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On the Weibull distribution appeared in chaotic systems

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Various kinds of scaling laws are often observed in complex dynamical systems; for instance, SOC and punctuated equilibrium near the edge of chaos, 1/f spectrum, long time tails and anomalous diffusion in non-hyperbolic systems, where the self-similarity structures play essential roles in dynamical space and induce the breaking of central limit theorem for gaussian regime.

In the present paper we discuss three complex dynamical systems with non-gaussian scaling regime described by the Weibull distribution. Though the universality of Weibull distribution functions has not yet been made clear, but it is surmised that the Weibull regime is omnipresent in the systems under consideration.

1. Hamiltonian systems

Weibull distributions in hamiltonian dynamics were studied in (1): Mixmaster universe model [Prog. Theor. Phys. **98** No.6 (1997), 1225], and in (2) Cluster formation [Prog. Theor. Phys. **103** (2000), 519 ; Suppl. No.139 (2000), 1]. In particular, it was shown that the Arnold diffusion can be explained in terms of the universality of Log-Weibull distributions.

Here we discuss a new simplified model of random potential scattering, where Weibull distributions are numerically obtained.

2. Time dependent non-stationary chaotic map

It is known that the modified Bernoulli map reveals strong intermittency and anomalous large deviation properties [Prog. Theor. Phys. **99** (1989), 149 and **90** No.3 (1993), 547].

Here we consider the time dependent process where the system parameter is varying in the course of time. Surprising results are the followings ; the time dependent intermittency obeys the Weibull distribution in the intermediate long time scale, but in the intrinsic long time scale it obeys the Log-Weibull distribution.

3. A model of financial market

It is known in econophysics studies that the distribution of returns (change of price) obeys the stable (*Lévy*) distribution, but many theoretical models for financial market do not always display the stable law.

Here we study a multi-agent model [Chaos, Solitons & Fractals **11** (2000), 1077 ; 1739, Physica A **287** (2000), 507], and point out that the return distributions are well adjusted by the Weibull distribution function in wide parameter range.

Quantum chaos: Universal versus system-specific fluctuations

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It is by now well established that spectral fluctuations of quantum systems whose underlying classical motion is chaotic exhibit universalities. On the other hand, it is also well known that there are parameters characterizing a particular chaotic system. The purpose of the two lectures will be to illustrate, partly by studying particular examples, what makes some properties system-dependent and other universal. The general framework will be periodic orbit and random matrix theories.

Emphasis will be put on two properties not so widely discussed so far: i) spectral spacing autocorrelations, ii) total energies of (non-interacting) fermion systems. Besides a general discussion, two cases will be treated in detail: 1) zeros of the Riemann zeta function (an example of a 'chaotic' system for which all 'classical' information is well known), 2) the fluctuation of the binding energy of atomic nuclei (a system which is mainly regular but for which there is evidence that a (small) chaotic part is present).

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Soliton–like solutions of higher order wave equations of the KdV type

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In this work we study second and third order approximations of water wave equations of the KdV type. First we derive analytical expressions for solitary wave solutions for some special sets of parameters of the equations. Remarkably enough, in all these approximations, the form of the solitary wave and its amplitude–velocity dependence are identical to the sech^2–formula of the one–soliton solution of the KdV. Next we carry out a detailed numerical study of these solutions using a Fourier pseudospectral method combined with a finite–difference scheme, in parameter regions where soliton–like behavior is observed. In these regions, we find solitary waves which are stable and behave like solitons in the sense that they remain virtually unchanged under time evolution and mutual interaction. In general, these solutions sustain small oscillations in the form of radiation waves (trailing the solitary wave) and may still be regarded as stable, provided these radiation waves do not exceed a numerical stability threshold. Instability occurs at high enough wave speeds, when these oscillations exceed the stability threshold already at the outset, and manifests itself as a sudden increase of these oscillations followed by a blowup of the wave after relatively short time intervals.

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Localized oscillations in 1-dimensional nonlinear media and homoclinic orbits of invertible maps

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In recent years, a very interesting phenomenon has captured the imagination of many nonlinear scientists: The occurrence of stable, localized oscillations in 1 - and 2 - dimensional lattices. These oscillations have been termed discrete breathers, in analogy to similar solutions found in certain completely integrable continuous systems, like the sine Gordon and the Nonlinear Schrödinger partial differential equations.

In these lectures, we will first review the physical models in which discrete breathers were first discovered and studied. We will then outline the work of Aubry and MacKay who rigorously established the existence of stable discrete breathers in a wide class of infinite chains of linearly coupled anharmonic oscillators. It is interesting that, in many cases, breathers are indeed a discrete phenomenon, as they are not expected to exist in the continuum limit. Furthermore, they have also been recently observed in several experiments, notably some involving arrays of coupled Josephson junctions.

We shall demonstrate that discrete breathers correspond, in fact, to homoclinic orbits at the intersections of invariant manifolds of a saddle point, lying at the origin of a 2N - dimensional map in Fourier space. Exploiting this geometric approach, we will show that discrete breathers can be accurately approximated and even classified using ideas of symbolic dynamics.

In this way, a great variety of such forms (also called multibreathers) can be constructed most of which are found to be linearly unstable. Thus, developing methods for computing homoclinic orbits of invertible maps allows us to obtain accurate representations of discrete breathers and stabilize (or destabilize) them using techniques of continuous feedback control.

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Efficient quantum computing of complex dynamics

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In interacting many body systems such as nuclei, complex atoms, quantum dots, and quantum spin glasses, the interaction leads to quantum chaos characterized by ergodicity of eigenstates and level spacing statistics as in Random Matrix Theory. In this regime, a quantum computer eigenstate is composed by an exponentially large number of quantum register states and the computer operability is destroyed. Here we model an isolated quantum computer as a two-dimensional lattice of qubits (spin halves) with fluctuations in individual qubit energies and residual short-range inter-qubit couplings. We show that above a critical inter-qubit coupling strength, quantum chaos sets in and this results in the interaction induced dynamical thermalization and the occupation numbers well described by the Fermi-Dirac distribution. This thermalization destroys the noninteracting qubit structure and sets serious requirements for the quantum computer operability. We then construct a quantum algorithm which uses the number of qubits in an optimal way and efficiently simulates a physical model with rich and complex dynamics. The numerical study of the effect of static imperfections in the quantum computer hardware shows that the main elements of the phase space structures are accurately reproduced up to a time scale which is polynomial in the number of qubits. The errors generated by these imperfections are more significant than the errors of random noise in gate operations.

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M-theory and particle physics

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Recent developments in string and M-theory are reviewed with an emphasis on particle physics implications. Aspects of non-perturbative extended objects - branes are introduced. The focus is on the role these objects play in the construction of new four-dimensional solutions of string theory with the structure of the standard model and three families of quarks and leptons. A beautiful relationship of these constructions to purely geometric one, as an M-theory compactified on special holonomy spaces is highlighted.

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Chaos and what to do about it: An overview

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That deterministic dynamics leads to chaos is no surprise to anyone who has tried pool, billiards or snooker - that is what the game is about - so we start our course about what is chaos and what to do about it by a game of pinball. This might seem a trifle trivial, but a pinball is to chaotic dynamics what a pendulum is to integrable systems: thinking clearly about what is "chaos" in a pinball will help us tackle more difficult problems, such as computing diffusion constants in deterministic gases, or computing the Helium spectrum.

We all have an intuitive feeling for what a pinball does as it bounces between the pinball machine disks, and only highschool level Euclidean geometry is needed to describe the trajectory. Turning this intuition into calculation will lead us, in clear physically motivated steps, to almost everything one needs to know about deterministic chaos: from unstable dynamical flows, Poincaré sections, Smale horseshoes, symbolic dynamics, pruning, discrete symmetries, periodic orbits, averaging over chaotic sets, evolution operators, dynamical zeta functions, Fredholm determinants, cycle expansions, quantum trace formulas and zeta functions, and to the semiclassical quantization of helium.

Reference

Read chapter 1 and appendix A of P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, `http://www.nbi.dk/ChaosBook/`.

Dynamics, qualitative

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Confronted with a potentially chaotic dynamical system, we analyze it through a sequence of three distinct stages; diagnose, count, measure. First, we determine the intrinsic *dimension* of the system - the minimum number of degrees of freedom necessary to capture its essential dynamics. If the system is very turbulent (its attractor is of high dimension) we are, at present, out of luck. We know only how to deal with the transitional regime between regular motions and weak turbulence. In this regime the chaotic dynamics is restricted to a space of low dimension, the number of relevant parameters is small, and we can proceed to the second step; we *count* and *classify* all possible topologically distinct trajectories of the system. If successful, we can proceed with the third step: investigate the *weights* of the different pieces of the system.

In this lecture qualitative dynamics of simple stretching and mixing flows is used to introduce Smale horseshoes and symbolic dynamics, and the topological dynamics is encoded by means of transition matrices/Markov graphs.

We learn how to count and describe itineraries. While computing the topological entropy from transition matrices/Markov graphs, we encounter our first zeta function.

By now we have covered for the first time the whole distance from diagnosing chaotic dynamics to computing zeta functions. Historically, these topological zeta functions were the inspiration for injecting statistical mechanics into computation of dynamical averages; Ruelle's zeta functions are a weighted generalization of the counting zeta functions.

Reference

Read chapters 2, 3, 10 and 11 of P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, `http://www.nbi.dk/ChaosBook/`.

Global dynamics

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This lecture is the core of the course: we discuss the necessity of studying the averages of observables in chaotic dynamics, and cast the formulas for averages in a multiplicative form that motivates the introduction of evolution operators.

In chaotic dynamics detailed prediction is impossible, as any finitely specified initial condition, no matter how precise, will fill out the entire accessible phase space (similarly finitely grained) in finite time. Hence for chaotic dynamics one does not attempt to follow individual trajectories to asymptotic times; what is possible (and sensible) is description of the geometry of the set of possible outcomes, and evaluation of the asymptotic time averages. Examples of such averages are transport coefficients for chaotic dynamical flows, such as the escape rate, mean drift and the diffusion rate; power spectra; and a host of mathematical constructs such as the generalized dimensions, Lyapunov exponents and the Kolmogorov entropy. We shall now set up the formalism for evaluating such averages within the framework of the periodic orbit theory. The key idea is to replace the expectation values of observables by the expectation values of generating functionals. This associates an evolution operator with a given observable, and leads to formulas for its dynamical averages.

If there is one idea that you should learn about dynamics, it happens in this lecture(s) and it is this: there is a fundamental local - global duality which says that (global) eigenstates are dual to the (local) periodic geodesics. For dynamics on the circle, this is called Fourier analysis; for dynamics on well-tiled manifolds this is called Selberg trace formulas and zeta functions; and for generic nonlinear dynamical systems the duality is embodied in trace formulas, zeta functions and spectral determinants that we will now introduce. These objects are to dynamics what partition functions are to statistical mechanics. The bold claim is that once you understand this, classical ergodicity, wave mechanics and stochastic mechanics are nothing but special cases, to be worked out at your leisure.

The strategy is this: Global averages such as escape rates can be extracted from the eigenvalues of evolution operators. The eigenvalues are given by the zeros of appropriate determinants. One way to evaluate determinants is to expand them in terms of traces, log det = tr log. The traces are evaluated as integrals over Dirac delta functions, and in this way the spectra of evolution operators become related to periodic orbits.

The rest of the course is making sense out of this objects and learning how to apply them to evaluation of physically measurable properties of chaotic dynamical systems.

Reference
Read chapters 5, 6, 7, 8 and 9 of P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, `http://www.nbi.dk/ChaosBook/`.

Cycle expansions: Semiclassical quantum mechanics

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In last lecture we have derived a plethora of periodic orbit trace formulas, spectral determinants and zeta functions. Now we learn how to expand these as cycle expansions, series ordered by increasing topological cycle length, and evaluate average quantities like escape rates. These formulas are exact, and, when the winds are kind, highly convergent. The pleasant surprise is that the terms in such expansions fall off exponentially or even faster, so that a handful of shortest orbits suffices for rather accurate estimates of asymptotic averages.

The course now shifts gear to recent advances in the periodic orbit theory of chaotic, non-integrable systems, and the modern generalization of the De Broglie - Bohr quantization of hydrogen atom.

Instead of quantizing by suspending standing-wave configurations on stable Keplerian orbits, one suspends the standing-wave configurations on the infinity of unstable orbits. Such unstable periodic orbits are observed experimentally in the helium atom, the hydrogen in strong external fields, and other systems.

This is what could have been done with the old quantum mechanics if physicists of 1910's were as familiar with chaos as you by now are. The Gutzwiller trace formula together with the corresponding spectral determinant, the central results of the semiclassical periodic orbit theory, are derived.

The helium atom spectrum can then be computed via spectral determinants.

Reference
Read chapters 13, 21 and 22 of P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, `http://www.nbi.dk/ChaosBook/`.

Trace formulas for stochastic evolution operators

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Intuitively, the noise inherent in any realistic system washes out fine details and makes chaotic averages more robust. Quantum mechanical $\hbar$ resolution of phase space implies that in semi-classical approaches no orbits longer than the Heisenberg time need be taken into account. We explore these ideas in some detail by casting stochastic dynamics into path integral form and developing perturbative and nonperturbative methods for evaluating such integrals. In the weak noise case the standard perturbation theory is expansion in terms of Feynman diagrams. Now the surprise; we can compute the same corrections faster and to a higher order in perturbation theory by integrating over the neighborhood of a given saddlepoint exactly by means of a nonlinear change of field variables. The new perturbative expansion appears more compact than the standard Feynman diagram perturbation theory; whether it is better than traditional loop expansions for computing field-theoretic saddlepoint expansions remains to be seen, but for a simple system we study the result is a stochastic analog of the Gutzwiller trace formula with the $\hbar$ corrections so far computed to five orders higher than what has been attainable in the quantum-mechanical applications.

Resume

A motion on a strange attractor can be approximated by shadowing the orbit by a sequence of nearby periodic orbits of finite length. This notion is here made precise by approximating orbits by primitive cycles, and evaluating associated curvatures. A curvature measures the deviation of a longer cycle from its approximation by shorter cycles; the smoothness of the dynamical system implies exponential (or faster) fall-off for (almost) all curvatures. The technical prerequisite for implementing this shadowing is a good understanding of the symbolic dynamics of the classical dynamical system. The resulting cycle expansions offer an efficient method for evaluating classical and quantum periodic orbit sums; accurate estimates can be obtained by using as input the lengths and eigenvalues of a few prime cycles.

To keep exposition simple we have here illustrated the utility of cycles and their curvatures by a pinball game. Glancing back, we see that the formalism is very general, and should work for any average over any chaotic set which satisfies two conditions: 1. the weight associated with the observable under consideration is multiplicative along the trajectory; 2. the set is organized in such a way that the nearby points in the symbolic dynamics have nearby weights.

Reference

Read articles in the "Chaotic Field Theory" section of
`http://www.nbi.dk/~predrag/papers/preprPOT.html`
and the take-home problem set for the next millennium in P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, `http://www.nbi.dk/ChaosBook/`.

Relaxation dynamics described by nonlinear Fokker-Planck equations: applications to human movement sciences

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Mesoscopic stochastic descriptions of many particle systems have frequently been used in the context of mean field nonlinear Fokker-Planck equations (Desai and Zwanzig 1978; Kuramoto 1984) and nonlinear Fokker-Planck equations related to nonextensive entropies (Plastino and Plastino 1995). We discuss a recent attempt to obtain a unified description for stochastic relaxation processes that are characterized by mean field interactions, on the one hand, and nonextensivity, on the other hand (Frank 2001b).

First, we consider the stationary case. To this end, we introduce the concept of inverse distortion functions (Frank, Daffertshofer 1999). For mean field models we obtain from inverse distortion functions implicit descriptions of stationary distributions that involve transcendent equations and can be used to address the phenomenon of multistability. For nonextensive systems we obtain cut-off and power-law distributions.

Second, we examine the transient case. We briefly discuss exact time-dependent solutions of systems related to the Sharma-Mittal entropy (Frank, Daffertshofer 2000). Then, H-theorems are developed on the basis of (i) inverse distortion functions and (ii) free energy measures (Shiino 1987,2001; Bonilla et al. 1998; Frank 2001a,b,2002; Kaniadakis 2001).

Third and finally, we study the stability of stationary distributions for a stochastic mean field model that is in line with the field theoretical description of neural activity proposed by Haken (1996) and Jirsa and Haken (1996) and can describe neural activity during rhythmic finger movements (Frank et al. 2000). We focus on two approaches: the transcendent equation approach and Lyapunov's direct method (Frank et al. 2001).

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Generalized extended self-similarity in turbulence and its scaling hypothesis

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One of eminent statistical characteristics of homogeneous, isotropic developed turbulence in the three dimensional system is the self-similar energy cascade in the wavenumber space. This results in the power law behavior $S_q(r) \sim r^{\zeta(q)}$ for the velocity structure function $S_q(r)$, the q-th moment of the velocity difference at two positions separated by r. $\zeta(q)$ is a universal function of q. Several years ago, it was found that even when the turbulence is not fully developed, i.e., the Reynolds number is not extremely high and the the scaling range where the above scaling law holds is not wide enough, scaling behaviors of $S_q(r)$ which has an extended form of that in developed turbulence holds. They are called the extended self-similarity (ESS) and the generalized extended self-similarity (GESS). I will talk about a phenomenological derivation of ESS and GESS, proposing a new scaling hypothesis on the basis of the large deviation theory of probability theory. Furthermore, using numerical and experimental data, I will examine the validity of the present approach.

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Noise-induced pattern dynamics and intermittency

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Intermittency is a quite ubiquitous phenomenon in nonlinear dynamics. The intermittency observed when a particular dynamical state undergoes the instability is called the modulational (often called the on-off) intermittency. Recently, an experimental confirmation of the on-off intermittency in the electrohydrodynamic convection in nematics under dichotomous noise was reported by John *et al.*. An eminent statistics of the observation is the intermittent generation of convective pattern.

In my talk, in order to elucidate the experiment I will first propose a phenomenological nonlinear stochastic model which has the structure of the Swift-Hohenberg equation for local convection variable with fluctuating threshold. Then, I will discuss results of numerical integration of the model equation associated with the intermittent emergence of convective pattern. Detailed analysis on the statistics of the intermittent pattern dynamics will be addressed.

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Hamiltonian chaos and statistical mechanics

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In systems of statistical mechanics, the chaotic dynamics is characterized by Lyapunov exponents which are of the order of the inverse of the intercollisional time between the particles. This time scale is the one of kinetics. Instead, the relaxation toward the thermodynamic equilibrium occurs on the longer time scale of hydrodynamics which is determined by transport properties such as diffusion, viscosity, or heat conductivity.

The connection between the chaotic and transport properties can be established thanks to the escape-rate theory or a newer theory which allows us to directly construct the hydrodynamic modes of diffusion and reaction-diffusion [1,2]. These hydrodynamic modes turn out to present fractal properties with a fractal dimension given in terms of the transport coefficients. The fractal character of the hydrodynamic modes results from the stretching and folding of nonequilibrium inhomogeneities induced by the chaotic dynamics. This mixing naturally leads to the entropy production expected from nonequilibrium thermodynamics [3].

The first lecture will be devoted to the relationship between chaos, transport properties, and entropy production. The second lecture will present the results of recent work on chaos in systems composed of many interacting particles [4].

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Experimental modeling of chaotic fields

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Physical description of natural laws is based on evolution equations of fields but their analytical formulation is often not possible for very complex chaotic phenomena [1]. In the lecture we show how a method of chaotic time series prediction can be generalized to statistical modeling of chaotic fields [2, 3]. For this purpose we assume that a record of the field is provided by an experiment and that the field amplitude $\phi(\mathbf{s})$ at a point of observation $\mathbf{s}$ is related to amplitudes in a surrounding region. We represent the field values in the surrounding region by the vector $\mathbf{g}(\mathbf{s})$, and describe the field evolution by the mapping equation

$$\phi(\mathbf{s}) = G(\mathbf{g}(\mathbf{s})), \tag{1}$$

in which the function G is estimated statistically. For this purpose N samples of the joint state vector $\{(\phi(\mathbf{s}_i), \mathbf{g}(\mathbf{s}_i)) = (\phi_i, \mathbf{g}_i); i = 1 \ldots N\}$ are first extracted from the given record. As an optimal non-parametric estimator of the field at point $\mathbf{s}$ we employ the conditional average, which is expressed by

$$\hat{\phi}(\mathbf{s}) = \mathrm{E}[\phi(\mathbf{s})|\mathbf{g}(\mathbf{s})] = \frac{1}{N}\sum_{n=1}^{N} B_n(\mathbf{g}(\mathbf{s}))\phi_n \tag{2}$$

Here $B_n(\mathbf{g}(\mathbf{s})) = w(\mathbf{g}(\mathbf{s}) - \mathbf{g}_n)/\sum_{k=1}^{N} w(\mathbf{g}(\mathbf{s} - \mathbf{g}_k))$ describes a similarity between the given vector $\mathbf{g}(\mathbf{s})$ and a sample $\mathbf{g}_n$, while w denotes a kernel function, such as Gaussian. The vector $\mathbf{g}(\mathbf{s})$ is considered as a given condition and is comprised from field values in the surrounding of point $\mathbf{s}$. During the calculation of the conditional average the surrounding can be arbitrary selected, which is an advantage of non-parametric estimator. To calculate a field distribution in some domain, the field must be first specified in a sub-domain. From given values the field distribution in the surrounding of sub-domain can be estimated by Eq. 2. The estimated values are then considered as given ones and the procedure of field estimation is iteratively continued. In the lecture optimal statistical methods for selection of surrounding region of a point $\mathbf{s}$ and self-organized determination of number N are explained. Various examples of estimated chaotic filed distributions, such as profiles of a rough surface [3, 4], charge density and electron temperature in turbulent ionization waves in plasma, etc, are demonstrated. The performance of the proposed statistical modeling is described by comparing correlation functions and spectra of experimentally recorded and estimated fields.

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Turbulent diffusion

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Dust, aerosoles, smoke and other air pollutions spread, in still air, by molecular diffusion. In addition there can be advection by atmospheric flow, either well directed in jets or winds, or nondirected in air turbulence. Diffusional spreading by turbulence is extraordinary effective. Instead of the well known linear growth of the mean square particle distance with time t

$$\sigma_t^2 = 6Kt \ , \ \text{regular diffusion} \ ,$$

σ_t^2 grows much faster under turbulent advection, according to a cubic t dependence

$$\sigma_t^2 = c\epsilon t^3 \ , \ \text{turbulent diffusion} \ .$$

This was discovered by Lewis Fry Richardson (1926, 1929) when he performed his ingenious study of the turbulent diffusivity K_{turb} . He presented data on impressively many scales. A bulk of later measurements confirmed his results, as e.g. balloon campaignes, cf. Lundgren (1981).

The anomalously fast particle distance growth could be explained from fluid dynamics (Navier-Stokes equations) by Grossmann and Procaccia (1984), Effinger and Grossmann (1984), Grossmann (1990) in mean field approximation. New results on dynamical Lagrangian time correlation decay (numerical, Grossmann and Wiele 1997 and analytical, Daems et al. 1999) stimulated efforts to even determine the absolute magnitude as characterized by the prefactor c in addition to the scaling exponent (Grossmann 2002).

In particular the memory effects in the time correlation decay have turned out to be very important. And turbulent intermittency implies additional scale dependence of the turbulent diffusivity (Grossmann 2002). A survey on the past development and on the most recent surprising findings is offered in the lecture.

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Turbulent correlation decay

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The appropriate tool to study the **dynamics** of a statistical system is its (stationary) time correlation function. This has also proven to be true for turbulent fluid flow. The observable quantities of interest are here the Lagrangian, scale dependent eddies $v(r)$, i.e., the Eulerian velocity differences $\mathbf{v}(\mathbf{r};\mathbf{x},t) = \mathbf{u}(\mathbf{x}+\mathbf{r},t) - \mathbf{u}(\mathbf{x},t)$. These objects are statistically time t and space $\mathbf{x}$ independent in stationary and homogeneous turbulence, but depend besides on scale r on the time lapse τ between two observations of an r-eddy at times t and $t+\tau$. By means of a continued fraction expansion the time correlation function can be uniquely expressed in terms of the static, stationary, time independent structure functions; but all orders of those are needed.

Analysis of the dynamical time correlation function $D(r,\tau) = \langle v(r)v(r,\tau)\rangle$ is presented. $D(r,\tau)$ was first studied in 1-pole approximation (Grossmann and Thomae 1982), because no estimate of higher order stationary structure functions was available then. Since Grossmann and Wiele (1997) provided numerical data for large Reynolds number, highly turbulent flow, it became clear that the memory effects surprisingly *reduce* the decorrelation time. The additional effects of turbulent intermittency were elucidated analytically in Daems et al. (1999). The main, rather unexpected results are:

i. The static multifractality of turbulent flow destroys dynamical scaling despite good scaling of stationary moments, i.e., of power laws in r of all p^{th} order structure functions. ii. The deviations from dynamical scaling are a direct measure of the strength of intermittency. iii. The scale dependence of the correlation decay rate can be expressed approximately in terms of the turbulent structure function of 2^{nd} order (Grossmann 2002).

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Turbulent heat convection

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Turbulent heat transport in fluid layers heated from below is one of the most intensely studied fluid flow problems. In 1900 Bénard detected interesting pattern formation in this system, in 1916 Rayleigh calculated the underlying instability. If the heating is increased further, chaotic motion and finally turbulence is observed in the bulk , surrounded by Blasius type boundary layers near the plates and walls. In the $50th$ and $60th$ power laws for the heat transport as a function of the thermal driving were suggested for the Rayleigh-Bénard system in accordance with other scaling behavior in turbulence,

$$Nu \propto Ra^{\beta} Pr^{\beta'} .$$

The Nusselt number Nu is the effective heat current including turbulent advection, nondimensionalised with the molecular heat current. The Rayleigh number Ra measures the thermal driving due to the temperature difference Δ across the fluid layer of height L, originating from buoyancy in the gravitational field g by thermal expansion α, which is counteracted by the fluid's kinematic viscosity ν and its thermal diffusivity κ,

$$Ra = \frac{\alpha g L^3 \Delta}{\nu \kappa} .$$

The Prandtl number $Pr = \nu/\kappa$ weighs the ratio of the space and time scales according to *molecular* momentum (ν) and energy (κ) transport. Recent measurements of increasing accuracy (Libchaber group (Castaing et al. 1989), Cioni et al. (1997), Chavanne et al. (1997), Niemela et al. (2000), Xu et al. (2000), Ahlers et al. (2001), Xia et al. (2002)) led to various unexpected results. A unifying theory explains all these data (Grossmann and Lohse 2000, 2001, 2002). In particular the theory's prediction is confirmed that simple power laws are insufficient to describe Nu versus Ra adequately. In the lecture the theory of heat convection in a very extended $Ra - Pr-$parameter space of up to 10 orders of magnitude is presented and discussed.

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Introduction to decoherence

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Quantum superpositions tend to decohere to mixtures, due to dissipative environmental influence. In particular, two superposed wave packets loose their relative coherence the faster the larger is their distance d. For sufficiently large d, the relative phase of packets is lost before any deformation of the shapes of the individual packets and any change of their distance become noticeable.

I shall illustrate the phenomenon by discussing recent efforts to take decoherence under experimental control (diffraction of Fullerenes in Vienna; superpositions of coherent states of microwave resonator modes in Paris; superpositions of wave packets of *Be* ions in Paul traps in Boulder; superpositions of states with counterpropagating supercurrents in Delft and Stony Brook).

All of these experiments observe superpositions of packets whose distance d is larger than the individual width λ, but the ratio d/λ achieved is still so moderate that the environment imposed decoherence time $\tau_{\rm dec}$ is, while shorter than the time scale $\tau_{\rm diss}$ of dissipative changes of d, still longer than typical oscillation periods $\tau_{\rm sys}$ of the isolated system, i.e. $\tau_{\rm sys} < \tau_{\rm dec} < \tau_{\rm diss}$. The appropriate theoretical treatment is thus based on Fermi's Golden Rule or, equivalently, Markovian master equations.

I illustrate golden rule type decoherence for the damped harmonic oscillator, using the simple master equation familiar from quantum optics. The important prediction is $\tau_{\rm dec}/\tau_{\rm diss} = (\lambda/d)^2$. The underlying perturbative treatment of the system-environment interaction requires the self-consistency condition $\tau_{\rm sys} \ll \tau_{\rm dec}$.

Emergence of classical behavior in the macroworld: meso- and macroscopic superpositions

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The golden-rule prediction breaks down once the distance d between the superposed wave packets is so large, compared to the quantum scale of reference λ, that $\tau_{\mathrm{dec}} \lesssim \tau_{\mathrm{sys}}$. The golden rule can therefore not be invoked to explain the notorious absence of quantum interference effects from the macroworld. In the limit $\tau_{\mathrm{dec}} < \tau_{\mathrm{sys}}$ decoherence obviously no longer is a weak-damping phenomenon. A simple solution of the system $\oplus$ environment Schrödinger equation becomes possible when $\tau_{\mathrm{dec}} \ll \tau_{\mathrm{sys}}$, the limit of relevance for superpositions of macroscopically distinct wave packets. The simplicity of that limit rests on the fact that in the full Hamiltonian $H = H_{\mathrm{sys}} + H_{\mathrm{bath}} + H_{\mathrm{int}}$ the free-system part H_{sys} becomes an effectively small perturbation. The decoherence time scale is then found to obey the power law $\tau_{\mathrm{dec}} \propto \hbar^{\mu}/d^{\nu}$ with positive exponents μ, ν. That law is a universal one, independent of the character of the system and the environment. It is only based on the interaction Hamiltonian H_{int} additively involving a large number of degrees of freedom such that the central limit theorem holds for the reservoir means met with.

After treating the universal asymptotics of the limit $\tau_{\mathrm{dec}} \ll \tau_{\mathrm{sys}}$ I shall briefly discuss the crossover from that interaction dominated regime to golden-rule type decoherence. The crossover is system specific; I shall rely on an exactly solvable model system, an harmonic oscillator coupled to a reservoir which itself consists of harmonic oscillators.

Quantum measurement

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Textbook wisdom has it that superpositions $|\psi\rangle = \sum_i c_i|\psi_i\rangle$ of eigenstates ψ_i of some observable of a microscopic object "collapse", under measurement of that observable, to a mixture, $|\varphi|\rangle\langle\varphi| \stackrel{\text{collapse}}{\longrightarrow} \sum_i |c_i|^2|\varphi_i\rangle\langle\varphi_i|$, with probabilities $|c_i|^2$ as in the original pure state but all coherences $c_i^*\ c_j$ for $i \neq j$ gone. Such collapse can be understood as due to unitary time evolution of a tripartite system comprising, besides the micro-object, a macroscopic pointer (idealizable as having a single degree of freedom), and a many-freedom environment. The Hamiltonian must allow micro-object and pointer to become entangled as

$$
\begin{aligned}
\varphi^{\text{obj}}(0)\psi^{\text{point}}(0) &= \left(\sum_i c_i\varphi_i\right)\psi^{\text{point}}(0) \\
&\rightarrow \textstyle\sum_i c_i\varphi_i\psi_i^{\text{point}} \qquad ,
\end{aligned}
$$

with the various pointer states ψ_i^{point} corresponding to macroscopically distinct pointer displacements. Concomitantly, the many-freedom environment will decohere the superposition to the mixture $\sum_i |c_i|^2|\varphi_i\ \rangle\langle\varphi_i| \otimes |\psi_i^{\text{point}}\rangle\langle\psi_i^{\text{point}}|$; inasmuch as the pointer states $|\psi_i^{\text{point}}\rangle$ are macroscopically distinct, the decay of the coherences $c_i^*\ c_j$ will occur with $\tau_{\text{dec}} \ll \tau_{\text{syst}}, \tau_{\text{diss}}$, i.e. might appear as practically instantaneous. By disregarding all information about (i.e. tracing over) the pointer, one has the textbook-wisdom mixture for the micro-object.

I shall present exactly solvable models both for both processes involved, entanglement and decoherence.

Brain dynamics

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Lecture 1: Some basic facts about the brain

The human brain is a highly complex system that is composed of about hundred billion neurons each of which may interact with up to ten thousand other neurons. The brain serves many purposes as recognition of our surrounding, steering of movement, emotions, etc. Roughly speaking, the brain consists of individual areas that serve specific purposes. But these areas are again strongly interconnected. The neurons can fulfill the tasks only by a high degree of cooperation. But who or what steers the neurons? Since two decades I have been propagating the idea that the brain acts by self-organization; a point of view that is accepted more and more in the scientific community. In particular, concepts of synergetics can be applied to brain function. Some typical aspects will be illustrated by examples from visual perception, such as recognition of faces and facial expressions, perception of ambiguous figures, hysteresis, etc.

Lecture 2: Some forms of integrate and fire models

I first present some basic features about the structure and dynamics of individual neurons including cell body, axons, dendrites, and synapses. The signal processing via spikes will be discussed. Then I will study the interplay between dendritic currents and axonal spikes (pulses) via the light-house model. The spikes are described by a phase angle, increasing in the course of time in analogy to a rotating light beam from a light- house through which the intervals between spikes are determined. The rotation speed depends on the inputs from other neurons. After elimination of the dendritic currents, we find a special form of an integrate and fire model, whereupon I will discuss a number of different forms that take into account arbitrary strengths of synapses and time-lags.

Lecture 3: Phase locking

I analytically study the solutions of a network of integrate and fire neurons of a general form. In particular, I study the circumstances under which phase locking becomes possible and discuss cases of coexistence between phase-locked neurons and other neurons.

Lecture 4: Associative memory

The general equations of the integrate and fire neurons under different sensory inputs are time-averaged over short intervals. It is shown how by suitable choices of the synaptic connections these models give rise to associative memory. Associative memory means that a set of incomplete data is completed by the system to a well-defined set depending on the partly given data. A connection with Kaniza figures is established, as well as with other models of associative memory.

Lecture 5: Activity patterns

Again using suitable time-averages instead of the equations for spikes, rate equations for spike sequences as well as for dendritic currents are established. Such equations are related to the Nunez equations in a simplified form and have been derived along different lines by Jirsa and Haken. When the dendritic currents are eliminated, we reobtain the Wilson- Cowan equations that in turn allow the derivation of spatio-temporal patterns of brain activity in terms of firing rates. If, on the other hand, the axonal pulses or pulse-rates are eliminated, we obtain a new type of equation first derived by Jirsa and Haken for the dendritic currents, which, in turn, are responsible for the electric and magnetic fields measured in EEGs and MEGs.

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`http://www.theo1.physik.uni-stuttgart.de/en/mitarbeiter/haken/`

Intermediate quantum-level statistics by means of quaternion representation of distributions

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There has been a long-standing subject in quantum-chaos studies to achieve a framework of energy-level statistics which interplolates between Poisson for integrable systems and those of Gaussian random matrix ensembles (GOE, GUE and GSE) for fully chaotic systems by means of a parameter [1]. In 1985, Yukawa [2] gave an excellent idea to construct such a one-parameter family of N-level distributions, based on the so-called level dynamics [3]: one considers an $N \times N$ Hermitian matrix of the form $H = H_0 + tV$ (a perturbation of H_0 by another V with perturbation strength t which is regarded as the"time"). His result can be summarized by a special form

$$P_{N,\beta}(x_1, x_2, .., x_N) = C_{N,\beta} \prod_{j<k} \left(\frac{(x_j - x_k)^2}{a^2 + (x_j - x_k)^2} \right)^{\beta/2} \qquad \beta = 1 \text{ GOE},\ 2 \text{ GUE},\ 4 \text{ GSE}. \tag{1}$$

Here, the parameter that appears is a which interpolates the two limits as

$$a \to 0 \quad \text{Poisson limit}, \qquad a \to \infty \quad \text{Gaussian limit (Wigner-Dyson limit)}. \tag{2}$$

A further treatment of the distribution (1) to deduce concrete statistics (the spacing distribution, long-range 2-level correlations, etc.) is a hard problem, and have so far produced little fruits. However, for the unitary symmetry class $\beta = 2$, a fully analytic method was devised by Forrester [4]: he showed that for this class expression (1) can be rewritten as a determinant

$$W_N^{(\beta=2)}(x_1, .., x_N) \equiv a^{-N} \prod_{1 \le j < k \le N} \left(\frac{(x_j - x_k)^2}{a^2 + (x_j - x_k)^2} \right) = \det \left[\frac{1}{a - i(x_j - x_k)} \right]_{j,k=1,..,N}, \tag{3}$$

where the right-hand side allows an infinite sum over N with coefficient ζ, called the grand canonical series with fugacity ζ. In the present talk, it will be extended to the other two classes i.e. orthogonal and symplectic classes by using *quaternion representation* [5] which replaces the two opposite limits in (2) by $\zeta \to 0$ and $\zeta \to \infty$, respectively. It is based on our recent work [6].

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Dynamics of nonlocally coupled oscillators I

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The dynamics of large assemblies or extended fields of coupled nonlinear elements depends crucially on the interaction range involved. Most foregoing studies have been confined to the limiting cases of either local coupling or global coupling, while systematic studies of more general nonlocally coupled systems remain few. In this series of lectures, I will discuss some remarkable features in the dynamics which can arise peculiarly to nonlocally coupled systems, working mainly with oscillatory dynamics. The first lecture will be devoted to general discussions on what are essentially new with nonlocally coupled systems as contrasted with locally coupled systems. Some points to be discussed include the following:

1. Effective nonlocality in coupling arising from locally coupled systems (typically reaction-diffusion systems) as a result of elimination of some variables.
2. Proposal of a 3-component reaction-diffusion model as a canonical model covering the local, global and nonlocal regimes of the effective coupling, thus providing an ideal model for the study of the effects of nonlocality on the dynamics.
3. Pointing out the fact that nonlocal coupling has its own asymptotic regeme, not being merely something intermediate between the local and global.
4. Applicability of center-manifold reduction and phase reduction leading to some new forms of universal equations.
5. General conditions under which the effects of nonlocality become overt.
6. Mean-field picture applicable to nonlocally coupled systems.
7. Onset of spatial discontinuity of patterns as a general feature of nonlocally coupled systems.

Dynamics of nonlocally coupled oscillators II

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In this second lecture, the impact of the nonlocality in coupling will be demonstrated by showing a variety of curious dynamics exhibited by the canonical model introduced in the first lecture or by its reduced forms. Our emphasis will be placed upon how the types of behavior observed are peculiar to the coupling nonlocality and how they can naturally be understood in terms of the general notions developed in the first lecture. In particular, the mean-field picture valid for nonlocally coupled elements turns out quite useful for interpreting certain features of behavior. Some problems involving stochasticity are also discussed for which a theory can be formulated by virtue of the same mean-field picture. The types of behavior discussed are:

A. Self-sustained pacemakers in monostable excitable media whose origin is completely different from those proposed in the past.

B. Two-dimensional spiral waves with a strongly turbulent core which is initiated by a breakdonw of synchronization of a small group of central oscillators to the periodic internal forcing.

C. Coexistence of coherent and incoherent domains. This is a generalization of case B.

D. Spatio-temporal chaos with multi-scaling properties for which the pattern is fractalized with its fractal dimension changing continuously with the system parameter.

E. *Soft-mode turbulence* which occurs right at the Turing instability in the presence of a Goldstone mode.

F. *Sawtooth turbulence* characterized by persistent creations and annihilations of wave sources and sinks.

G. Transmission of waves in random media for which a statistical theory can be formulated in terms of a Boltzmann type kinetic equation.

Frequency Map Analysis, theory and practice

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Introduction. Frequency Map Analysis is a numerical method based on refined Fourier techniques which provides a clear representation of the global dynamics of many multi-dimensional systems, and which is particularly adapted for systems of 3-degrees of freedom and more. This method relies heavily on the possibility of making accurate quasiperiodic approximations of quasiperiodic signal given in a numerical way.

In these lectures, we will describe the basis of the frequency analysis method. This is intended to be of practical use for any researcher who is willing to explore and use the frequency map analysis methods for the understanding of the dynamics of Hamiltonian systems.

Applications to several examples will be provided, in Solar System, or Particle Accelerator Dynamics.

1. Quasi periodic approximation

2. Convergence of the quasi periodic approximation, asymptotic expansion

3. Numerical examples

4. Frequency Map Analysis

5. Application of Frequency Map Analysis in Solar System and Particle Accelerator dynamics

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Dynamics and structure of granular flow through a vertical pipe

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Static, dynamic and statistical properties of granular materials are one of the most important topics in current science and its application to technology. In some situations granules behave like ordinary fluid or ordinary solid. On the other hand, a variety of unusual motions peculiar to granules can be observed, such as size segregation, bubbling, standing waves and localized excitations under vertical vibrations, avalanche and other unusual motions in a rotating mill, chute flow down a slope and a fluidized bed due to air injected inside a box containing granules. Emergence of density waves of granules flowing through a vertical pipe is also a typical and the simplest example of unusual features of granular motion.

We have shown how density waves of granular particles (ordinary sand) emerge, while they flow through a vertical glass pipe, by controlling air flow out of a flask attached to the bottom-end of the pipe. When the cock attached to the flask is fully open, air is dragged by falling granules and flows together with them. No density waves are observed for this situation. As the cock is gradually closed, however, the pressure gradient of air inside the pipe becomes gradually large, inducing the velocity difference between granules and air. As a result, density waves emerge from the lower part of the pipe. The smaller the rate of air flow, i.e., the more the cock is closed, the higher the onset point (along the pipe) of density waves. The onset of density waves is characterized by the growth of the lower frequency part of the power spectra of time-series signals of density waves. The power spectra of density waves display a clear power-law form $P(f) \sim f^{-\alpha}$ with the value of the exponent $\alpha = 1.33 \pm 0.06$, which is very close to 4/3. The value of α is robust even under the medium flow or variation of the pipe diameter, as far as density waves can be seen.

Very recently we have also controlled the flow rate of granules under the condition that the cock is completely closed, i.e., medium air does not flow with granules. When the flow rate of granules is small, they flow homogeneously with no density waves. The power spectra of the flow exhibit white-noise-like behavior. This is an expected result, just as raindrops fall homogeneously. What we observed rather unexpectedly is the following. If you gradually increase the flow rate of granules, density waves emerge suddenly at some threshold value of the flow rate. Above this threshold the power spectra exhibit clear power-law form with the same exponent $\alpha = 4/3$ robustly.

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Pattern formation in bacterial colonies

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We have studied the growth mechanism and morphological change in colony formation of bacteria from the viewpoint of physics of pattern formation. Even very small number of bacterial cells, once they are inoculated on the surface of an appropriate medium such as semi-solid and nutrient-rich agar plate and incubated for a while, repeat the growth and cell division many times. Eventually the cell number of the progeny bacteria becomes huge, and they swarm on the medium to form a visible colony. The colony changes its form sensitively with the variation of environmental conditions. This implies that although usual bacteria such as *Escherichia coli* are regarded as single cell organisms, they never make their colony independently and randomly but somehow collaborate multicellularly. We have thus tried to extract some simple and universal behavior in growth from such complex bacterial systems.

Here we varied only two parameters to investigate the colony growth; concentrations of nutrient C_n and agar C_a in a thin agar plate as the incubation medium. Other parameters specifying experimental conditions such as temperature were kept constant. We mainly used a typical bacterial species *Bacillus subtilis*. Otherwise the experimental procedures are standard. It was found that colonies show characteristic patterns in the specific regions of values of C_n and C_a in the morphological diagram and the patterns change drastically from one region to another. They were classified into five types; fractal DLA-like, compact Eden-like, concentric ring-like, simple disk-like and densely branched DBM-like. We have experimentally elaborated characteristic properties for each of these colony patterns.

We have also examined colony formation of a species *Proteus mirabilis*, which forms concentric-ring-like colonies that look much more regular than those produced by *Bacillus subtilis*. The colony grows cyclically with the interface repeating an advance (migration) and a stop (consolidation) alternately. Our experimental results suggest that macroscopically the most important factor for its repetitive growth is the cell population density, i.e., that there seem to be higher threshold of the cell population density to start migrating and lower one to stop migrating.

We have tried to construct a phenomenological model which produces characteristic colony patterns observed in our experiments. The basic idea is that the main features of individual biological organisms are, if focusing on their population behavior, reproduction and active motion. Our modeling is, therefore, based on the reaction-diffusion-type approach for the population density of bacterial cells and the concentration of nutrient.

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Optimal fluctuations and the control of chaos

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Large fluctuations are responsible for many important physical phenomena, including e.g. stochastic resonance and transport in Brownian ratchets. They usually proceed along *optimal paths*. Starting from Boltzmann (1904), a huge body of theory was developed during the last century; the modern understanding dates from Onsager and Machlup (1953). The introduction of the prehistory probability distribution established optimal paths as physical observables (Dykman et al, 1992), and the corresponding optimal force driving the fluctuations was measured for the first time by Luchinsky (1997). Recent developments, centered on nonequilibrium systems, will be discussed, including extensions of the work has to encompass escape from chaotic attractors (Khovanov et al, 2000; Luchinsky et al, 2002). In particular, it has been established that fluctuational escape from a chaotic attractor involves the system passing between unstable saddle cycles – thus paving the way for an analytic theory. Measurements of the optimal force can be used to determine the energy-optimal control function needed to effect escape in the deterministic system in the absence of fluctuations.

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Quantized turbulence

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Turbulence in superfluids – e.g. the superfluid states of liquid ^{4}He and ^{3}He, the electron gas in superconductors, the nucleonic fluids in neutron stars, and Bose-Einstein condensates in laser-cooled gases – is quantized. It consists of a tangle of vortex lines, each element of which is identical to every other in any given system. Apart from its intrinsic scientific interest it is of importance because (a) being in some ways a very simple form of turbulence one can hope to understand in considerable detail, and (b) it is the state believed to be created during a fast passage through a second order phase transition. Two ongoing research programmes on superfluid turbulence will be reviewed and discussed. First, the initial experiments (Davis et al, 2000) on the decay of turbulence in superfluid ^{4}He at mK temperatures will be considered. The vortices are created with a electrostatically-driven vibrating grid, and detected by the use of negative ions travelling near the Landau critical velocity in isotopically pure ^{4}He. Preliminary results indicate that the vortex decay rate apparently becomes temperature-independent below about 70 mK. It is believed (Vinen, 2000) that the corresponding decay mechanism may involve a Kolmogorov cascade, Kelvin waves and, ultimately, phonon creation. Secondly, the status of superfluid helium experiments modelling the GUT transition in the early universe 10^{-35} s after the Big Bang (Dodd et al, 1998) will be reviewed.

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Synchronization phenomena in the kidney

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The pressure and flow regulation in the individual functional unit of the kidney (the nephron) tends to operate in an unstable regime. For normal rats, the regulation displays regular self-sustained oscillations, but for rats with high blood pressure the oscillations become chaotic. The lecture explains the mechanisms responsible for this behavior and discusses the involved bifurcations. Experimental data show that neighboring nephrons adjust their pressure and flow regulation in accordance with one another. For rats with normal blood pressure, in-phase as well as anti-phase synchronization can be observed. For spontaneously hypertensive rats, indications of chaotic phase synchronization are found. Accounting for a hermodynamics as well as for a vascular coupling between nephrons that share a common interlobular artery, the lecture presents a model of the interaction of the pressure and flow regulation between adjacent nephrons. It is shown that this model, with physiologically realistic parameter values, can reproduce the different types of experimentally observed synchronization.

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Mosekilde E, Maistrenko Yu and Postnov D 2002 *Chaotic Synchronization – Applications to Living Systems* (World Scientific, Singapore)

Chaotic synchronization of time-continuous oscillators

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Considering two coupled identical Rössler oscillators the lecture first discusses the necessary and sufficient conditions for stability of the synchronized chaotic state. The lecture continues to examine the transitions through which low periodic orbits embedded in the synchronized chaotic state lose their transverse stability and produce the characteristic picture of riddled basins of attraction. We also discuss the distinction between local and global riddling and illustrate the further development of the asynchronous periodic orbits.

A similar approach is applied to a model of two interacting biological cells. Considering a prototypic model of the bursting oscillations in insulin producing pancreatic cells, we first present one- and two-dimensional bifurcation diagrams of the individual cell. These diagrams reveal a squid-formed area of chaotic dynamics in parameter space with period-doubling bifurcations on one side and saddle-node bifurcations on the other. The transition from this structure to the so-called period-adding structure is found to involve a subcritical period-doubling and the emergence of type-III intermittency. Finally, the lecture addresses the issue of the robustness of the synchronized chaotic state to a mismatch of the parameters between the interacting oscillators.

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Quantum transport in chaotic quantum dots: orbit bifurcations, Arnold diffusion and fractals

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I talk about three interesting sub-themes bridging between nonlinear dynamics and quantum transport in mesoscopic billiards.

Firstly, triangular antidot lattices are investigated. We analyze the semiclassical conductivity of fully-chaotic triangular antidots in the low but intermediate magnetic field. Taking into account both a smooth classical part evaluated by the mean density of states and an oscillation part evaluated by periodic orbits, we find that the resistivity of the system yields a monotonic decrease with respect to the magnetic field. But when including the effect of orbit bifurcation due to the overlapping of a pair of periodic orbits, several distinguished peaks of resistivity appear. The theoretical results nicely explain both the locations and intensities of the anomalously large peaks observed in the experiment by NEC group (Phys. Rev. B51(1995)4649) [1].

Then, we shall proceed to investigation of open three-dimensional (3-d) quantum dots. Mixed phase-space structures of 3-d billiards show the Arnold diffusion that cannot be seen in 2-d billiards. A semiclassical conductance formula for ballistic 3-d billiards is derived. We find that, for partially- or completely-broken ergodic 3-d billiards such as SU(2) symmetric billiards, the dependence of the conductance on the Fermi wavenumber is dramatically changed by the lead orientation. As a symmetry-breaking weak magnetic field is applied, the conductance shows a tendency to grow. We conclude: In contrast to the 2-d case, the anomalous increment of the conductance should include a contribution arising from the (classical) Arnold diffusion as well as the (quantum) weak localization correction [2].

Finally, within a formalism of the semiclassical Kubo formula for conductivity, we give a periodic-orbits picture for the fractal magneto-conductance fluctuations recently observed in submicron-scale phase coherent ballistic billiards [3]. The self-similar conductance fluctuations are shown to be caused by the self-similar unstable periodic orbits which are generated through a sequence of isochronous pitchfork bifurcations of straight-line orbits oscillating towards harmonic saddles. The saddles are universally created right at the point of contact with the leads or at certain places in the cavity as a consequence of the softwall confinement. Our mechanism is able to explain all the fractal-like magneto-conductance fluctuations in general softwall billiards [3].

Many other interesting themes in this field will be described in [4].

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Some classes of self-similar planar fractals

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We present some well known classes of planar fractals, based on regular polygons, and introduce a new class of fractals, appearing from the construction of simple branching trees. Most of the studied objects are self-similar in a strong sense. Therefore, the self-similar dimension $d_S = \frac{\ln N}{\ln k}$, where N is the total number of congruent sub objects and k is the coefficient of similarity, is introduced and its properties are studied.

Besides the dimension d_S, various other characteristics of the presented fractals are examined, for instance, when the overlapping occurs, what is the equation of the boundary curve, and what is the density of the embedded object. We also explain the concept of an iterated function system and give the IFS-codes for some of the studied fractals. Most of the results can be generalized to the 3-dimensional space (starting, for instance, with regular solids) and into the higher dimensions.

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From independent particle towards collective motion in a few electron lattice model with Coulomb repulsion

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A basic question in quantum many body theory is to know how one goes from independent particle motion towards collective motion when one decreases the density n_s of a system of charged particles repelling each other via a U/r Coulomb repulsion. The motivation to re-visit nowadays this question comes from the possibility to create two dimensional gases of charges in high quality field effect devices and to decrease by a gate the carrier density n_s down to a very dilute limit. Conductance measurements for different densities as a function of the temperature, of the bias voltage, of a parallel magnetic field, etc, show the existence of an unexpected low temperature metallic behavior when one goes towards the dilute limit. This raises the question of the existence of an intermediate metallic phase between two insulating phases of different nature: the Fermi glass of Anderson localized states at large n_s and the pinned Wigner solid at low n_s. This question is numerically investigated using mesoscopic lattice models with and without disorder.

In **lecture 1**, the recent experimental results motivating to re-visit the Fermi-Wigner crossover will be reviewed.

In **lecture 2**, detailed exact numerical studies of a few electron mesoscopic lattice models will be presented, showing a specific intermediate regime between the weak and strong coupling limits.

In **lecture 3**, the conjecture of an intermediate phase between the Fermi liquid and the Wigner solid, first proposed by Andreev-Lifshitz, will be discussed in relation with the numerical results.

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Transport of strongly correlated electrons

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One of the central open theoretical problems in the solid state physics is the understanding of strongly correlated electrons, where the properties are dominated by strong electron-electron repulsion and Pauli exclusion principle. We discuss the quantum electronic transport in such systems [1], in particular electrical conductivity, spin diffusion and heat conductivity. The concept of charge stiffness is introduced which makes qualitative distinction between conductors and insulators in the quantum ground state, while at finite temperatures it leads to possibilities of usual resistors, but also of anomalous ideal conductors and ideal insulators [2,3]. It is shown that the singular transport appears in many integrable systems of interacting fermions, even when the current is not a conserved quantity. The evidence comes from the relation with level dynamics [3], the existence of conserved quantities [4], from exact results as well as from numerical studies of small correlated systems using exact diagonalization method and finite-temperature Lanczos method [5]. Several open theoretical problems in this connection will be addressed: a) necessary ingredients for the quantum dissipationless transport, b) transport in systems close to integrability, and c) the existence of ideal insulators at finite temperatures.

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Stability of quantum motion and correlation decay I & II

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In this two-hour lecture we will make a review of the theory and the (numerical) experiments on the behavior of *quantum fidelity* of classically chaotic and regular hamiltonian systems.

We will derive a simple and general relation between the fidelity of quantum motion, characterizing the stability of quantum dynamics with respect to arbitrary static perturbation of the unitary evolution propagator, and the integrated time auto-correlation function of the generator of perturbation. Quite surprisingly, this relation predicts the slower decay of fidelity the faster decay of correlations. In particular, for non-ergodic and non-mixing dynamics, where asymptotic decay of correlations is absent, a qualitatively different and faster decay of fidelity is predicted on a time scale $\propto 1/\delta$ as opposed to mixing dynamics where the fidelity is found to decay exponentially on a time-scale $\propto 1/\delta^2$, where δ is proportional to the strength of perturbation. A detailed discussion of a semi-classical regime of small effective values of Planck constant $\hbar$ is given where classical correlation functions can be used to predict quantum fidelity decay. Note that the correct and intuitively expected classical stability behavior is recovered in the classical limit $\hbar \to 0$, as the two limits $\delta \to 0$ and $\hbar \to 0$ do not commute. In addition we also discuss a non-trivial dependence on the number of degrees of freedom and the role of the thermodynamic limit.

The theoretical predictions will be demonstrated mainly in two families of models: (i) a quantized kicked top and a quantized pair of coupled kicked tops where the semiclassical regime is emphasized, and (ii) kicked Ising spin 1/2 chain where the thermodynamic regime is emphasized. We also need to stress that these results have important implications for the stability of quantum computation, and may be used in order to optimize the accuracy of quantum algorithms.

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Chaotic resonances in quantum many-body dynamics

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The questions about the mechanisms and the conditions for the relaxation to equilibrium in the thermodynamic limit of a generic isolated hamiltonian system constitute important open problems in statistical mechanics. In this talk we consider an established technique in describing relaxation of strongly chaotic single-particle classical systems, namely the concept of Perron-Frobenius-Ruelle resonance spectrum, and use it for the dynamical description of non-integrable quantum many-body systems in thermodynamic limit.

We define a quantum Perron-Frobenius master operator over a suitable normed space of translationally invariant states adjoint to the quasi-local C^* algebra of quantum lattice gasses (e.g. spin chains), whose spectrum determines the exponents of decay of time correlation functions. The gap between the leading eigenvalue and the unit circle signals the exponential mixing (universal asymptotic exponential decay of arbitrary time correlation functions), whereas closing the gap typically corresponds to a transition to non-ergodic dynamics, which may as a consequence, lead to important anomalous transport properties. In particular, the conservation laws of completely integrable quantum lattices represent degenerate eigenvalue 1 eigenvectors of the Perron-Frobenius operator.

Theoretical ideas are applied and validated in a generic example of kicked Ising spin 1/2 chains, namely a one dimensional spin 1/2 lattice with nearest neighbor Ising interaction kicked with periodic pulses of a tilted homogeneous magnetic field. We show that the 'chaotic eigenmodes' corresponding to leading Perron-Frobenius-Ruelle eigenvalue resonances have fractal structure in the basis of local operators.

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Introduction to quantum chaos of generic Hamiltonian systems

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We shall review the basic aspects of complete integrability and complete chaos (ergodicity) in classical Hamiltonian systems, as well as all the cases in between, the generic, mixed type systems, where KAM Theory is applicable, and shall illustrate it using the billiard model systems.

Then we shall proceed to the quantum chaos and its stationary properties, that is the structure and the morphology of the solutions of the underlying Schroedinger equation which in case of 2-dim billiards is just the 2-dim Helmholtz equation. We shall discuss the statistical properties of chaotic eigenfunctions, the statistical properties of the energy spectra, and show arguments and results in support of the so-called universality classes of spectral fluctuations, where in the fully chaotic case the Random Matrix Theory (RMT) is applicable.

First we discuss the universality classes of spectral fluctuations (GOE/GUE for ergodic systems, and Poissonian for integrable systems). We explain the problems in the calculation of the invariant (Liouville) measure of classically chaotic components, which has recently been studied by Robnik et al (1997) and by Prosen and Robnik (1998). Then we describe the Berry-Robnik (1984) picture, which is claimed to become exact in the strict semiclassical limit $\hbar \to 0$. However, at not sufficiently small values of $\hbar$ we see a crossover regime due to the localization properties of stationary quantum states where Brody-like behaviour with the fractional power law level repulsion is observed in the corresponding quantal energy spectra.

We shall mention the rich variety of applications in the domain of physics.

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Introduction to computational algebra

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Consider a system of polynomials

$$\begin{array}{l} f_1(x_1, x_2, \ldots, x_n) = 0, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ f_k(x_1, x_2, \ldots, x_n) = 0 \end{array} \tag{1}$$

where $f_1, \ldots, f_k$ are polynomials with coefficients from some field k (usually, k is the field of real or complex numbers). In algebraic geometry the solution space of system (1) is called *the variety.* There are many numerical algorithms for solving non-linear systems such as (1). These algorithms solve for one solution at a time, and find an approximation to the solution. They ignore the geometric properties of the solutions space (the variety), and do not take into consideration possible alternate descriptions of the variety (using a different system of polynomials). However recently efficient computational algorithms have been developed which enable us to get algebraic and geometric information about the *entire* solution space of system (1). They are based on the Gröbner bases theory worked out by B.Buchberger around the middle of 60th of last century. The idea of the methods is to find the "best" representation of the corresponding variety. To illustrate this recall that the Gauss-Jordan elimination method transforms a system of linear equations into the so-called row echelon form. The system thus obtained has exactly the same solutions (the variety) as the original system, but it is trivial to solve. The other example is the system (1) with f_i being polynomials in a single variable, $f_i = f_i(x)$. In this case there is a polynomial $f(x)$ such that the system (1) is equivalent to the equation $f(x) = 0$. The polynomial $f(x)$ is the greatest common divisor of $\{f_1, \ldots, f_k\}$ and it can be found using the Euclidean Algorithm. Although the Gauss-Jordan elimination method cannot be directly expanded to the case of non-linear polynomials and the Euclidean Algorithm – to the case of multivariable polynomials, it turns out that an expansions of these methods is possible, and, in a sense, the Gröbner bases theory is a generalization of the Gauss-Jordan method and the Euclidean Algorithm to the case of non-linear multivariable polynomials.

In the lecture we give an introduction to the Gröbner bases theory, discuss some algorithms implemented in computer algebra systems (e.g. in Mathematica) and consider a few applications to the theory of dynamics systems, in particular, to the investigation of the time-reversible systems of ODE.

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Synchronization of irregular oscillators: from theory to data analysis

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In the classical sense, synchronization of coupled oscillating systems means appearance of certain relations between their phases and frequencies due to weak coupling. After giving brief introduction into the classical theory we review its recent extension to the case of chaotic systems. We discuss how the phase and mean frequency can be determined and consider synchronization effects in two- or many-oscillator systems.

Next, we discuss how synchronization theory can be used in data analysis. In particular, we consider how the phases and frequencies can be estimated from time series and how the intensity and directionality of interaction can be estimated. The methods are illustrated by the results of the investigation of cardiorespiratory interaction in humans.

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Ladder operators and moment problems

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Ladder operator formalisms typically arise in factorization approaches to Schrödinger operators. There, they serve to an algebraic understanding of spectral problems like finding suitable eigenvalues of the linear operators under consideration. Also when it comes to describing supersymmetric Schrödinger operators in quantum mechanics, the concept of lowering and raising operators turns out to have an important meaning. So far, typical scenarios when ladder operators arise are briefly sketched. In recent contributions it has become apparent that methods involving ladder operators can also be used to deal with moment problems in context of special functions in analysis. We give several examples for this application. It remains a fascinating task and also a kind of challenge to investigate the interactions between related analytic and stochastic structures. For instance, the role of discrete Hermite polynomials and their connections with discrete martingale theory has to be understood in detail.

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Adaptivity or chaoticity of strongly driven natural multiphase systems?

Synergetics as a transdisciplinary set of paradigms explaining amphidynamic behaviour vacillating between procedural coherency (”order”) and ”Self-Organised Criticality” (”pseudochaotic coordination”)

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Subsequent to Edward LORENZ'es, David RUELLE's and Florin TALKEN's secular discoveries of ”pseudochaotic coordination” in hydro-aero-dynamic systems, the strange coexistence of indeterminate kinematics and apparently deterministic dynamics has become the topic of intensive, computer based research in the physical sciences. Using the mere phenomenological analogy between evolution of systems with sensitive dependency on the initiating conditions, positive LYAPUNOV exponents and fractal multi-dimensionality, sterile extrapolations from idealising physicalism to profound natural sciences became popular, inter alia by search for ”chaoticity” in the time series of cardio-vascular, endocrinological and/or skeletomuscular reactions. All of these are long known in their microscopic details and behavioural traits of ”re-exitable membrane channels”, the most important element for ”non-linear-non-equilibrium phase transitions”. These were systematically studied since the days of SHERRINGTON, one of the founding fathers of physiological synergetics around the turn of the 19th century. Hermann HAKEN, here cooperating with Hans Peter KOEPCHEN, the leading authority of the autonomous nervous system in the 1990ies: the two authors postulated that neurodynamic behavioural traits are characterised by only transient, self-limiting phases of coherent performance. To paraphrase the latter in a intuitive manner, they proposed to use the term ”quasi attractor” in describing such short lived emergence and subsequent submergence of procedural coherency: methods to display this ”natural behaviour” have now been developed.

Using time series obtained in awake human subjects exposed to cold environment, in patients undergoing psychomotor relaxation, in patients and volunteers undergoing pain stimuli and in subjects undergoing a specific regimen of bicycle ergometry, multiple base-line time series (skeletomotor cardiovascular, respiratory, continuous skin galvanic response) were analysed by a comprehensive algorithm based on primary MORLET-wavelet analysis, ARMA-procedures (moving average) and were then plotted as time frequency plots (”prosodograms” depicting in intuitive manner the emergence and submergence of preferred attractors). The combined data clearly corroborated the basic assumptions of the HAKEN-KOEPCHEN paradigm: in addition, clear indicators of n:m synchronisation (see Lecture TASS) became evident in even short lived coherency separated by likewise clearly detectable transient. In proposing that a moratorium should be it placed on the future publication of dynamic portraits from ”single base-line” recordings, it can be anticipated that the ”adaptive nature” of normal physiological reactions will become evident to the educated (and soon the general public) ending a ”short lived historical transient” where natural adaptivity was misconceived as ”chaos”.

Self-limiting passive discharge followed by transfer blockade:

On putative microscopic causes of ”non-linear” reactions in multiphase systems operated under robustly sustained dysequilibrated boundary conditions (BERTALANFFian ”flow equilibria”)

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HAKEN’s proposal that rapidly emerging and subsequently submerging ”dominance” of preferred ”attractor” behaviour were the cause of apparent ”chaoticity” of biological systems is guiding the project of physiological synergetics: it can be put it into the proper perspective by postulating, that owing to well known behavioural traits of ensembles of membrane channels on the microscopic, and of ensembles of effector neurons and inhibitor neurons, rapid ”phase synchronisation” on the one hand, and automatic re-inhibition on the other forms the basis of functional adaptivity.

These concepts also apply - under boundary conditions with steep potential gradients mimicking those prevailing in vivo - , to prebiotic systems capable of undergoing ”drive dependent consensualisation of a priori independent movements” (our rheological definition for the vague term ”self-organisation”). In studying a wide spectrum of prebiotic (putatively) chaotic prebiotic systems (ranging form sand pile kinematics and hour glass behaviour, over dripping faucets and water clocks, the holocoherent BENARD-MARANGONI hyper-stability and various new versions of the BELOUSOV-ZHABOTINSKI reaction as paradigmatic example of self-organised catalytic activity, the HAKEN-KOEPCHEN-quasi attractor concept could be verified. In the latter, the well known ”periodicity” could be enhanced, blurred or even abolished completely by the appropriate choice of ”setting” providing ”sinks” for products (CO2, electrons). Lastly, calcium waves in isolated cells (beating myocardiocytes) and in suspension of sarcoplasmatic reticulum in agar were studied: they all showed identical behaviour, i.e. autowaves due to self-limiting discharge, refractorisation with restitution of the ”kinetic threshold” allowing ”critical slowing” and ”enthalpy peaking” as basis for BRILLOIN’s negentropy principle of information.

Using disarmingly simple cellular automata simulating eruptive, self-limiting discharge and variable length of refractory periods, the above described (putatively universal) behavioural traits could be modelled, the resulting patterns displaying the very same ”apparent kinematic indeterminancy” (due to spatio-temporal in-homogeneities) which can be easily corrected by choosing the proper combination (and homogeneity) of parameters reflecting the well known determinants for resonance prone behaviour, namely generalised inertance, generalised capacitive resilience and generalised inhibitance. We propose that, eruptivity abounds in ”nature” (in highly ”non-linear” reactions) due to the multiphase nature of natural materials, especially when systems are driven into the strongly dysequilibrated modes of operation first identified by BERTALANFFY as ”cause” of sustained transfer of energy and matter ”feeding the negentropy” postulated by SCHRÖDINGER for living systems.

Clustering of passive tracers in free-surface flows

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Experimental and numerical studies of turbulent fluid motion in a free surface are presented. The flow is realized experimentally on the surface of a tank filled with water stirred by a vertically oscillating grid positioned well below the surface (Goldburg et al 2001). The effect of surface waves appears to be negligible so that the flow can numerically be realized with a flat surface and stress-free boundary conditions above three-dimensional volume turbulence (Eckhardt and Schumacher 2001).

The two-dimensional free surface flow, $\mathbf{v}(x, y, t)$, is unconventional: it is not incompressible, i.e. $\partial_x v_x + \partial_y v_y \neq 0$, and neither kinetic energy, nor squared vorticity (enstrophy) are conserved in the limit of zero fluid viscosity and of absence of external driving as it is the case for "usual" two-dimensional turbulent flows (Lesieur 1990). According to both experiment and numerical simulation, statistical properties of the surface flow are closer to those of three-dimensional turbulence.

The dynamics of passive Lagrangian tracers that are advected in such flows is dominated by rapidly changing patches of the surface flow divergence. Single particle and pair dispersion show different behavior for short and large times: on short times particles cluster exponentially rapidly until patches of the size of the divergence correlation function are depleted; on larger times the pair dispersion is dominated by subdiffusive hopping between clusters. We also find that the distribution of particle density is algebraic, and not lognormal as predicted for flows that are delta-correlated in time (Klyatskin and Saichev 1997). The latter so-called Kraichnan flows are rather synthetic but allow for making analytical progress. Our results can be traced back to the exponential distribution of the divergence field of the surface flow. Very recently, physical mechanisms for the formation of rain drops were discussed by Balkovsky et al (2001). They considered the motion of tracers that have inertia (Maxey and Riley 1983) but are advected in an incompressible turbulent Kraichnan flow. The relation of our findings to this problem is discussed.

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Chaos, integrability, entanglement and decoherence

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We shall give an unexpected view of the relation of decoherence and entanglement to the integrability or chaoticity of the underlying classical systems.

This view stems from the fact, that we shall consider separately the dependence on the dynamics from the dependence of the initial state, and not limit ourselves to the usual coherent states, as initial conditions. This has two reasons: First we believe that the phenomenon can be better understood in this fashion and second in the context of quantum computing we are certainly more interested in the evolution of a random initial state, than in the one of a Gaussian packet. We shall use both random matrix methods and correlation function techniques, as presented in the lectures of Dr. Prosen, to illuminate the problem at hand. We shall give close attention to the question, when decoherence follows the trend of the corresponding autocorrelation function (including fidelity in echo situations), and when not. We shall also inquire, if other correlation functions become relevant in situations where the autocorrelation function does not explain the behaviour of decoherence or entanglement.

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Quantum chaos in the mixed phase space and the Julia set

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Phase space of multi-dimensional Hamiltonian systems is generally composed of infinitely many invariant components. Chaotic trajectories have the largest dimension as an invariant set, while the periodic orbits have the lowest. Coexistence of qualitatively different ergodic components, which are usually intermingled in a self-similar way in the phase space, characterizes a generic situation which is so complicated that our understanding is far from accomplished. The orbits in classical mechanics are always confined on the corresponding invariant set by definition, in particular, except in case of ideal chaotic systems, there are orbits with positive measure that move only on the limited subspace whose dimension is less than that of full phase space.

On the other hand, the wavepacket of quantum mechanics is not forced to stay on a certain limited classical manifold, but spreads over or shares different invariant subsets simultaneously. The spreading is a consequence of the wave effect which is the most marked difference between classical and quantum mechanics. There is not any obstacle in principle preventing the transition between arbitrary two points in the phase space and the quantum wavepacket can penetrate into any kinds of barriers. Such a classically forbidden process does not have classical counterparts. The penetration into the energy barrier is especially called *tunneling*, which is understood as the most typical quantum effect and plays important roles in many physical and chemical phenomena. The existence of chaos in the phase space crucially affects the nature of tunneling (Shudo & Ikeda 1995, 1998).

In this lecture, after introducing recent developments of the theory of multi-dimensional complex dynamical systems, with some technical tools necessary to construct the theory (Bedford & Smillie 1991a, 1991b 1992), we will give numerical and mathematical evidences which show that the orbits on the Julia set in the complex phase space can just be regarded as the *classical counterparts* of the quantum wave effects (Shudo, Ishii & Ikeda 2001). More precisely, arbitrary two regions in the phase space are necessarily connected via the orbits on the Julia set *even in the mixed system.* This remarkable property, which is absent in the real classical dynamics, comes from the transitivity of the Julia set, which has rigorously been proved for the complex Hénon map, and conjectured for the standard and semi-standard map. The measure whose support gives the Julia set is a unique ergodic measure, and unstable saddles are dense on it. Our arguments are based on the complex semiclassical description of quantum forbidden processes.

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Complication of linear spatial socio-economies

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The purpose of this lecture is to provide an explanation of the process of complication, i.e. the deepening and evolution of complexity in evolving complex linear systems. The complication means the transfer from complex structure to much more complex structure in the evolution of complex systems. The main feature of the evolution of a complex system is the emergence of new properties which did not exist in previous trends and which add new information to the system. The growth of complexity means the appearance and increase of information and therefore the decrease of the (Shannon) entropy. Even the simplification of the system is the part of process of complication, since simplification is clearing the room for the further adoption of new information. This clearing presents the essential force acting against the modern information explosion and playing the important role in the process of self-organization. Spread of information within the complex system presents the essence of the process of complication. This spread shows itself through the partial adoption of new information and through the path dependent process of self-organization within socio-spatial complex system. In this study we will concentrate ourselves only on the forms of complication and self-organization in linear socio-economic systems, leaving behind the innovation diffusion and bifurcation analysis. The concept of complication is pointed out on the deficiency of purely economic considerations of socio-economic systems and stresses the necessity to widen the concept of "Homo Oeconomicus" to the concept of "Homo Socialis". Such a widening is radical in the study of complex socio-economic processes because of the important difference between the economic and socio-economic rationality: the traditional identification of economic rationality of "Homo Oeconomicus" as the optimization is complementary to socio-economic rationality of "Homo Socialis" as parsimony. In our lecture we will apply the paradigm of complexity and complication to several main branches spatially connected with the augmentation and development of flows, networks and superposition of their hierarchies in linear systems. We are using the concept of complication as the unifying frame for theories of linear spatial analysis of complex socio-economic systems: the Push-Pull theory of Migration Streams, the theory of Central Place hierarchies, the spatial production cycles and trade feedback loops, the Dynamic Input-Output Analysis and the theory of the Fields of Influence of changes in Input-Output systems, the classical Key Sector Analysis, the Structural Q-analysis and the Miyazawa model of income distribution within Input-Output systems and their "onion skins" extensions.

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Determinism and noise in cardiovascular dynamics

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Signals derived from the human cardiovascular system are well known to exhibit highly complex, nearly periodic, oscillatory behaviour whose nature is something of an enigma and still the subject of vigorous debate. The variation of cardiac frequency with time, known as heart rate variability (HRV), has been intensively investigated using both deterministic and stochastic methods. It has, for example, been variously described as chaotic, fractal, stochastic, and subject to 1/f fluctuations and it was proposed that the state of the system can be classified by the slope of its power spectrum on a log-log plot.

We illustrate some problems in characterising slow modes in real measurements and show that oscillatory dynamics under the influence of strong noise, coupled with a limited time of observation, can lead to a $1/f$-like behaviour. We review and describe some recent experiments that illuminate the problem and discuss a combination of almost periodic and stochastic frequency modulation as a signature of the system dynamics.

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Interactions in the cardiovascular system

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In a healthy subject in repose a volume of blood equivalent to the total amount in the body returns to the heart every minute. Several oscillatory processes characterise cardiovascular dynamics within this circulation time. Cardiac and respiratory activities act on higher frequency scales, with characteristic frequencies of 1 Hz and 0.2 Hz, respectively. The lowest frequency component, at around 0.01 Hz, has been associated with the activity of the layer of endothelial cells forming the inner surfaces of all blood vessels.

The characteristic frequencies of all the cardiovascular oscillations are found to vary in time, apparently because the oscillatory processes mutually interact. Hales and Ludwig independently described the modulation of cardiac frequency by respiration, in 1773 and 1847 respectively, a process that today is known as respiratory sinus arrhythmia. The occurrence of episodes of synchronization between the cardiac and respiratory rhythms has also been demonstrated.

Recent developments of methods based on dynamical and information theory are facilitating studies of synchronization and of the directionality of couplings between the cardiovascular oscillations. We review and discuss their characteristics in health and disease. The characteristics of the cardiac and respiratory interaction during paced respiration are used to illustrate the role of directionality of coupling in the interplay between synchronization and modulation.

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Microwave experiments in chaotic and disordered systems

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In a sequence of two talks microwave experiments on spectra, line widths, and field distributions in various closed and open microwave resonators are presented with special emphasis on universal features common to all chaotic systems.

1. The random-superposition-of-plane-waves approach

According to a conjecture of Berry (1977) at any point in a chaotic billiard the wave function may be described by a random superposition of plane waves,

$$\psi(r) = \sum_n a_n e^{\imath \mathbf{k}_n \mathbf{r}_n} ,$$

where the modulus $k = |\mathbf{k}_n|$ of the incoming waves is fixed, but directions $\mathbf{k}_n/k$ and amplitudes a_n are considered as random. As an consequence of the central-limit theorem the approach predicts Gaussian distributions for the wave function amplitudes, or, equivalently, Porter-Thomas distributions for their squares. Such distributions have been observed for the first time for wave functions of chaotic billiards (McDonald and Kaufman 1988), and subsequently in numerous simulations and experiments on chaotic and disordered systems.

In this lecture microwave experiments are presented, exploiting further consequences of the Berry conjecture. Results for field distributions and spatial correlation functions in three-dimensional Sinai resonators (Dörr *et al* 1998) are presented. For spectral level dynamics in a disordered system with the position of one impurity as the parameter the approach allows to calculate velocity distributions and velocity autocorrelation functions which are in complete agreement with the experiment (Barth *et al* 1999). In open billiards and billiards with broken time-reversal symmetry the distributions of currents and vortices, as well as the vortex distance distribution are measured and compared with the prediction from the Berry conjecture (Barth and Stöckmann, Vraničar *et al*).

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2. Open microwave billiards as scattering systems

Whenever a microwave experiment is performed, the system has to be opened either by attaching wave guides or introducing antennas. This has the unavoidable consequence that the system is perturbed, and the measurement always yields an unwanted combination of properties of the system and the apparatus. A tailor-made approach to cope with this situation is provided by scattering theory. For the case of isolated resonances an expression for the matrix elements of the scattering matrix is obtained,

$$S_{ij} = \delta_{ij} - 2\imath\gamma \sum_n \frac{\psi_n(r_i)\psi_n(r_j)}{k^2 - k_n^2 + \frac{\imath}{2}\Gamma_n},$$

which is a direct equivalent of the Breit-Wigner formula known from nuclear physics for many years (Stein *et al* 1995, see chapter 6 of Stöckmann 1990 for details). In the equation $\psi_n(r_i)$ is the value of the wave function of the billiard (with Dirichlet boundary conditions at the wall, and Neumann ones at the opening) at the coupling position r_i. γ is a parameter describing the coupling to the wave guide or the antenna.

This correspondence of microwave billiards with atomic nuclei can be used to check predictions from theory which are unaccessible in nuclear physics. As an example the first unambiguous demonstration of resonance trapping is presented, namely the phenomenon that with increasing coupling strength the widths of the resonances do not increase unlimited but finally decrease again (Persson *et al* 2000). If the transmission through a cavity with a number of incoming and outgoing channels is measured as a function of frequency, irregular fluctuations are observed, an equivalent to the Ericson fluctuations observed in nuclear scattering processes. The distribution of these fluctuations was studied in an open microwave billiard in dependence of the number of channels, both for systems with and without time-reversal symmetry, and the results were compared with random matrix predictions (Schanze *et al* 2001). A new parameter comes into play if absorption is involved, which is unavoidable in experiments anyway, but has been considered by theory only recently (Beenakker and Brouwer 2001). Again the experiment is able to verify the theoretical predictions perfectly (Méndez *et al*).

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Quantum chaos on graphs

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Quantum graphs have recently been introduced as model systems to study quantum problems with chaotic classical limit (Kottos and Smilansky 1997). The most fascinating features of quantum graphs are that they can be constructed easily and almost at will covering a variety of classical limits such as chaotic dynamics, scattering or diffusive behaviour. Yet, the quantum mechanics can be formulated in terms of unitary propagators on finite Hilbert spaces. In addition, quantum graphs show many of the phenomena observed in more general quantum systems such as universality of the spectral statistics or Anderson localisation.

I will review recent developments on quantum graphs and generalise these concepts to quantum propagation on arbitrary, directed graphs (Tanner 2000). In its simplest version, the wave dynamics on the graph is solely determined by the topology of the graph given by the adjacency matrix, metric properties, that is, the length of the edges, and dynamical properties entering as (complex) transition amplitudes describing transitions between edges at the vertices. Necessary and sufficient conditions for a graph to be 'quantisable' can be given (Pakoński et al 2002).

A specific quantum graph can in a natural way be associated with an ensemble of unitary matrices (Tanner 2001). The 'classical' dynamics on the graph can be interpreted as a Markov chain defined on the graph with stochastic transition matrix $\mathbf{T}$ obtained from the unitary propagator on the graph (Kottos and Smilansky 1997, Pakoński et al 2001). I will formulate a conjecture linking universality of the statistical properties of the unitary matrix ensemble after ensemble average to the spectral gap of the stochastic transition matrix. More precisely, it is expected that the matrix ensemble follows random matrix statistics in the limit of large network size, if the spectral gap Δ, that is, the minimal distance of eigenvalues of $\mathbf{T}$ from the unit circle, scales like (Tanner 2001)

$$\lim_{N\to\infty} \frac{\log N}{\Delta(N)N} = 0$$

where N is the number of edges in the graph. Some examples will be presented. Results on correlation functions for eigenvalues and spectral determinants will be discussed (Tanner 2002).

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Classical and quantum spectra of intermittent dynamics

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Intermittency is a typical phenomenon on the transition from order to chaos. The existence of intermittent behaviour in dynamical systems can in general be traced back to the presence of marginal stability either at a single periodic orbit or along the boundary of stable islands. This leads to nearly regular together with strongly chaotic dynamics in connected components of the phase space. Intermittency is typically accompanied by algebraic decay of correlation and long tail memory effects. In this talk, I will present techniques to calculate the spectra of Frobenius-Perron operators for intermittent systems in terms of periodic orbits. Implications for a periodic orbit quantisation of intermittent dynamics using Gutzwiller's trace formula will be discussed briefly.

Periodic orbits, which approach the marginal stable regime in phase space are characterised by a vanishing Lyapunov exponent

$$\lambda_p = \frac{\log \Lambda_p}{T_p} \to 0, \qquad \text{for} \quad T_p \to \infty$$

with Λ_p the largest eigenvalue of the Monodromy matrix along the orbit. The algebraic decay in the eigenvalues Λ_p induces divergences in trace formula, which need to be removed systematically (in addition to the usual problem of overcoming convergence problems due to the exponential proliferation of periodic orbits). This leads to periodic orbit expansions of the trace of the Frobenius Perron operator in terms of families of periodic orbits converging towards the marginal stable region.

The talk is mainly based on the chapter on intermittency in the web-book by Cvitanović *et al.* I will introduce the method for a specific 1d-piecewise linear map for which the algebraic decay behaviour can be calculated explicitly. Generalisations to arbitrary uni-model maps will be given. The stadium billiard will serve as an example to discuss modifications of the method for semiclassical periodic orbit formulas (Tanner 1997).

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Development of demand-controlled deep brain stimulation-techniques based on stochastic phase resetting

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Pathological cerebral synchronization may severely perturb brain function as observed in several neurological diseases like Parkinson's disease and essential tremor. In patients that do no longer respond well to drug therapy, depth electrodes are chronically implanted in target areas located in the thalamus or the basal ganglia. To suppress the pathologically synchronized firing, a permanent high-frequency ($>$ 100 Hz) stimulation is performed (Benabid et al. 1991, Blond et al. 1992). Although the therapeutic effects are impressive, there are nevertheless significant drawbacks: (i) The energy consumption of the permanent stimulation is quite high. Thus, the generator (plus battery) has to be exchanged after 1-3 years by means of an operation. (ii) Even more important is the fact that the permanent high-frequency input is an unphysiological type of stimulation which causes the stimulated target areas to adapt. In a number of patients the amplitude of the stimulation has to be increased over the years, in order to maintain the tremor suppressive effect. With increasing stimulation amplitude, however, the probability of the occurence of severe side effects (like dysarthria, dysaesthesia, cerebellar ataxia, psychotic symptoms) increases.

With methods from synergetics (Haken 1983) and statistical physics (Kuramoto 1984) the concept of phase resetting (Winfree 1984) was extended to populations of interacting oscillators subjected to random forces (Tass 1999). This stochastic phase resetting approach has lead to the development of demand-controlled deep brain stimulation techniques (Tass 2001a-2001c, 2002a, 2002b). The latter work in a completely different way compared to the standard high-frequency stimulation: While the standard technique probably simply suppresses the neuronal firing in the target area (Wielepp et al. 2001), the novel techniques only desynchronize the firing whenever it gets pathologically synchronized. In this way, the novel methods intent to bring the neurons' dynamics as close to the physiological state (i.e. to the uncorrelated firing) as possible. The talk is about both theory and first experimental results.

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Synchronization tomography and phase resetting tomography:

Three-dimensional anatomical localization of spontaneous and stimulus-locked synchronization in the human brain

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Cerebral synchronization processes play an essential role under both physiological (Freeman 1975) and pathological (Llinás and Jahnsen 1982, Bergman et al. 1994) conditions. To detect and localize phase synchronization and stochastic phase resetting dynamics in the human brain non-invasively with magnetoencephalography novel methods have been developed:

1. *Synchronization tomography* (Tass et al. 2002): First, the cerebral current source density is reconstructed in each cerebral voxel (i.e. volume element) for each time t by means of the magnetic field tomography (MFT) (Ioannides et al. 1990). Next, the phase synchronization analysis (Tass et al. 1998) is applied to each voxel and to external reference signals such as muscular activity. In this way brain/brain- and brain/muscle phase synchronization are determined. It turns out that phase synchronization is a fundamental coordination principle in cerebral motor control (Tass et al. 2002).

2. *Phase resetting tomography:* The cerebral current source density is reconstructed with MFT. Next, a stochastic phase resetting analysis (Tass 1999, 2002a, 2002b) is applied to each voxel as well as to all pairs of voxels and external signals. This enables the detection of transient stimulus-locked response clustering and transient stimulus-locked synchronization and desynchronization. In contrast, standard techniques like cross-trial averaging or cross-trial cross-correlation are not able to detect such processes and may even produce artifacts.

The talk is about the theoretical background of the novel methods, their application to experimental data, and their diagnostic relevance.

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Clustering in granular gas I: Birth and sudden death of a cluster

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The key feature of a granular gas, making it fundamentally different from any ordinary molecular gas, is its tendency to form clusters (Goldhirsch and Zanetti, 1993; Kudrolli et al., 1997). This can be traced back to the inelasticity of the collisions between the particles. In applications such as conveyor belts and sorting machines, the clustering is an unwanted and very costly effect. Here we study the phenomenon in the setting of the so-called Maxwel Demon experiment (Eggers, 1999).

Granular material in N connected compartments is brought into a gaseous state through vertical shaking. For sufficiently strong shaking the particles are uniformly distributed over the compartments, but if the shaking intensity is lowered this uniform distribution gives way to a clustered state. The clustering transition is experimentally shown to be of 2nd order for $N = 2$ and of 1st order for $N \geq 3$. In particular, the latter is *hysteretic*, involves long-lived transient states, and exhibits a striking lack of time reversibility (Van der Weele et al., 2001; Van der Meer et al., 2001).

In the strong shaking regime, a cluster breaks down very abruptly (sudden death) and in its further decay shows anomalous diffusion, with the length scale going as $t^{1/3}$ rather than the standard $t^{1/2}$ (Van der Meer et al., 2002). We focus upon the self-similar nature of this process. The observed phenomena are all accounted for within a dynamical flux model.

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Clustering in granular gas II: David vs. Goliath and other competitive effects

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There are many extensions of the Maxwell Demon experiment. First we consider a bi-disperse mixture consisting of large and small particles (Mikkelsen et al., 2002). This is done with an eye to practical applications, where granular material is rarely mono-disperse. For moderate shaking the material clusters into the compartment which initially contained most of the large particles: Goliath wins. For very mild shaking, however, the cluster goes into the compartment originally dominated by small particles: David wins. These experimental observations are quantitatively explained within a bi-disperse version of the flux model.

Second, we study a system in which the compartments are arranged in the form of a staircase, resembling an industrial conveyor belt. The central topic here is the competition between the clustering effect and the natural tendency of the particles to stream downwards. When a cluster is formed, one can get rid of it by shaking sufficiently hard. The ensuing transition to the desired uniform flow is found to be a self-similar process involving Burgers-like shockwaves (Kloosterman et al., 2002).

Finally, we discuss two related clustering phenomena from other fields: the traffic jam problem (Helbing, 2001) and the formation of sand ripples at the beach (Andersen et al., 2001). Both turn out to be well described by flux models markedly similar to our own.

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Dissipation

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Dissipation is the irreversible transfer of energy into a reservoir, having a large number of degrees of freedom. I will consider the case where the reservoir is a system of nearly independent fermions. This includes dissipation by electrical conduction and dissipation in the dynamics of the nucleus. The lectures will demonstrate that this is a rich subject for investigation, with significant problems outstanding.

Lecture 1: Physical applications, and the standard approach (the Kubo formula). Energy diffusion as an alternative approach to dissipation. Classical-quantum correspondence. Limitations of the Kubo formula approach.

Complex quantum systems. Random matrix theory, and universality hypotheses. Dimensionless parameters describing response of complex systems. Parametric random matrix models. Matrix element sum-rules, and semiclassical estimates. Some parametric statistics.

Lecture 2: Estimates for energy diffusion constant in Landau-Zener and Kubo formula regimes. Predictions of various anomalous effects. Numerical experiments testing these predictions. Theoretical arguments reconciling random matrix and semiclassical predictions.

Much of the material is covered in *Parametric Random Matrices: Static and Dynamic Applications* , M. Wilkinson, in '*Supersymmetry and Trace Formulae*, eds. I. V. Lerner, J. P. Keating and D. E. Khmelnitskii, New York: Plenum, p.369-399, (1999). Several new results will be discussed.