Multiplicity of Limit Cycle Attractors in a Coupled Heteroclinic Cycle System

Extended Abstract for

LET'S FACE CHAOS THROUGH NONLINEAR DYNAMICS

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We study a coupled oscillator system in which each oscillator has a heteroclinic cycle attractor instead of a limit cycle attractor or phase oscillator. We employ a replicator system as the heteroclinic oscillator. The equation of the system is given as

$$\frac{dx_i}{dt}^{(u,v)} = x_i^{(u,v)}\Big(\sum_j^4 a_{ij}x_j^{(u,v)} - \sum_{j,k}^4 a_{jk}x_j^{(u,v)}x_k^{(u,v)}\Big)$$
$$+D(x_i^{(u+1,v)} + x_i^{(u,v+1)} + x_i^{(u-1,v)} + x_i^{(u,v-1)} - 4x_i^{(u,v)}), \tag{1}$$

with the parameter matrix

$$(a_{ij}) = \begin{pmatrix} 0 & -2 & -1 & 1 \\ 1 & 0 & -2 & -1 \\ -1 & 1 & 0 & -2 \\ -2 & -1 & 1 & 0 \end{pmatrix}. \tag{2}$$

A phase space of each replicator system is illustrated in Fig.1. Oscillators are distributed on a two-dimensional square lattice and coupled with the nearest neighbors diffusively. We adopt a open boundary condition (Fig.2).

With a large coupling constant, this system is taken as a partial differential replicator system and well-known rotating spiral wave patterns are observed (Fig.3-1). But the coupling constant is sufficiently small and the discreteness of the space is effective, we find many kinds of spatially disordered patterns (Fig.3-2,3-3).

We investigate these disordered patterns and show that they are limit cycle attractors in the ambient phase space (i.e. not chaotic) (Fig.4), and many limit cycles exist dividing the phase space as their basins (Fig.5). In addition, these patterns are constructed with a local law of difference of phases between the oscillators. The number of patterns grows exponentially with increasing of the number of oscillators (Fig.6).

Reference

[1] Hofbauer, J. and Sigmund, K. (1998), *Evolitionary Games and Population Dynamics*, Cambridge: Cambridge Univ. Press.

[2] Tachikawa, M. , in preparation.

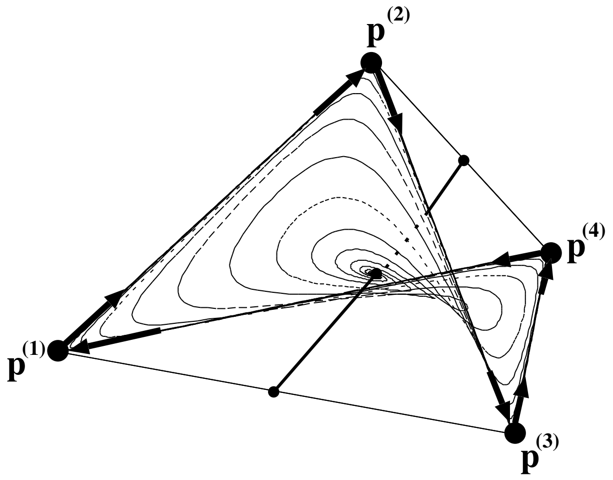

Fig.1 : A phase space of our replicator system and some trajectories that approach a heteroclinic cycle attrctor.

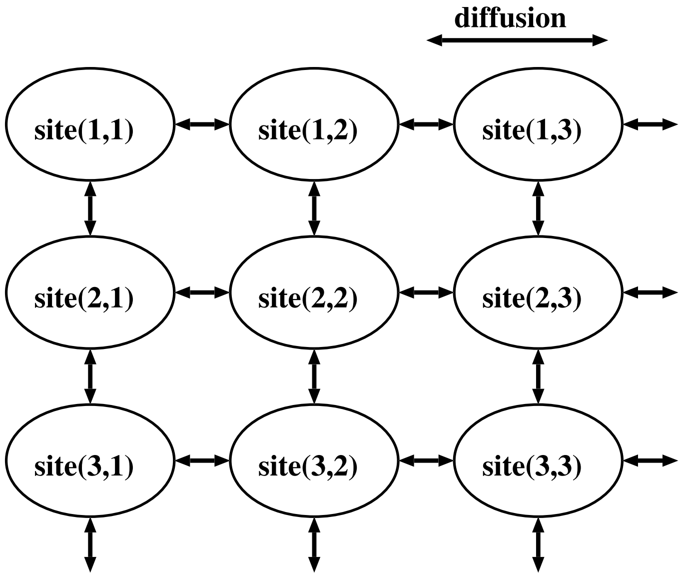

Fig.2 : A square lattice distribution.

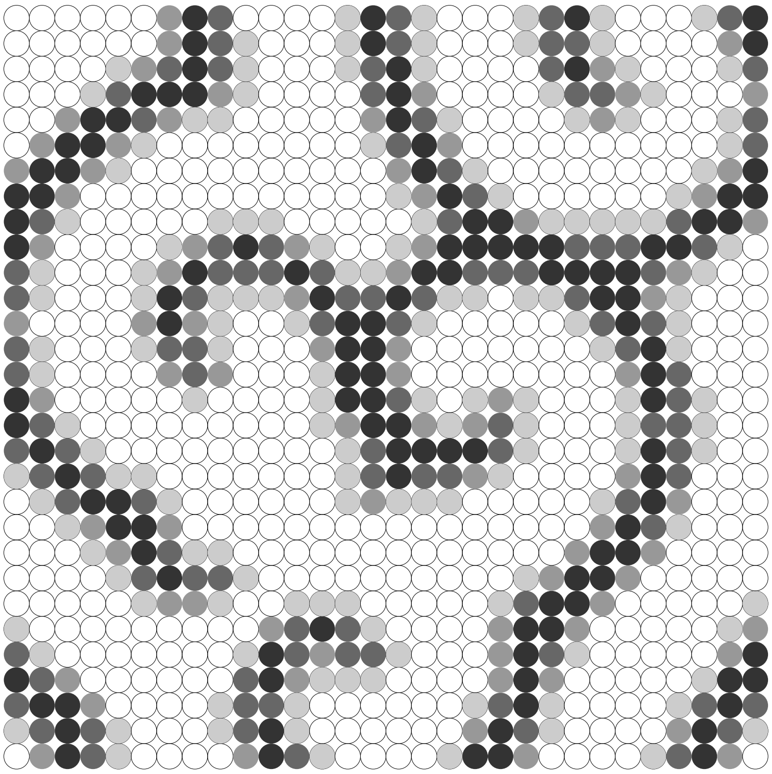

Fig.3-1 : A snapshot of distribution of x_1 (grey scale),
D=0.1, 30×30 sites system. A rotating spiral pattern is observed.

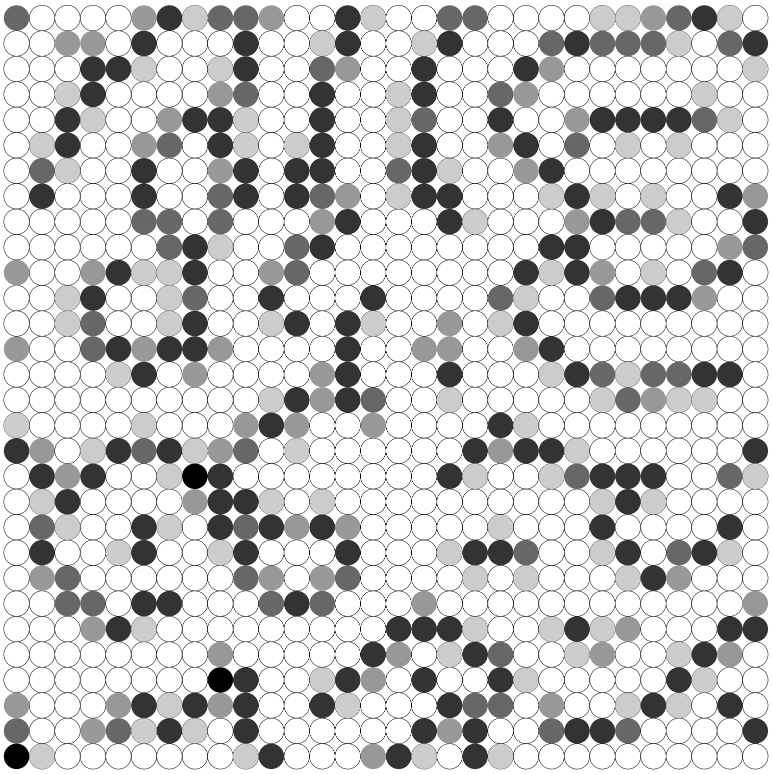

Fig.3-2 : A snapshot of distribution of x_1 (grey scale), D=0.01, 30×30 sites system.

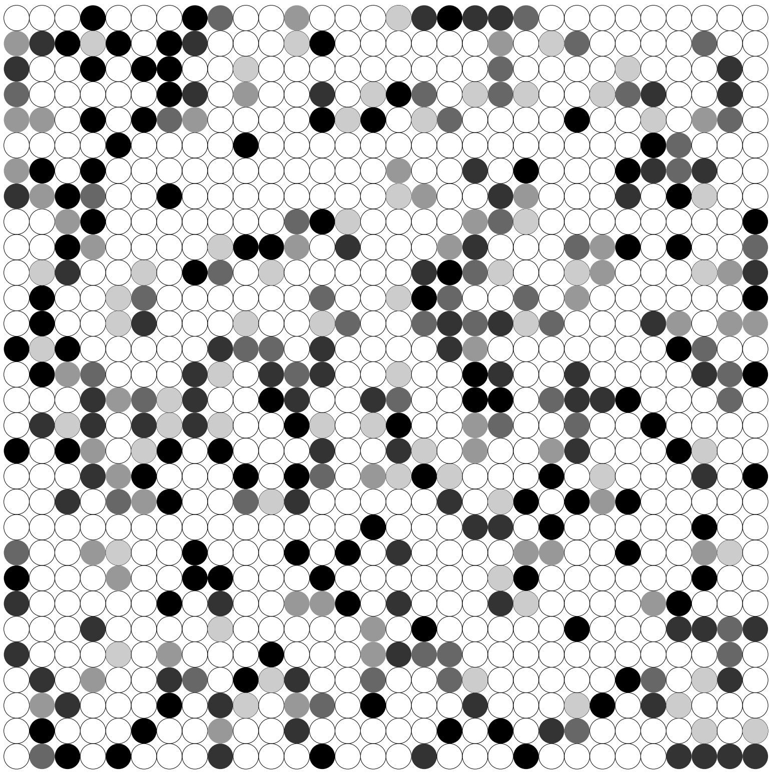

Fig.3-3 : A snapshot of distribution of x_1 (grey scale), D=0.001, 30×30 sites system.

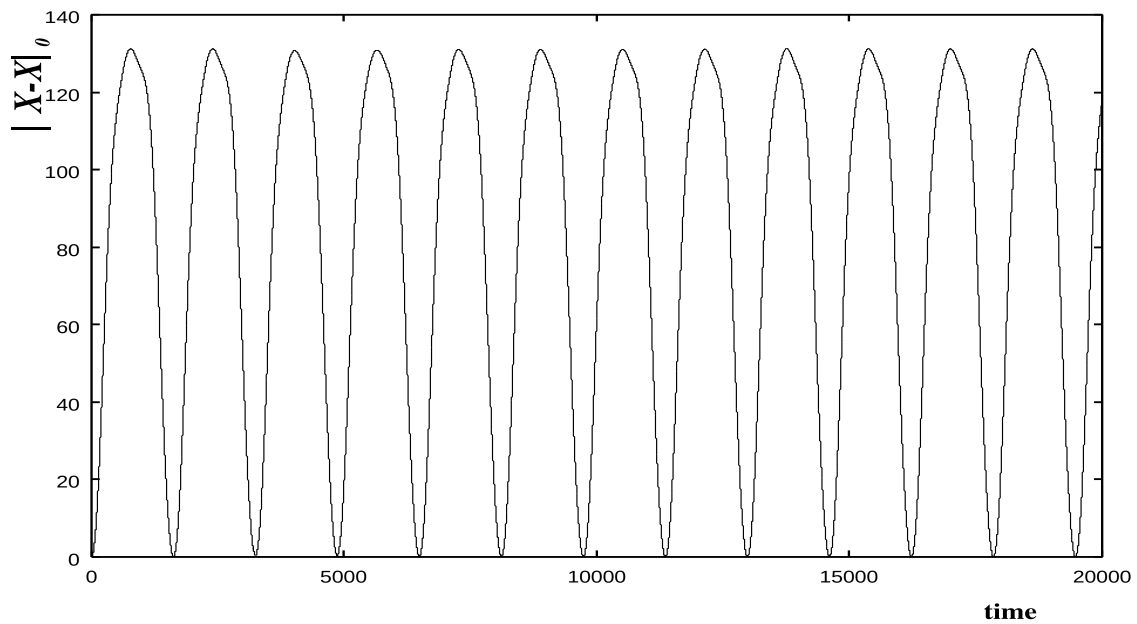

Fig. 4 : A limit cycle oscillation of a disordered pattern,
D=0.01, 10×10 sites system.

X is an orbit in the ambient phase space and $X_0(\stackrel{\text{def.}}{=} X(t_0))$ is a point on the orbit after it approaches an attractor sufficiently close. This figure shows that the orbit returns the same point in the phase space repeatedly. This property of such orbit is stable against small noise.

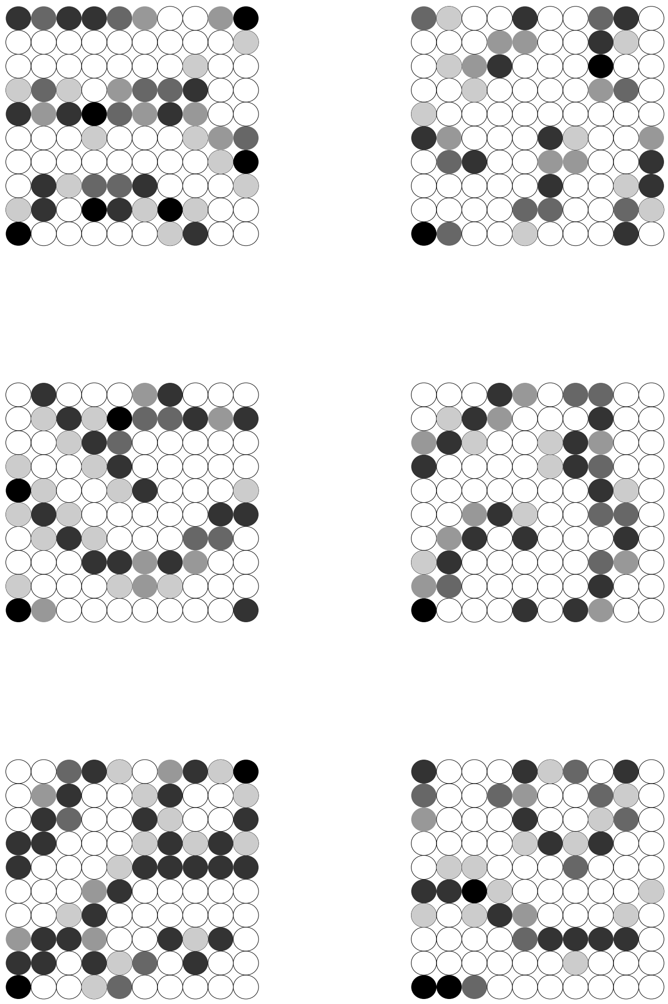

Fig. 5 : 6 snapshots of distributions of x_1 from different initial conditions (grey scale), D=0.01, 10×10 sites system.

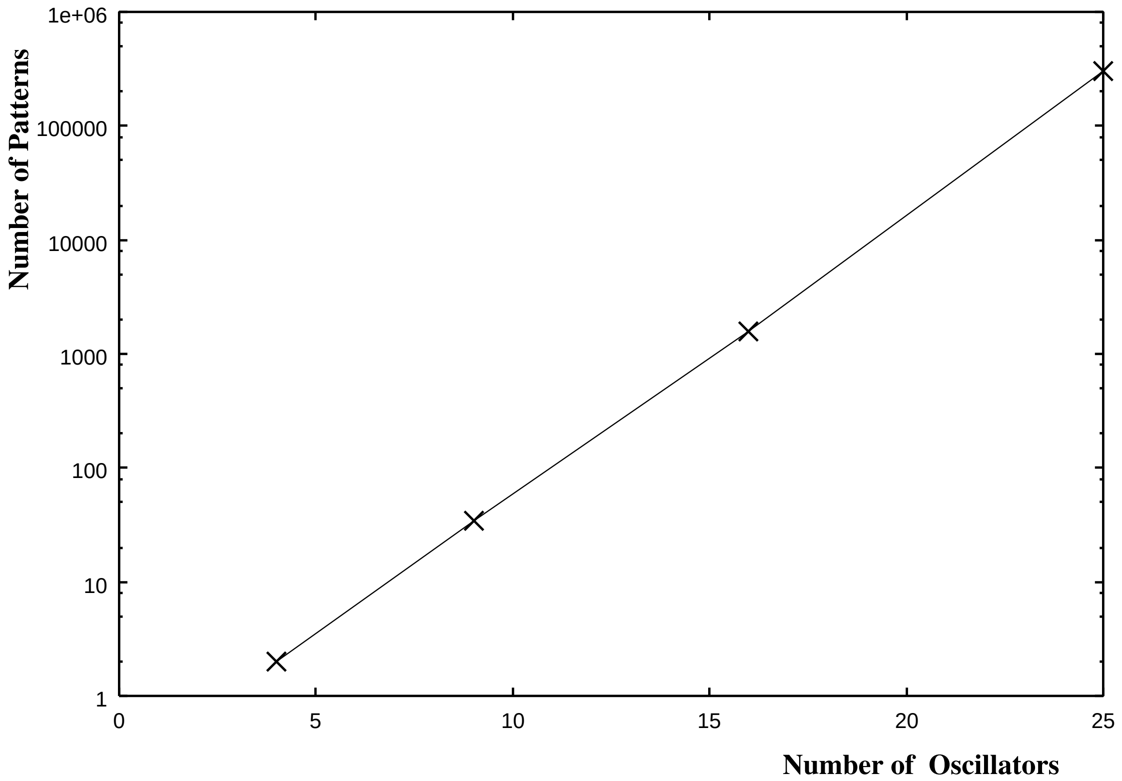

Fig. 6 : Growth of the number of attracting patterns with increasing of the number of oscillators. Values of $2^2, 3^2, 4^2, 5^2$ oscillators are plotted.