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A method to enhance noise reduction for data generated from a known differential equation

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There are several papers which show the use of principle component analysis to reduce noise from a discretely sampled data. This method consists of embedding the data in higher dimensions and then using the singular value decomposition of the resulting matrix, retaining only the dominant components in the data. This method does involve throwing away some information and is not very effective if the sampling interval is not very small. If we know the underlying differential equation, we can find expressions for the neglected components and restore them. We first develop a theoretical basis for the procedure and then illustrate it in the case of some data generated using the Duffing's equation.

Breathers and homoclinic orbits

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Homoclinic orbits are presented as a useful tool for obtaining breather solutions (time periodic, spatially localized) of one dimensional nonlinear lattices (Klein Gordon, FPU and mixed). We present how the relationship between homoclinic orbits and breathers is formed, and advantages of using this method for obtaining breathers over the traditional method of continuation.

Stochastic resonance: Some new results

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Stochastic resonance is a mechanism which was originally proposed to explain the close-to-periodic appearance of Ice Ages. It has been observed in a large number of physical and biological systems. Its main feature is that small additive noise may considerably enhance a system's response to a weak, periodic signal. I will report on joint work with Barbara Gentz (WIAS, Berlin). We introduce a new approach, which aims at describing the behaviour of "typical" paths, instead of merely computing the probability density. This allows us to give precise results on the dependence of transition probabilities on noise intensity, and provides an alternative measure of the output's "periodicity" to the usual signal-to-noise ratio. Various important quantities have a nontrivial power-law dependence on the small parameters. I will also mention some related results on dynamical hysteresis.

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On periodic orbits picture of fractal magnetoconductance in quantum billiards

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Starting from evaluating a simple soft-wall multichannel billiard and using the semiclassical Kubo formula for conductivity, we gave a periodic-orbits picture for the exact self-similar magnetoconductance fluctuations obtained in the phase coherent ballistic quantum billiards experiments. The applicability of Kubo formula to these single devices are justified, since the dots are coupled to large source and drain so that due to level broadening, the energy level inside the dots can be considered as effectively continuum. In our picture the soft-wall boundary is experimentally essential. We claimed that the exact self-similar magnetoconductance fluctuations is due to the self-similar periodic orbits generated through a sequence of isochronous pitchfork bifurcations of straight-line liberating orbits oscillating toward harmonic saddles. The saddles are naturally created right at the point of contact with the leads or at certain places in the cavity as a consequence of the sofwall confinement. We will show that the function we obtained, approximately satisfies a scaling relation, which generates its self-similar properties. In fact, its spectrum can be approximated by the famous Weierstrass spectrum with an upper cut. In addition, we would like to show that the Hurst exponent of the fractal is independent of the detailed geometrical shapes of the cavity and determined only by the local information on the saddles. On this issue we would like to confirm the recent experimental findings that the fractal dimension of the fluctuations are determined only by the quantumness of the system. We choose to discuss the Hurst exponent which is defined as the ratio of the logarithmic values of the scaling constants in the y-direction and x-direction, instead of the fractal dimensions, since they are easy to calculate and to be confirmed experimentally for an exact self-affine fractal. Finally, using the same reasoning and again using the semiclassical Kubo formula for conductivity, we would like to show that even in the situation when the conductance fluctuations versus the applied weak magnetic field show fractal behaviour, the conductance fluctuations as a function of the Fermi energy are not fractal-like.

Experimental test of a trace formula for chaotic 3D microwave cavities

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We have measured resonance spectra in a superconducting microwave cavity with the shape of a three-dimensional generalized Bunimovich stadium billiard and analyzed their spectral fluctuation properties. The experimental length spectrum exhibits contributions from periodic orbits of non-generic modes and from unstable periodic orbits of the underlying classical system. It is well reproduced by our theoretical calculations based on the trace formula derived by Balian and Duplantier for chaotic electromagnetic cavities. Analysis of the spectral statistics reveals that there is a partial decoupling between TE- and TM-modes which decreases with increasing frequency.

Optical wave chaos in a macroscopic open resonator

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In recent years, microwave resonators have lead to tremendous experimental progress in the field of wave chaos. Most notably, stadium type billiards have been spectacularly successful. *Optical* wave chaos has not benefited from this, and progress here has been modest. The primary reason for this is the inherent difficulty in producing and controlling stadium-type resonators in the optical range. Recently, we have put forward a radically new approach to optical wave chaos, using a system that is highly macroscopic, therefore offers unsurpassed control, and allows a wealth of different experimental techniques. We have demonstrated that an open, three-mirror, folded optical cavity can show chaotic dynamics [1]. The key ingredient in this resonator is a *curved* folding mirror, introducing, even at very modest numerical aperture, considerable aberrations through its use at non-normal incidence. These aberrations cause the paraxial approximation to be violated, making the wave equation describing the intra-cavity field non-separable. The *strength* of these aberrations crucially influences the chaos in the system. These aberrations, as introduced by the folding mirror, may be modified in two distinct ways: by varying the folding angle of the resonator (changing the angle of incidence on the curved folding mirror), and by varying the effective numerical aperture of the system. In line with expectations, increasing the folding angle of the resonator from $0°$ to $90°$ shows a smooth transition from a non-chaotic to a chaotic resonator. Also, increasing the effective numerical aperture, thereby preferentially exciting modes that have appreciable amplitude far from the optical axis of the resonator, leads to increased chaotic behaviour. We expect that this highly versatile and promising approach will boost research into optical wave chaos. At the same time, it may serve as an excellent model system to study many intriguing phenomena in wave chaos in general.

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Towards chaos criterion in quantum field theory

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Chaos criterion for quantum field theory in terms of functional integral is proposed. Its correspondence with known criteria in semi-classical regime of finite dimension quantum mechanics is shown. It is demonstrated for real scalar field forth power interaction Lagrangian that chaos corresponds to spontaneous symmetry breakdown, degeneration of vacuum state and confinement for lattice formulation.

Modelling of chaos enforced instanton tunnelling

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Chaos assisted tunnelling has been attracting much attention. Instanton technique mainly used in quantum field theory describes quantum tunnelling between vacua. It seems to be important to investigate influence of chaos on properties of instanton transitions. In this work on the example of model system we demonstrate that chaos enforces the rate of instanton tunnelling and increases density of instanton gas in dilute instanton gas approximation.

Chaotic wavefunction statistics in quantum map and billiard

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In the chaotic quantum map, the overlap distributions between the eigenstates $|n>$ and a probe state $|k>$ are well described by the random matrix theory (RMT) in most cases. However, if the probe state is localized on a fixed point, the overlap distribution shows a strong deviation from the RMT expectation, which is a result of the **scarring phenomena** in chaotic eigenstates [1]. In our recent paper [2], a joint-probability distribution has been developed for the overlaps with the eigenstates $|k>$ of harmonic oscillator which act as the probe states. This is a kind of scarred RMT, and has explained successfully the scarring effect in tunneling rate distributions. This joint-probability distribution contains all information of wavefunction statistics near the fixed point, and has an interesting structure which is directly related to the corresponding classical dynamics. We find that one linear combination of $|k>$'s is an optimized probe state. Following Vergini *et al* [3] we call this the **scar state**, and the statistics of its overlap with chaotic states shows maximum deviation from RMT. This scar state shows that the scarring appears along the stable and unstable manifolds. We show that this joint distribution works very well not only in the perturbed cat map, but also in the boundary function of billiard. This means that the chaotic wavefunction statistics are characterized by the Lyapunov exponent and the angle between the stable and unstable manifolds of the fixed point. In the billiard case, we investigate the relation between the statistics of the boundary functions and those of the real eigenfunctions. This study gives the way to see which eigenstate should be scarred or not in a statistical way.

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Spatial pattern formation in Turing systems

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In the 1950s British mathematician Alan Turing proposed a simple reaction-diffusion system describing chemical reactions and diffusion to account for morphogenesis [1]. Due to the complexity of nature, researchers have not yet succeeded in developing a Turing system based model that would describe morphogenesis in essence, although specific examples such as the skin coloring of animals have been modeled using Turing systems.

Turing systems show a very rich behavior from the pattern formation point of view, which means that by numerically solving these mathematically defined systems we obtain a variety of spatial patterns in two dimensions and structures in three dimensions, varying from spots to stripes and from lamella to chaotic structures. In this study we have obtained results for a three-dimensional system, which, to the authors' knowledge, has never before been studied using numerical simulations [2]. We have also studied the basic characteristics of a three-dimensional Turing system and compared them with those of a two-dimensional one. In 3D the morphological development becomes more interesting and complex. We have also studied the transition between two and three dimensional morphologies. In addition, the robustness of Turing structures has been investigated against Gaussian random noise.

Our motivation for studying the Turing systems is biological. We have shown how connections between certain points can be grown by using a Turing system with sources of chemicals. The resulting connected network has many interesting properties, and in the future work we will concentrate on a Turing system and an active random walker model combined to explain some of the features of neural patterning, i.e., how neurons establish connections to other neurons.

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Spectral statistics of R-R intervals in ECG

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The statistical distribution of R-R intervals and Delta-3 statistics provide useful information on the structure of fluctuation-patterns in heart-rate dynamics. We compare the spectral analysis of R-R intervals with the prediction of the random matrix theory.

Pseudointegrable systems with growing surface roughness

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We study numerically the level statistics of pseudointegrable billards with different degrees of surface roughness, expressed by the different genus numbers g and calculate the eigenvalues and the eigenfunctions under Dirichlet and Neumann boundary conditions.

We study the distance distribution $P(s)$ of the eigenvalues and related quantities such as half of the second moment $I_0 = (1/2)\int s^2 P(s)ds$, the spectral rigidity Δ_3 and the number variance Σ^2. The considered systems possess genus numbers between $g = 2$ and $g \approx 700$. Their shapes range from very simple two-step systems to systems with many small teeth at the boundary and fractal drums with complicated scaling surface roughness. The eigenfunctions are analyzed according to their localization volume $V_{\rm loc}$ (inverse participation ratio), their amplitude distribution $P(\psi)$ and the momentum space.

For lower energies we can distinguish several energy windows with different behavior. In some windows, the values of I_0 come close to the value of the Poisson distribution $I_0 = 1$. The eigenfunctions in these regimes are either localized (with $V_{\rm loc} \leq 0.2\,V_0$, where V_0 is the surface of the system) or relatively regular functions with $V_{\rm loc} \approx 4/9 V_0$, the value of a sine or cosine function. In other energy windows, the values of I_0 come closer to the Wigner limit of $I_0 \approx 0.637$ and the eigenfunctions approach chaotic functions with a Gaussian distribution $P(\psi)$.

Except for some unusual nearly-symmetric system shapes, we find an asymptotic energy regime above some non-universal limiting energy value. In the asymptotic regime, the genus number g seems to be the determining parameter that governs the level statistics. With increasing g, the behavior of the systems in the asymptotic regime changes from Poisson-like to Wigner-like behavior.

3D BEC solitons under transverse confinement

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The Bose-Einstein condensate (BEC) of a dilute gas of bosons is well described by the 3D Gross-Pitaevskii equation (3D GPE), that is a nonlinear Schrodinger equation. By imposing a transverse confinement the BEC can travel only in the cylindrical axial direction. We show that in this case the BEC with attractive interaction admits a 3D bright soliton (shape invariant) solution which generalizes the text-book one, that is valid in the 1D limit (1D GPE). We investigate stability and collective oscillations of this Bose-condensed bright soliton. By comparing the results of the 3D GPE with those of the 1D GPE, we find a good agreement only for weakly-interacting bright solitons. In particular, contrary to the 1D case, the 3D bright soliton exists only below a critical attractive interaction which depends on the extent of confinement. Analyzing the macroscopic quantum tunneling of the bright soliton on a Gaussian barrier we find that its interference in the tunneling region is strongly suppressed with respect to non-solitonic case, moreover the reflected and transmitted matter waves are not solitonic. Finally, we show that the collapse of the soliton is induced by the scattering on the barrier or by the collision with another matter wave when the density reaches a critical value, for which we derive an accurate analytical formula.

Influence of stochastic conditions on self-organization in irradiated materials

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The theoretical investigation for the role of nonlinear couplings between elements of damage structure of irradiated crystal in its self-organization and influence of external fluctuations of defect generation rate on this process is developed. It is shown that nonlinear couplings determine kind and parameters of the arising structure or terminal behavior namely concentration decomposition in alloys, oxygen impurity redistribution and its nonhomogeneous precipitation in silicon, breaks on curves of the creep-rate dependence on temperature and dose, creep and swelling rate oscillations. Parameters of the arising structures and conditions of its development are obtained. It is determined that the fluctuations of irradiation conditions are most strongly apparent in dynamic behavior of system. As a result new unstable regions and new dynamic behavior that do not have deterministic analog appear. The cause of it is strong interaction between fluctuations that is accompanied by their essential increase. Fluctuations in irradiated materials are substantially nonequilibrium, since the main cause for generation of defects, i.e. irradiation, is an external factor. Unlike the equilibrium internal fluctuations, they are not inversely proportional to a certain power of the characteristic size of the system. On the contrary, significant deviations of the density of point defects from its average in the process of irradiation are quite common. Especially great is the role of random external disturbances in systems with nonlinear links, in processes of radiation-induced transformations, which have a threshold nature and exhibit branching points, where qualitatively different ways of evolution are possible. On the one hand, the stochastic nature of defects is related to the fluctuations of the flux of impinging particles (neutrons, ions, etc.). On the other hand, the formation of defects in real crystal correlates with the disposition of various imperfections in it, such as dislocation, grain boundaries, precipitates, etc., which are distributed in the crystal randomly.

Experiments on chaotic systems with a mixed phase space

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Experimental and theoretical studies devoted to investigation of chaotic systems with a mixed phase space such as H atoms interacting with strong microwave fields and quantum billiards are reported. We describe combined experimental-theoretical study to realize and control the behavior of nonlinear common (joint) resonances that occur when excited H atoms are driven by linearly polarized microwave field consisting of two commensurate frequencies. We show how the relative phase of the field can be used to affect the resonances in the mixed phase space and, by this means, control the ionization probability. We also study the regime of quantum cantori in quarter-stadium billiards. Experimentally, the stadium billiard is simulated by a thin quarter-stadium microwave cavity. The eigenfunctions of the quarter-stadium billiard with the parameter $\varepsilon = 0.1$ are reconstructed in the cantori regime $N = 7 - 63$ using a field perturbation technique and a circular wave expansion method. We show that in the quantum cantori regime the rescaled localization length of the eigenfunctions does not depend on average on N. Support by KBN grant No. 2 P03B 023 17 is acknowledged.

Determining the ordered or chaotic nature of orbits in Hamiltonian systems by the smaller alignment index (SALI) method

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We apply the method of the Smaller Alignment Index (SALI) for determining the ordered or chaotic nature of orbits, on some examples of Hamiltonian systems and symplectic maps. The computation of the SALI for a sample of initial conditions allows us to distinguish easily between regions in the phase space where ordered or chaotic motion occurs. The computation of SALI is performed rather easily: for a given orbit we follow the evolution in time of two different initial deviation vectors computing the norms of the difference (parallel alignment index) and the addition (antiparallel alignment index) of the two normalized vectors. The time evolution of the smaller alignment index reflects clearly the chaotic or ordered nature of the orbit. In general the SALI tends to zero for chaotic orbits, while it fluctuates around non-zero values for ordered orbits. The relationship of this index with the Lyapunov exponent is also discussed.

Periodic trace formulae for elastodynamic resonators

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We report on progress in Gutzwiller periodic orbit theory for elastodynamical resonators. The equations we solve are the conventional equations from continuum mechanics which are used to model phonon properties of mesoscopic structures as well as macroscopic bodies such as cars and bridges. The wave chaos is quantified in Gutzwiller theory which predicts the mode density using periodic classical orbits.

The complexity arises from the vectorial character of the elastic wave equation and the boundary conditions which for free boundaries leads to wave splitting and a multitude of surface waves. Nevertheless when these are incorporated a Gutzwiller description of the spectral density as a sum over periodic orbits arise [1]. Formally from this the diagonal approximation can be made giving random matrix behaviour as observed also in experiments.

The talk will concentrate on solid bodies with simple shapes as for example discs and rectangles. Appropriate trace formulae for the density of modes differ here from the Gutzwiller trace formula in that sums over isolated orbits are replaced by sums over families of orbits.

We shall pay particular attention to orbits with incidence angles beyond the critical as these turn out to be less attenuated than ordinary ray splitting orbits. Furthermore surface orbits will be included as some for elastodynamics are virtually undamped at high frequencies. Similar surface orbits are also seen for impedance boundary condition in electrodynamics. Such behaviour does not occur in conventional scalar quantum mechanics where surface orbits are only important at low frequencies.

The ray splitting itself can be thought to introduce mixing in otherwise regular systems or more precisely a much larger number of periodic orbits arise. As a consequence even for simple shapes spectral statistics from integrable to chaotic can be observed contrary to scalar quantum mechanics.

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Steady plane Stokes flow with long range correlation, fractal Fourier spectrum, and anomalous transport

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We demonstrate a class of two-dimensional steady viscous flows which have singular continuous (fractal) Fourier spectra. Such flows represent a novel intermediate stage between order and Lagrangian chaos: the motion of individual fluid particles in them is neither entirely correlated nor completely disordered. We describe qualitatively and numerically the solutions of the two-dimensional Stokes equations for a flow of a viscous incompressible fluid past a periodic array of solid cylinders. The equations are solved on a square with periodic boundary conditions; the central part of the domain is occupied by the solid body with no-slip condition on its border. For the observable associated with the moving fluid particle (e.g. instantaneous velocity of the particle), the temporal autocorrelation function decays; the decay rate is prescribed by the power law. Estimation of the Fourier spectrum indicates that the spectral measure is supported by the multifractal set. Furthermore, this simple steady two-dimensional flow pattern exhibits anomalous transport properties. Existence of this unusual state is caused by the power-like singularities of passage times which develop along the particle paths near the surface of the solid body.

Pulses in high-order dissipative models of active systems

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Many important problems of applied mathematics and physics may be reduced to analyses of single evolution PDE with dissipation (like the Kuramoto-Sivashinsky equation in chemical physics and fluid mechanics). In order to balance internal energy influx such equations include high-order (> 2) spatial derivatives responsible for the energy dissipation. We consider two high-order dissipative models of that kind, one of which describes spinning combustion front, and the other describes extended elementary particle. The equations have similar properties. They are invariant with respect to constant shift of function of interest and contain nonlinear internal source of energy. As a result of a balance between the source and dissipation, nonlinear dissipative structures emerge in the form of pulses. The first equation in question, which is fourth-order, leads to stable standing pulse representing extended elementary particle, that is particle with finite size (not just physical point as in classical quantum mechanics). This model develops the work of Sivashinsky (Nuovo Cimento, 77A, 1983, 21) where the classical Hamilton-Jacobi (HJ) equation was extended using fourth-order dissipation. That approach, however, led to unstable pulse. We show that the Sivashinsky equation can be modified further to include nonlinear terms in order to ensure stability of the pulse. Remarkably, the original aim of his model was to put forward possible explanation of quantum randomness by deterministic law. This aim was achieved, because the extended HJ equation had global chaotic solutions. Whether or not chaos exists within our model is not clear yet. The second equation in question, which is sixth-order, models spinning combustion waves. It has fundamental solution in the form of travelling pulse. We present and discuss the results of numerical analyses of the models.

Spectral statistics of a spin-1/2 coupled quartic oscillators

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The quantum spectral statistical properties, i.e. the nearest neighbor level spacing distribution (NNSD), the mode fluctuation distribution (MFD), the spectral rigidity etc. of a spin-1/2 particle in two- and three-dimensional coupled quartic oscillator potentials are numerically studied. Changing coupling parameters the system is continuously transformed from an integrable one to chaotic ones. In the chaotic regime, selecting non-zero parameters, various kinds of the Gaussian ensembles(GEs): GOE, GUE and GSE can be achieved [1]. Especially, it is a rare example for the GSE. We have been interested in the Gaussian ensembles in the aspect of the intermediate system between the integrable one and the GEs. In order to have reliable statistics of quantum levels, it is necessary to evaluate thousands of energy levels from the ground state without missing any. We compute the quantum energy levels by numerical diagonalization of the truncated matrix of the Hamiltonian in the basis of harmonic oscillators. Most of our calculations start with the system of the superposition of two or three truncated harmonic oscillators [2,3]. The method with the spherical Bessel functions and the surface harmonics has been also developed. Several types of the interpolation formula of the Poissonian and the Wigner-like distribution of each GEs for the NNSD are examined. It is found that the MFD is more sensitive to the integrability of the system than the chaoticity as in the case of a spin-less particle. The distribution of $n(>1)$ nearest neighbors is also examined [4].

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Neutrally bouyant particles and bailout embeddings in three-dimensional flows

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We use the bailout embeddings of three-dimensional volume-preserving maps to study qualitatively the dynamics of small spherical neutrally buoyant impurities suspended in a time-periodic incompressible fluid flow. The accumulation of impurities in tubular vortical structures, the detachment of particles from fluid trajectories near hyperbolic invariant lines, and the formation of nontrivial three-dimensional structures in the distribution of particles are predicted.

Semiclassical analysis of Wigner functions

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The Wigner functions are quantum analogues of the classical phase space density. They are functions of both momenta and coordinates and are typically obtained from wavefunctions that are usually given as functions of only either coordinates or momenta. The motivation for this work was the question whether using the Wigner functions one can perform the formalism of quantum mechanics in the full phase space without referring to the actual wavefunction. While the formal equations for such a procedure are well known in the form of Moyal brackets, these can hardly be used directly as, in the general case, they represent a differential equation of an infinite order. While a naive semiclassical expansion of the Wigner function does make the equations in the Moyal approach to become of finite order for any order of the Planck's constant, such an approach fails immediately after the lowest order solution, which itself is the delta distribution on the invariant component of the classical phase space of the system. The Moyal bracket can, however, be rewritten in the form of an integral equation. In analogy to the usual WKB semiclassical approach for the wavefunction we expand the phase of the Wigner function in terms of the powers of the Planck's constant. If we now perform the analysis of the integral equation around the stationary point as given by the lowest order contribution to the phase of the Wigner function, we obtain a set of recursive equations that, at each order of the Planck's constant, contain only a finite number of terms. Unlike in the previously mentioned naive Moyal approach this set of equations allows us to find a solution to all orders of the Planck's constant. One should also note that such a separation does not happen if one inserts the phase expansion directly into the Moyal bracket. We apply this approach to the harmonic oscillator. We find the energy eigenvalues to all orders of the Planck's constant and hence obtain them exactly, while for the Wigner functions themselves we show that even the low order solutions match the exact ones very nicely and that they also agree with certain analytical approximations for Legendre polynomials.

‘Persistent currents’ and eigenfunctions in thin microwave resonators with broken time reversal symmetry

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A magnetic flux through a mesoscopic metallic ring gives rise to a persistent current which can be detected via characteristic oscillations of the magnetization depending on of the applied field [Lévy et al., Phys. Rev. Lett. **64**, 2074 (1990)]. In this paper a direct visualization of such persistent currents in a microwave analogue experiment is reported, making use of the analogy between the probability density current in the quantum-mechanical system and the Poynting vector in the corresponding electromagnetic one. To break time-reversal symmetry, a small ring of a ferrite material in a static external magnetic field was introduced into the resonator. In our analysis of the experimental data we employ the off-diagonal elements of the scattering matrix. Due to the small size of the ferrite compared to the resonator size the symmetry was partially broken in the sense that the real and the imaginary parts of the wave functions are not equally large on average. The statistical properties of the wave function's real and imaginary parts $\mathrm{Re}(\psi)$, $\mathrm{Im}(\psi)$, respectively, as well as the distributions of the total wave function's amplitudes $|\psi|$ and the statistical properties of the probability density currents were also examined and compared to the theoretical predictions.

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Can chaos improve stability of quantum algorithms?

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Stability of quantum computers to imperfections is of crucial importance for their proper working. Indeed, the power of a quantum computer relies on an interference and entanglement and can be as such very sensitive to perturbations, whether they result from the coupling with the environment or from the faulty algorithm itself. For successful realization of even modest-size quantum computer, understanding sensitivity to the perturbation is very important. As a measure of stability we choose to study fidelity. First, we expand fidelity in terms of correlation functions of a generator of perturbation. Decay of fidelity is related to the integral of a correlation function. Contrary to common intuition, we argue that for "chaotic" quantum systems, where correlation functions decay fast, the fidelity falls exponentially with time. In turn, for "integrable" systems with a non-decaying correlation function, the dependence is Gaussian. Although simple, this novel way of looking at fidelity greatly facilitates understanding of its behavior under perturbation. To demonstrate the above theory we apply it on a Quantum algorithm for discrete Fourier Transformation (QFT) algorithm. For a constant static random matrix perturbation on the whole Hilbert space of a computer, the fidelity falls with the number of qubits n as $\exp(-cn^3)$. By a simple change of an algorithm, so that the correlation function decays faster, we change this behavior into $\exp(-cn^2)$. For instance, this would mean improving fidelity in the case of $n = 100$ qubits from 0.1 for plain FFT algorithm to 0.9 for our improved one.