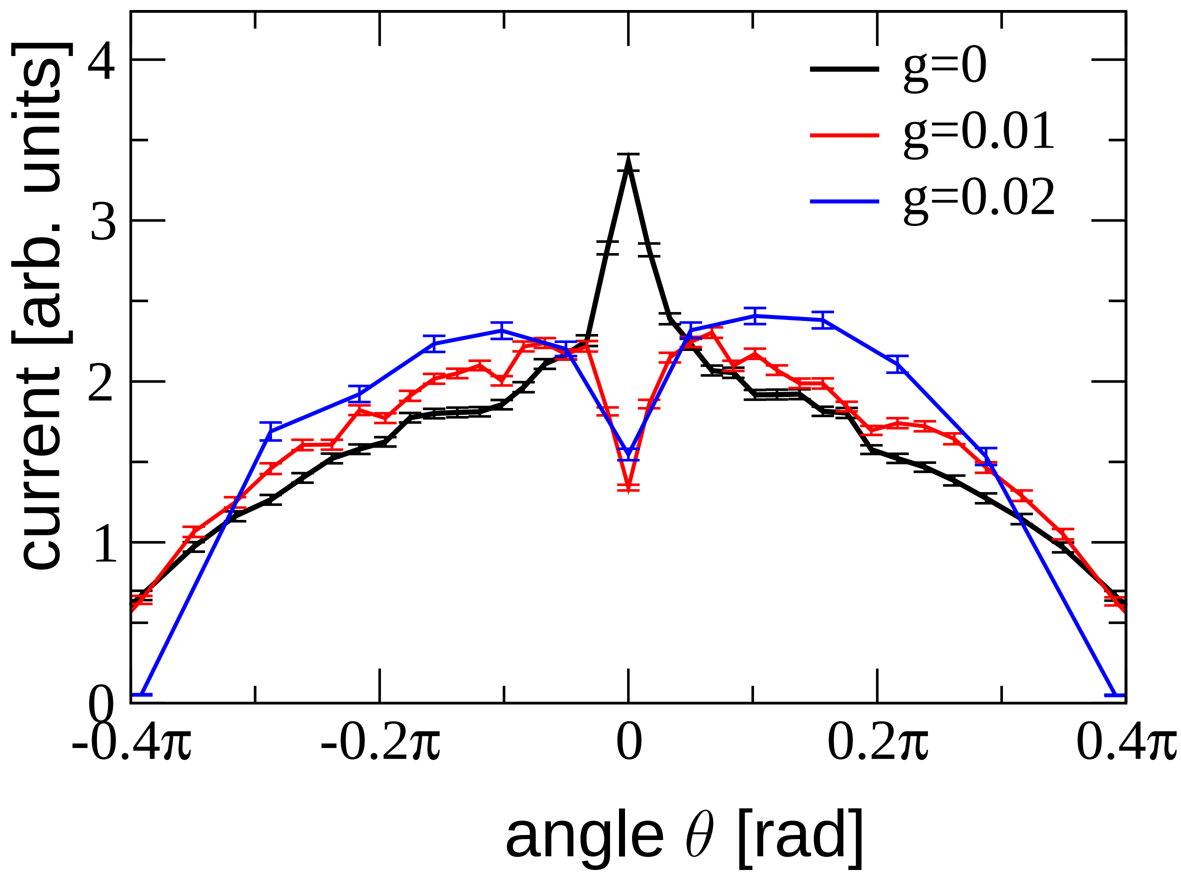

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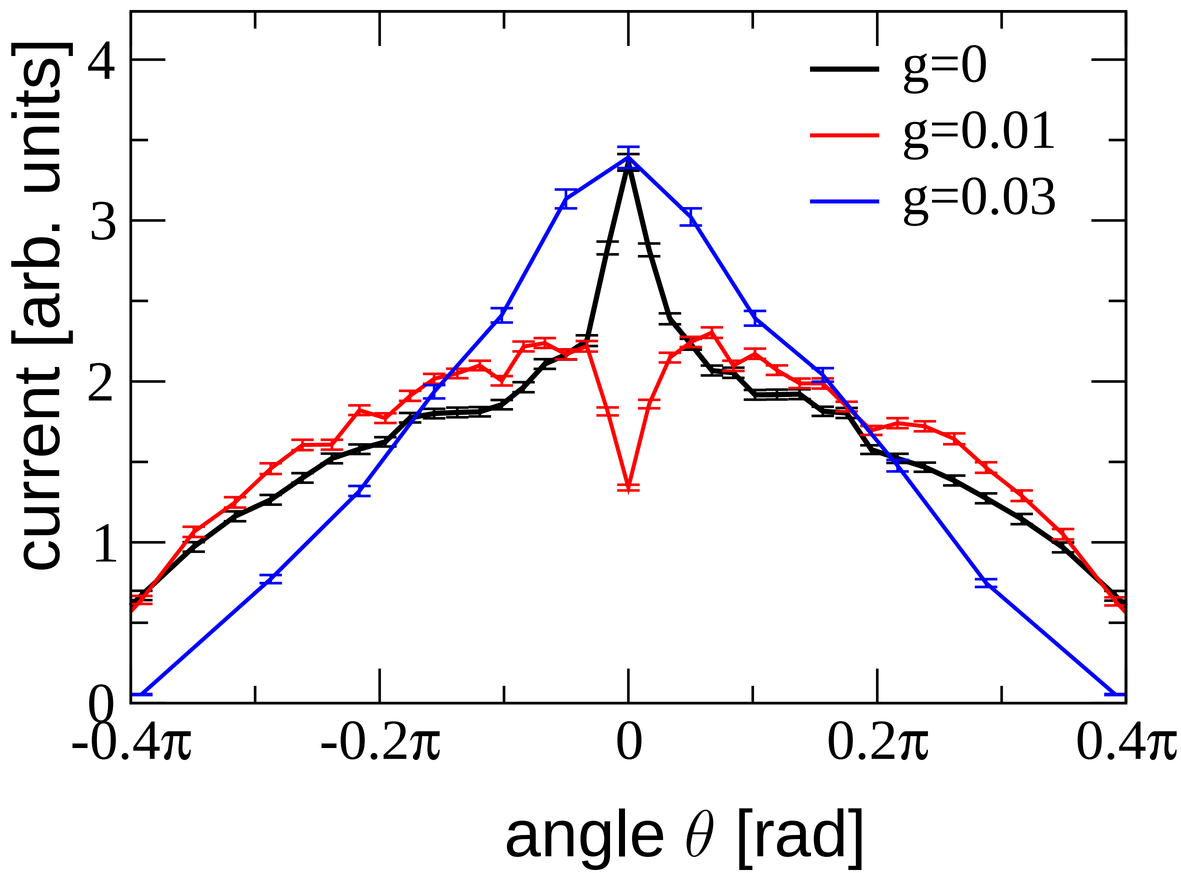
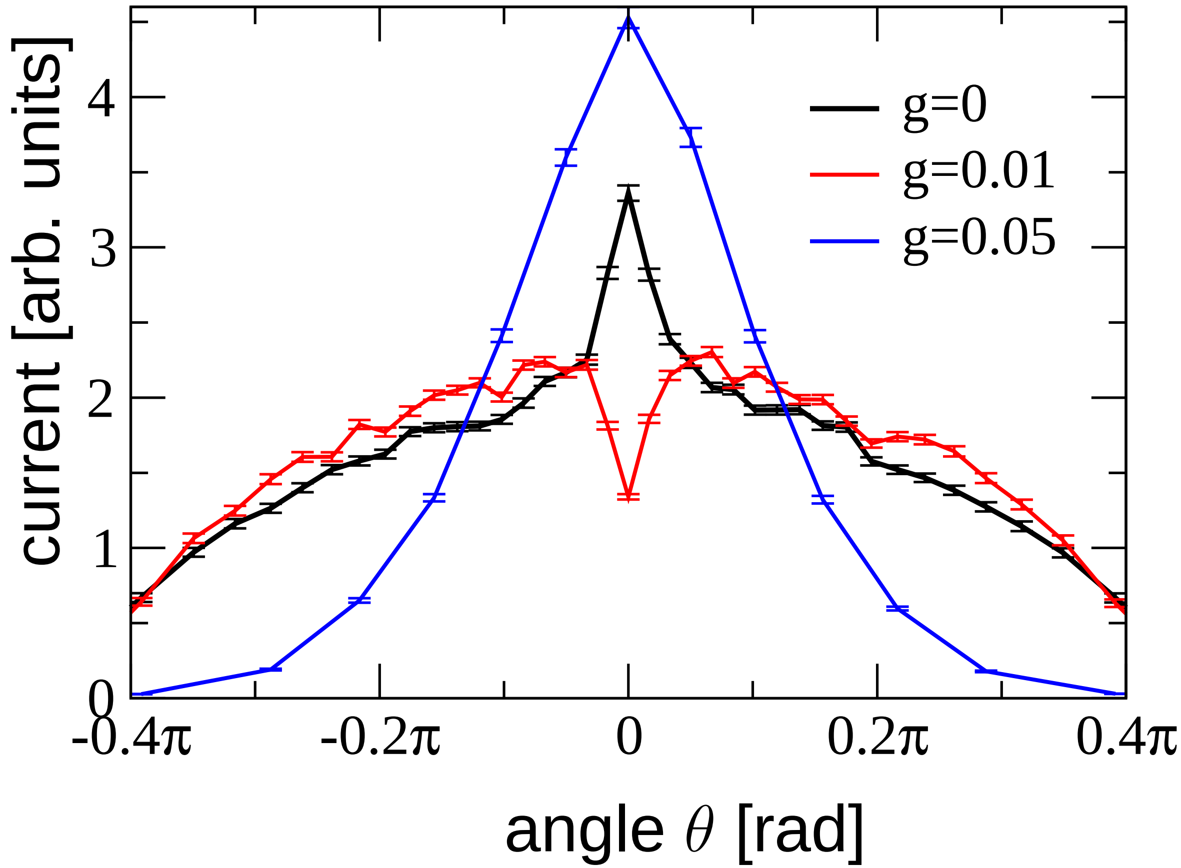


Coherent backscattering of the condensate



⟶ broad peak for stronger interaction:

regime of permanently time-dependent scattering

Conclusion

Interaction strongly affects the localization properties of propagating condensates in disorder potentials:

- Transport through 1D disorder potentials:
  - crossover from exponential (Anderson-type) to algebraic decrease of the transmission
  - correlated with appearance of time-dependent scattering

  T. Paul *et al.*, PRA 72, 063621 (2005); PRL 98, 210602 (2007)

- Transport through 2D disorder potentials:
  - coherent backscattering peak inverted in presence of weak interaction

  M. Hartung, T. Wellens, C. A. Müller, K. Richter, and P.S., arXiv:0804.3723 (PRL, in press)

The team

Tobias Paul

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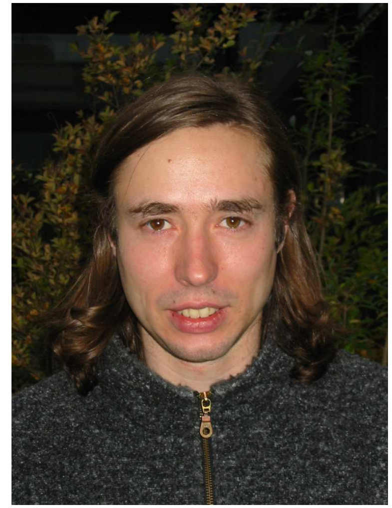

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