CAMTP

"Let's Face Chaos
through
Nonlinear Dynamics"

5th International
Summer School/Conference

at the University of Maribor
30 June - 14 July 2002

European Physical Society sponsored conference

Dedicated to the 75th Birthday of Professor Hermann Haken

Maribor **Slovenia**

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Foreword

The series of by now traditional international Summer Schools/Conferences "Let's Face Chaos through Nonlinear Dynamics" began in the year 1993 on the initiative of a group of undergraduate students of the various faculties at the University of Ljubljana, under the leadership of Ms. Maja Malus, a student of electrical engineering at the time (now a medical doctor at Harvard), under the scientific guidance of Professors Marko Robnik, Aneta Stefanovska and Igor Grabec. Since 1996 the Schools/Conferences are held exclusively at the University of Maribor, under the organization of CAMTP - Center for Applied Mathematics and Theoretical Physics, every three years. The 5th School/Conference is the largest, according to the number of invited lecturers (40) as well as the number of other participants (about 90), and also according to the richness of the scientific and cultural programme, which you can see in the following pages of this Book of Programme.

The character of our Schools/Conferences is strongly international, we have invited lecturers and participants from all over the world, from all continents, and the national component of the Slovenian participants in total (invitees and others) does not exceed 10%. They are strongly interdisciplinary, with the focus on the rich variety of problems in mathematics, natural sciences and engineering in the field of chaos, synergetics and theory of complex systems. As for the scientific level we believe that we are gathering the worldwide leadership and elite, not only among the invited speakers, but also quite pronounced in the other participants, most of them are very talented and productive young scientists from some of the best research groups in the world. So, our gatherings in Maribor have the following dimensions: High level science, internationality, interdisciplinarity, special attention to young students and scientists, promoting them and also helping them financially (especially for those coming from financially weak countries), and finally the cultural dimension which ties together science and life, in a cosmopolitan spirit, without any nationalisms, but rather in mutual respect of all cultures of the world. In fact, it is one of my by now quite popular sayings, that "The Science is the Culture of the World", it is a universal culture indeed, like music.

As the main organizer of the Schools/Conferences, I have made every effort to make your stay in Maribor scientifically as profitable as possible, also to make it culturally as much enjoyable as possible, hoping that you will not only obtain new knowledge, but also successfully present your own research work, and make new scientific collaborative links and creative friendships. This is the most important face of the Maribor gatherings, highly successful and appreciated so far.

Finally, I should like to emphasize the personal component of the 5th School/Conference, namely the fact that it is dedicated to the 75th Birthday of Professor Hermann Haken, the spiritual father and scientific founder and leader in the field of synergetics, the science of selforganizing phenomena in complex systems. We are all very much honoured by his presence in Maribor, he is visiting our Schools/Conferences in Maribor already for the third time (and this is his 5th visit to Slovenia), and it is a privilege for all of us to attend his brilliant lectures offering a pedagogical introduction to - as well as an account of - the current research work on synergetics. It is on Friday 12 July 2002, at 21:00 hours, that we shall start the official celebration of his 75th Birthday by a chamber music concert, by the Feguš Brothers String Quartet from Maribor, followed by a ceremony in the Hall of Knights of the Maribor Castle.

Last but not least, I should thank all the Members of the international Organizing Committee for their support and help: The Honorary Directors Giulio Casati, Predrag Cvitanović, Siegfried Grosssmann and Hermann Haken, and the Members: Yoji Aizawa, Tokyo, Tassos Bountis, Patras, Hiroshi Hasegawa, Tokyo, Igor Mozetič, Maribor, Jean-Louis Pichard, Saclay, Tomaž Prosen, Ljubljana, Andreas Ruffing, Munich and Aneta Stefanovska, Ljubljana. Among the local organizers my very special thanks go again to Igor Mozetič, from CAMTP, for his invaluable contribution in setting up all the computer databases, home pages, and information systems, and to Marko Vraničar, also from CAMTP, for his kind, reliable and perfect help in many important technical dimensions in the organizational work.

Our special gratitude must be acknowledged to our co-organizer University of Maribor represented by the Rector of the University of Maribor, Professor Dr. Ludvik Toplak, for providing the lecture halls and the facilities in the University Main Building, and to our respected general sponsors: Ministry of Education, Science and Sport of the Republic of Slovenia, The Nova Kreditna Banka Maribor, The European Physical Society, and other sponsors listed at the end of this book.

At the very end, thanks go to all participants for coming to Maribor and contributing to a productive and enjoyable friendly scientific atmosphere.

I wish you all a scientifically successful and culturally pleasant stay in Maribor, and of course, please enjoy the Maribor Festival Lent 2002, the fireworks, and all the cultural programme, the mountains, the excursions and trips, the Slovenian cuisine and wines, the concerts, and even Bolshoi Theatre from Moscow visiting Ljubljana!

Professor Dr. Marko Robnik
— Director of CAMTP —
— Director General of the Summer School/Conference —

Maribor, 16 June 2002

1st Week: 1 July - 6 July						
	MONDAY 1 July	TUESDAY 2 July	WEDNESDAY 3 July	THURSDAY 4 July	FRIDAY 5 July	SATURDAY 6 July
Chairman	*Robnik*	*Casati*	*Prosen*	*Seligman*	*Stefanovska*	*Chairman: Casati*
09:00 - 10:00	**Casati**	**Ruffing**	**Casati**	**Gaspard**	**Robnik**	Berglund 09:00-09:20
10:00 - 11:00	**Aizawa**	**Romanovski**	**Gaspard**	**Bountis**	**Aizawa**	Russ 09:20-09:40
11:00 - 11:30	– COFFEE & TEA –					Kuzmin 09:40-10:00
11:30 - 12:30	**Pichard**	**Pagon** 11:30-12:00 Kuvshinov 12:00-12:30	**Pichard**	**Pichard**	**Prosen**	Selyshchev 10:00-10:20
12:30 - 13:30	**Prosen**	**Grabec**	**Prelovšek**	**Prosen**	**Kuramoto**	Skokos 10:20-10:40
13:30 - 15:15	– LUNCH –					Sondergard 10:40-11:00
Chairman	*Aizawa*	*Pichard*	*Bountis*	*Gaspard*	*Kuramoto*	- COFFEE & TEA - 11:00-11:30
15:15 - 16:15	**Kuramoto**	**Bountis**	**Großmann**	**Großmann**	**Stefanovska**	*Chairman: Großmann*
16:15 - 16:45	– COFFEE & TEA –					Straube 11:30-11:50
16:45 - 17:45	**Tanner**	**Tanner**	**van der Weele**	**Fujisaka**	**Fujisaka**	Strunin 11:50-12:10
17:45 - 18:45	**van der Weele**	**Wilkinson**	**Wilkinson**	**Schumacher**	**Shudo**	Tomiya 12:10-12:30
	Angadi 18:45-19:05	19:00-20:30 DINNER	Dietz 18:45-19:05	Martinis 18:45-19:05	Veble 18:45-19:05	Tuval 12:30-12:50
	Budiyono 19:05-19:25	21:00-22:00 **Concert**	Dingjan 19:05-19:25	Lee S.-Y. 19:05-19:25	Žnidarič 19:05-19:25	13:05 LUNCH
	19:40 DINNER	22:00-23:30 **Reception**	19:40 DINNER	19:40 DINNER	19:40 DINNER	15:00-17:00 **Exc. City of Maribor**
						17:00-18:00 **Wine** 20:00 **Banquet**

2nd Week: 8 July - 13 July

	MONDAY 8 July	TUESDAY 9 July	WEDNESDAY 10 July	THURSDAY 11 July	FRIDAY 12 July	SATURDAY 13 July
Chairman	*Prosen*	*Tass*	*McClintock*	*Bohigas*	*Sonis*	*Cvitanović*
09:00 - 10:00	×	**Cvitanović**	**Cvitanović**	**Cvitanović**	**Cvitanović**	**Schmid-Schönbein**
10:00 - 11:00	**Haken**	**Haken**	**Haken**	**Haken**	**Haken**	**Tass**
11:00 - 11:30	– COFFEE & TEA –					
11:30 - 12:30	**Cvitanović**	**Stöckmann**	**Haake**	**Haake**	**Haake**	**Tass**
12:30 - 13:30	**Stöckmann**	**Bountis** 12:30-13:00 Bergamin 13:00-13:30	**Frank**	**McClintock**	**Matsushita**	**Bohigas**
13:30 - 15:15	– LUNCH –					
Chairman	*Stöckmann*	*Matsushita*	*Großmann*	*Haake*	*Stefanovska*	*Haken*
15:15 - 16:15	**Seligman**	**Großmann**	**Seligman**	**Mosekilde**	**Mosekilde**	**Bohigas**
16:15 - 16:45	– COFFEE & TEA –					
16:45 - 17:45	**Laskar**	**Laskar**	**Matsushita**	**Stefanovska**	**Schmid-Schönbein**	
17:45 - 18:45	**McClintock**	**Hasegawa**	**Nakamura**	**Rosenblum**	**Sonis**	
	19:00 DINNER	Leppänen 18:45-19:05	20:30 **Reception and festive Dinner**	Salasnich 18:45-19:05	19:00 DINNER	19:00 DINNER
		Vraničar 19:05-19:25		19:20 DINNER	21:00-22:00 **Concert**	21:00 **Cvetič**
		19:40 DINNER			22:00-24:00 **Haken**	

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Abstracts of Invited Lectures

On the Weibull distribution appeared in chaotic systems

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Various kinds of scaling laws are often observed in complex dynamical systems; for instance, SOC and punctuated equilibrium near the edge of chaos, 1/f spectrum, long time tails and anomalous diffusion in non-hyperbolic systems, where the self-similarity structures play essential roles in dynamical space and induce the breaking of central limit theorem for gaussian regime.

In the present paper we discuss three complex dynamical systems with non-gaussian scaling regime described by the Weibull distribution. Though the universality of Weibull distribution functions has not yet been made clear, but it is surmised that the Weibull regime is omnipresent in the systems under consideration.

1. Hamiltonian systems

Weibull distributions in hamiltonian dynamics were studied in (1): Mixmaster universe model [Prog. Theor. Phys. **98** No.6 (1997), 1225], and in (2) Cluster formation [Prog. Theor. Phys. **103** (2000), 519 ; Suppl. No.139 (2000), 1]. In particular, it was shown that the Arnold diffusion can be explained in terms of the universality of Log-Weibull distributions.

Here we discuss a new simplified model of random potential scattering, where Weibull distributions are numerically obtained.

2. Time dependent non-stationary chaotic map

It is known that the modified Bernoulli map reveals strong intermittency and anomalous large deviation properties [Prog. Theor. Phys. **99** (1989), 149 and **90** No.3 (1993), 547].

Here we consider the time dependent process where the system parameter is varying in the course of time. Surprising results are the followings ; the time dependent intermittency obeys the Weibull distribution in the intermediate long time scale, but in the intrinsic long time scale it obeys the Log-Weibull distribution.

3. A model of financial market

It is known in econophysics studies that the distribution of returns (change of price) obeys the stable (*Lévy*) distribution, but many theoretical models for financial market do not always display the stable law.

Here we study a multi-agent model [Chaos, Solitons & Fractals **11** (2000), 1077 ; 1739, Physica A **287** (2000), 507], and point out that the return distributions are well adjusted by the Weibull distribution function in wide parameter range.

Quantum chaos: Universal versus system-specific fluctuations

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It is by now well established that spectral fluctuations of quantum systems whose underlying classical motion is chaotic exhibit universalities. On the other hand, it is also well known that there are parameters characterizing a particular chaotic system. The purpose of the two lectures will be to illustrate, partly by studying particular examples, what makes some properties system-dependent and other universal. The general framework will be periodic orbit and random matrix theories.

Emphasis will be put on two properties not so widely discussed so far: i) spectral spacing autocorrelations, ii) total energies of (non-interacting) fermion systems. Besides a general discussion, two cases will be treated in detail: 1) zeros of the Riemann zeta function (an example of a 'chaotic' system for which all 'classical' information is well known), 2) the fluctuation of the binding energy of atomic nuclei (a system which is mainly regular but for which there is evidence that a (small) chaotic part is present).

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Soliton–like solutions of higher order wave equations of the KdV type

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In this work we study second and third order approximations of water wave equations of the KdV type. First we derive analytical expressions for solitary wave solutions for some special sets of parameters of the equations. Remarkably enough, in all these approximations, the form of the solitary wave and its amplitude–velocity dependence are identical to the sech^2–formula of the one–soliton solution of the KdV. Next we carry out a detailed numerical study of these solutions using a Fourier pseudospectral method combined with a finite–difference scheme, in parameter regions where soliton–like behavior is observed. In these regions, we find solitary waves which are stable and behave like solitons in the sense that they remain virtually unchanged under time evolution and mutual interaction. In general, these solutions sustain small oscillations in the form of radiation waves (trailing the solitary wave) and may still be regarded as stable, provided these radiation waves do not exceed a numerical stability threshold. Instability occurs at high enough wave speeds, when these oscillations exceed the stability threshold already at the outset, and manifests itself as a sudden increase of these oscillations followed by a blowup of the wave after relatively short time intervals.

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Localized oscillations in 1-dimensional nonlinear media and homoclinic orbits of invertible maps

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In recent years, a very interesting phenomenon has captured the imagination of many nonlinear scientists: The occurrence of stable, localized oscillations in 1 - and 2 - dimensional lattices. These oscillations have been termed discrete breathers, in analogy to similar solutions found in certain completely integrable continuous systems, like the sine Gordon and the Nonlinear Schrödinger partial differential equations.

In these lectures, we will first review the physical models in which discrete breathers were first discovered and studied. We will then outline the work of Aubry and MacKay who rigorously established the existence of stable discrete breathers in a wide class of infinite chains of linearly coupled anharmonic oscillators. It is interesting that, in many cases, breathers are indeed a discrete phenomenon, as they are not expected to exist in the continuum limit. Furthermore, they have also been recently observed in several experiments, notably some involving arrays of coupled Josephson junctions.

We shall demonstrate that discrete breathers correspond, in fact, to homoclinic orbits at the intersections of invariant manifolds of a saddle point, lying at the origin of a 2N - dimensional map in Fourier space. Exploiting this geometric approach, we will show that discrete breathers can be accurately approximated and even classified using ideas of symbolic dynamics.

In this way, a great variety of such forms (also called multibreathers) can be constructed most of which are found to be linearly unstable. Thus, developing methods for computing homoclinic orbits of invertible maps allows us to obtain accurate representations of discrete breathers and stabilize (or destabilize) them using techniques of continuous feedback control.

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Efficient quantum computing of complex dynamics

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In interacting many body systems such as nuclei, complex atoms, quantum dots, and quantum spin glasses, the interaction leads to quantum chaos characterized by ergodicity of eigenstates and level spacing statistics as in Random Matrix Theory. In this regime, a quantum computer eigenstate is composed by an exponentially large number of quantum register states and the computer operability is destroyed. Here we model an isolated quantum computer as a two-dimensional lattice of qubits (spin halves) with fluctuations in individual qubit energies and residual short-range inter-qubit couplings. We show that above a critical inter-qubit coupling strength, quantum chaos sets in and this results in the interaction induced dynamical thermalization and the occupation numbers well described by the Fermi-Dirac distribution. This thermalization destroys the noninteracting qubit structure and sets serious requirements for the quantum computer operability. We then construct a quantum algorithm which uses the number of qubits in an optimal way and efficiently simulates a physical model with rich and complex dynamics. The numerical study of the effect of static imperfections in the quantum computer hardware shows that the main elements of the phase space structures are accurately reproduced up to a time scale which is polynomial in the number of qubits. The errors generated by these imperfections are more significant than the errors of random noise in gate operations.

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M-theory and particle physics

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Recent developments in string and M-theory are reviewed with an emphasis on particle physics implications. Aspects of non-perturbative extended objects - branes are introduced. The focus is on the role these objects play in the construction of new four-dimensional solutions of string theory with the structure of the standard model and three families of quarks and leptons. A beautiful relationship of these constructions to purely geometric one, as an M-theory compactified on special holonomy spaces is highlighted.

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Chaos and what to do about it: An overview

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That deterministic dynamics leads to chaos is no surprise to anyone who has tried pool, billiards or snooker - that is what the game is about - so we start our course about what is chaos and what to do about it by a game of pinball. This might seem a trifle trivial, but a pinball is to chaotic dynamics what a pendulum is to integrable systems: thinking clearly about what is "chaos" in a pinball will help us tackle more difficult problems, such as computing diffusion constants in deterministic gases, or computing the Helium spectrum.

We all have an intuitive feeling for what a pinball does as it bounces between the pinball machine disks, and only highschool level Euclidean geometry is needed to describe the trajectory. Turning this intuition into calculation will lead us, in clear physically motivated steps, to almost everything one needs to know about deterministic chaos: from unstable dynamical flows, Poincaré sections, Smale horseshoes, symbolic dynamics, pruning, discrete symmetries, periodic orbits, averaging over chaotic sets, evolution operators, dynamical zeta functions, Fredholm determinants, cycle expansions, quantum trace formulas and zeta functions, and to the semiclassical quantization of helium.

Reference
Read chapter 1 and appendix A of P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, `http://www.nbi.dk/ChaosBook/`.

Dynamics, qualitative

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Confronted with a potentially chaotic dynamical system, we analyze it through a sequence of three distinct stages; diagnose, count, measure. First, we determine the intrinsic *dimension* of the system - the minimum number of degrees of freedom necessary to capture its essential dynamics. If the system is very turbulent (its attractor is of high dimension) we are, at present, out of luck. We know only how to deal with the transitional regime between regular motions and weak turbulence. In this regime the chaotic dynamics is restricted to a space of low dimension, the number of relevant parameters is small, and we can proceed to the second step; we *count* and *classify* all possible topologically distinct trajectories of the system. If successful, we can proceed with the third step: investigate the *weights* of the different pieces of the system.

In this lecture qualitative dynamics of simple stretching and mixing flows is used to introduce Smale horseshoes and symbolic dynamics, and the topological dynamics is encoded by means of transition matrices/Markov graphs.

We learn how to count and describe itineraries. While computing the topological entropy from transition matrices/Markov graphs, we encounter our first zeta function.

By now we have covered for the first time the whole distance from diagnosing chaotic dynamics to computing zeta functions. Historically, these topological zeta functions were the inspiration for injecting statistical mechanics into computation of dynamical averages; Ruelle's zeta functions are a weighted generalization of the counting zeta functions.

Reference
Read chapters 2, 3, 10 and 11 of P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, `http://www.nbi.dk/ChaosBook/`.

Global dynamics

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This lecture is the core of the course: we discuss the necessity of studying the averages of observables in chaotic dynamics, and cast the formulas for averages in a multiplicative form that motivates the introduction of evolution operators.

In chaotic dynamics detailed prediction is impossible, as any finitely specified initial condition, no matter how precise, will fill out the entire accessible phase space (similarly finitely grained) in finite time. Hence for chaotic dynamics one does not attempt to follow individual trajectories to asymptotic times; what is possible (and sensible) is description of the geometry of the set of possible outcomes, and evaluation of the asymptotic time averages. Examples of such averages are transport coefficients for chaotic dynamical flows, such as the escape rate, mean drift and the diffusion rate; power spectra; and a host of mathematical constructs such as the generalized dimensions, Lyapunov exponents and the Kolmogorov entropy. We shall now set up the formalism for evaluating such averages within the framework of the periodic orbit theory. The key idea is to replace the expectation values of observables by the expectation values of generating functionals. This associates an evolution operator with a given observable, and leads to formulas for its dynamical averages.

If there is one idea that you should learn about dynamics, it happens in this lecture(s) and it is this: there is a fundamental local - global duality which says that (global) eigenstates are dual to the (local) periodic geodesics. For dynamics on the circle, this is called Fourier analysis; for dynamics on well-tiled manifolds this is called Selberg trace formulas and zeta functions; and for generic nonlinear dynamical systems the duality is embodied in trace formulas, zeta functions and spectral determinants that we will now introduce. These objects are to dynamics what partition functions are to statistical mechanics. The bold claim is that once you understand this, classical ergodicity, wave mechanics and stochastic mechanics are nothing but special cases, to be worked out at your leisure.

The strategy is this: Global averages such as escape rates can be extracted from the eigenvalues of evolution operators. The eigenvalues are given by the zeros of appropriate determinants. One way to evaluate determinants is to expand them in terms of traces, log det = tr log. The traces are evaluated as integrals over Dirac delta functions, and in this way the spectra of evolution operators become related to periodic orbits.

The rest of the course is making sense out of this objects and learning how to apply them to evaluation of physically measurable properties of chaotic dynamical systems.

Reference

Read chapters 5, 6, 7, 8 and 9 of P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, `http://www.nbi.dk/ChaosBook/`.

Cycle expansions: Semiclassical quantum mechanics

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In last lecture we have derived a plethora of periodic orbit trace formulas, spectral determinants and zeta functions. Now we learn how to expand these as cycle expansions, series ordered by increasing topological cycle length, and evaluate average quantities like escape rates. These formulas are exact, and, when the winds are kind, highly convergent. The pleasant surprise is that the terms in such expansions fall off exponentially or even faster, so that a handful of shortest orbits suffices for rather accurate estimates of asymptotic averages.

The course now shifts gear to recent advances in the periodic orbit theory of chaotic, non-integrable systems, and the modern generalization of the De Broglie - Bohr quantization of hydrogen atom.

Instead of quantizing by suspending standing-wave configurations on stable Keplerian orbits, one suspends the standing-wave configurations on the infinity of unstable orbits. Such unstable periodic orbits are observed experimentally in the helium atom, the hydrogen in strong external fields, and other systems.

This is what could have been done with the old quantum mechanics if physicists of 1910's were as familiar with chaos as you by now are. The Gutzwiller trace formula together with the corresponding spectral determinant, the central results of the semiclassical periodic orbit theory, are derived.

The helium atom spectrum can then be computed via spectral determinants.

Reference

Read chapters 13, 21 and 22 of P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, `http://www.nbi.dk/ChaosBook/`.

Trace formulas for stochastic evolution operators

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Intuitively, the noise inherent in any realistic system washes out fine details and makes chaotic averages more robust. Quantum mechanical $\hbar$ resolution of phase space implies that in semi-classical approaches no orbits longer than the Heisenberg time need be taken into account. We explore these ideas in some detail by casting stochastic dynamics into path integral form and developing perturbative and nonperturbative methods for evaluating such integrals. In the weak noise case the standard perturbation theory is expansion in terms of Feynman diagrams. Now the surprise; we can compute the same corrections faster and to a higher order in perturbation theory by integrating over the neighborhood of a given saddlepoint exactly by means of a nonlinear change of field variables. The new perturbative expansion appears more compact than the standard Feynman diagram perturbation theory; whether it is better than traditional loop expansions for computing field-theoretic saddlepoint expansions remains to be seen, but for a simple system we study the result is a stochastic analog of the Gutzwiller trace formula with the $\hbar$ corrections so far computed to five orders higher than what has been attainable in the quantum-mechanical applications.

Resume

A motion on a strange attractor can be approximated by shadowing the orbit by a sequence of nearby periodic orbits of finite length. This notion is here made precise by approximating orbits by primitive cycles, and evaluating associated curvatures. A curvature measures the deviation of a longer cycle from its approximation by shorter cycles; the smoothness of the dynamical system implies exponential (or faster) fall-off for (almost) all curvatures. The technical prerequisite for implementing this shadowing is a good understanding of the symbolic dynamics of the classical dynamical system. The resulting cycle expansions offer an efficient method for evaluating classical and quantum periodic orbit sums; accurate estimates can be obtained by using as input the lengths and eigenvalues of a few prime cycles.

To keep exposition simple we have here illustrated the utility of cycles and their curvatures by a pinball game. Glancing back, we see that the formalism is very general, and should work for any average over any chaotic set which satisfies two conditions: 1. the weight associated with the observable under consideration is multiplicative along the trajectory; 2. the set is organized in such a way that the nearby points in the symbolic dynamics have nearby weights.

Reference

Read articles in the "Chaotic Field Theory" section of
`http://www.nbi.dk/~predrag/papers/preprPOT.html`
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Relaxation dynamics described by nonlinear Fokker-Planck equations: applications to human movement sciences

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Mesoscopic stochastic descriptions of many particle systems have frequently been used in the context of mean field nonlinear Fokker-Planck equations (Desai and Zwanzig 1978; Kuramoto 1984) and nonlinear Fokker-Planck equations related to nonextensive entropies (Plastino and Plastino 1995). We discuss a recent attempt to obtain a unified description for stochastic relaxation processes that are characterized by mean field interactions, on the one hand, and nonextensivity, on the other hand (Frank 2001b).

First, we consider the stationary case. To this end, we introduce the concept of inverse distortion functions (Frank, Daffertshofer 1999). For mean field models we obtain from inverse distortion functions implicit descriptions of stationary distributions that involve transcendent equations and can be used to address the phenomenon of multistability. For nonextensive systems we obtain cut-off and power-law distributions.

Second, we examine the transient case. We briefly discuss exact time-dependent solutions of systems related to the Sharma-Mittal entropy (Frank, Daffertshofer 2000). Then, H-theorems are developed on the basis of (i) inverse distortion functions and (ii) free energy measures (Shiino 1987,2001; Bonilla et al. 1998; Frank 2001a,b,2002; Kaniadakis 2001).

Third and finally, we study the stability of stationary distributions for a stochastic mean field model that is in line with the field theoretical description of neural activity proposed by Haken (1996) and Jirsa and Haken (1996) and can describe neural activity during rhythmic finger movements (Frank et al. 2000). We focus on two approaches: the transcendent equation approach and Lyapunov's direct method (Frank et al. 2001).

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Generalized extended self-similarity in turbulence and its scaling hypothesis

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One of eminent statistical characteristics of homogeneous, isotropic developed turbulence in the three dimensional system is the self-similar energy cascade in the wavenumber space. This results in the power law behavior $S_q(r) \sim r^{\zeta(q)}$ for the velocity structure function $S_q(r)$, the q-th moment of the velocity difference at two positions separated by r. $\zeta(q)$ is a universal function of q. Several years ago, it was found that even when the turbulence is not fully developed, i.e., the Reynolds number is not extremely high and the the scaling range where the above scaling law holds is not wide enough, scaling behaviors of $S_q(r)$ which has an extended form of that in developed turbulence holds. They are called the extended self-similarity (ESS) and the generalized extended self-similarity (GESS). I will talk about a phenomenological derivation of ESS and GESS, proposing a new scaling hypothesis on the basis of the large deviation theory of probability theory. Furthermore, using numerical and experimental data, I will examine the validity of the present approach.

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Noise-induced pattern dynamics and intermittency

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Intermittency is a quite ubiquitous phenomenon in nonlinear dynamics. The intermittency observed when a particular dynamical state undergoes the instability is called the modulational (often called the on-off) intermittency. Recently, an experimental confirmation of the on-off intermittency in the electrohydrodynamic convection in nematics under dichotomous noise was reported by John *et al.*. An eminent statistics of the observation is the intermittent generation of convective pattern.

In my talk, in order to elucidate the experiment I will first propose a phenomenological nonlinear stochastic model which has the structure of the Swift-Hohenberg equation for local convection variable with fluctuating threshold. Then, I will discuss results of numerical integration of the model equation associated with the intermittent emergence of convective pattern. Detailed analysis on the statistics of the intermittent pattern dynamics will be addressed.

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Hamiltonian chaos and statistical mechanics

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In systems of statistical mechanics, the chaotic dynamics is characterized by Lyapunov exponents which are of the order of the inverse of the intercollisional time between the particles. This time scale is the one of kinetics. Instead, the relaxation toward the thermodynamic equilibrium occurs on the longer time scale of hydrodynamics which is determined by transport properties such as diffusion, viscosity, or heat conductivity.

The connection between the chaotic and transport properties can be established thanks to the escape-rate theory or a newer theory which allows us to directly construct the hydrodynamic modes of diffusion and reaction-diffusion [1,2]. These hydrodynamic modes turn out to present fractal properties with a fractal dimension given in terms of the transport coefficients. The fractal character of the hydrodynamic modes results from the stretching and folding of nonequilibrium inhomogeneities induced by the chaotic dynamics. This mixing naturally leads to the entropy production expected from nonequilibrium thermodynamics [3].

The first lecture will be devoted to the relationship between chaos, transport properties, and entropy production. The second lecture will present the results of recent work on chaos in systems composed of many interacting particles [4].

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Experimental modeling of chaotic fields

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Physical description of natural laws is based on evolution equations of fields but their analytical formulation is often not possible for very complex chaotic phenomena [1]. In the lecture we show how a method of chaotic time series prediction can be generalized to statistical modeling of chaotic fields [2, 3]. For this purpose we assume that a record of the field is provided by an experiment and that the field amplitude $\phi(\mathbf{s})$ at a point of observation $\mathbf{s}$ is related to amplitudes in a surrounding region. We represent the field values in the surrounding region by the vector $\mathbf{g}(\mathbf{s})$, and describe the field evolution by the mapping equation

$$\phi(\mathbf{s}) = G(\mathbf{g}(\mathbf{s})), \tag{1}$$

in which the function G is estimated statistically. For this purpose N samples of the joint state vector $\{(\phi(\mathbf{s}_i), \mathbf{g}(\mathbf{s}_i)) = (\phi_i, \mathbf{g}_i); i = 1 \dots N\}$ are first extracted from the given record. As an optimal non-parametric estimator of the field at point $\mathbf{s}$ we employ the conditional average, which is expressed by

$$\hat{\phi}(\mathbf{s}) = \mathrm{E}[\phi(\mathbf{s})|\mathbf{g}(\mathbf{s})] = \frac{1}{N} \sum_{n=1}^{N} B_n(\mathbf{g}(\mathbf{s}))\phi_n \tag{2}$$

Here $B_n(\mathbf{g}(\mathbf{s})) = w(\mathbf{g}(\mathbf{s}) - \mathbf{g}_n) / \sum_{k=1}^{N} w(\mathbf{g}(\mathbf{s} - \mathbf{g}_k))$ describes a similarity between the given vector $\mathbf{g}(\mathbf{s})$ and a sample $\mathbf{g}_n$, while w denotes a kernel function, such as Gaussian. The vector $\mathbf{g}(\mathbf{s})$ is considered as a given condition and is comprised from field values in the surrounding of point $\mathbf{s}$. During the calculation of the conditional average the surrounding can be arbitrary selected, which is an advantage of non-parametric estimator. To calculate a field distribution in some domain, the field must be first specified in a sub-domain. From given values the field distribution in the surrounding of sub-domain can be estimated by Eq. 2. The estimated values are then considered as given ones and the procedure of field estimation is iteratively continued. In the lecture optimal statistical methods for selection of surrounding region of a point $\mathbf{s}$ and self-organized determination of number N are explained. Various examples of estimated chaotic filed distributions, such as profiles of a rough surface [3, 4], charge density and electron temperature in turbulent ionization waves in plasma, etc, are demonstrated. The performance of the proposed statistical modeling is described by comparing correlation functions and spectra of experimentally recorded and estimated fields.

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Turbulent diffusion

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Dust, aerosoles, smoke and other air pollutions spread, in still air, by molecular diffusion. In addition there can be advection by atmospheric flow, either well directed in jets or winds, or nondirected in air turbulence. Diffusional spreading by turbulence is extraordinary effective. Instead of the well known linear growth of the mean square particle distance with time t

$$\sigma_t^2 = 6Kt \text{ , regular diffusion ,}$$

σ_t^2 grows much faster under turbulent advection, according to a cubic t dependence

$$\sigma_t^2 = c\epsilon t^3 \text{ , turbulent diffusion .}$$

This was discovered by Lewis Fry Richardson (1926, 1929) when he performed his ingenious study of the turbulent diffusivity K_{turb} . He presented data on impressively many scales. A bulk of later measurements confirmed his results, as e.g. balloon campaignes, cf. Lundgren (1981).

The anomalously fast particle distance growth could be explained from fluid dynamics (Navier-Stokes equations) by Grossmann and Procaccia (1984), Effinger and Grossmann (1984), Grossmann (1990) in mean field approximation. New results on dynamical Lagrangian time correlation decay (numerical, Grossmann and Wiele 1997 and analytical, Daems et al. 1999) stimulated efforts to even determine the absolute magnitude as characterized by the prefactor c in addition to the scaling exponent (Grossmann 2002).

In particular the memory effects in the time correlation decay have turned out to be very important. And turbulent intermittency implies additional scale dependence of the turbulent diffusivity (Grossmann 2002). A survey on the past development and on the most recent surprising findings is offered in the lecture.

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Turbulent correlation decay

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The appropriate tool to study the **dynamics** of a statistical system is its (stationary) time correlation function. This has also proven to be true for turbulent fluid flow. The observable quantities of interest are here the Lagrangian, scale dependent eddies $v(r)$, i.e., the Eulerian velocity differences $\mathbf{v}(\mathbf{r};\mathbf{x},t) = \mathbf{u}(\mathbf{x}+\mathbf{r},t) - \mathbf{u}(\mathbf{x},t)$. These objects are statistically time t and space $\mathbf{x}$ independent in stationary and homogeneous turbulence, but depend besides on scale r on the time lapse τ between two observations of an r-eddy at times t and $t+\tau$. By means of a continued fraction expansion the time correlation function can be uniquely expressed in terms of the static, stationary, time independent structure functions; but all orders of those are needed.

Analysis of the dynamical time correlation function $D(r,\tau) = \langle v(r)v(r,\tau)\rangle$ is presented. $D(r,\tau)$ was first studied in 1-pole approximation (Grossmann and Thomae 1982), because no estimate of higher order stationary structure functions was available then. Since Grossmann and Wiele (1997) provided numerical data for large Reynolds number, highly turbulent flow, it became clear that the memory effects surprisingly *reduce* the decorrelation time. The additional effects of turbulent intermittency were elucidated analytically in Daems et al. (1999). The main, rather unexpected results are:

i. The static multifractality of turbulent flow destroys dynamical scaling despite good scaling of stationary moments, i.e., of power laws in r of all p^{th} order structure functions. ii. The deviations from dynamical scaling are a direct measure of the strength of intermittency. iii. The scale dependence of the correlation decay rate can be expressed approximately in terms of the turbulent structure function of 2^{nd} order (Grossmann 2002).

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Turbulent heat convection

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Turbulent heat transport in fluid layers heated from below is one of the most intensely studied fluid flow problems. In 1900 Bénard detected interesting pattern formation in this system, in 1916 Rayleigh calculated the underlying instability. If the heating is increased further, chaotic motion and finally turbulence is observed in the bulk , surrounded by Blasius type boundary layers near the plates and walls. In the $50th$ and $60th$ power laws for the heat transport as a function of the thermal driving were suggested for the Rayleigh-Bénard system in accordance with other scaling behavior in turbulence,

$$Nu \propto Ra^{\beta} Pr^{\beta'} \ .$$

The Nusselt number Nu is the effective heat current including turbulent advection, nondimensionalised with the molecular heat current. The Rayleigh number Ra measures the thermal driving due to the temperature difference Δ across the fluid layer of height L, originating from buoyancy in the gravitational field g by thermal expansion α, which is counteracted by the fluid's kinematic viscosity ν and its thermal diffusivity κ,

$$Ra = \frac{\alpha g L^3 \Delta}{\nu \kappa} \ .$$

The Prandtl number $Pr = \nu/\kappa$ weighs the ratio of the space and time scales according to *molecular* momentum (ν) and energy (κ) transport. Recent measurements of increasing accuracy (Libchaber group (Castaing et al. 1989), Cioni et al. (1997), Chavanne et al. (1997), Niemela et al. (2000), Xu et al. (2000), Ahlers et al. (2001), Xia et al. (2002)) led to various unexpected results. A unifying theory explains all these data (Grossmann and Lohse 2000, 2001, 2002). In particular the theory's prediction is confirmed that simple power laws are insufficient to describe Nu versus Ra adequately. In the lecture the theory of heat convection in a very extended $Ra - Pr-$parameter space of up to 10 orders of magnitude is presented and discussed.

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Introduction to decoherence

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Quantum superpositions tend to decohere to mixtures, due to dissipative environmental influence. In particular, two superposed wave packets loose their relative coherence the faster the larger is their distance d. For sufficiently large d, the relative phase of packets is lost before any deformation of the shapes of the individual packets and any change of their distance become noticeable.

I shall illustrate the phenomenon by discussing recent efforts to take decoherence under experimental control (diffraction of Fullerenes in Vienna; superpositions of coherent states of microwave resonator modes in Paris; superpositions of wave packets of Be ions in Paul traps in Boulder; superpositions of states with counterpropagating supercurrents in Delft and Stony Brook).

All of these experiments observe superpositions of packets whose distance d is larger than the individual width λ, but the ratio d/λ achieved is still so moderate that the environment imposed decoherence time $\tau_{\rm dec}$ is, while shorter than the time scale $\tau_{\rm diss}$ of dissipative changes of d, still longer than typical oscillation periods $\tau_{\rm sys}$ of the isolated system, i.e. $\tau_{\rm sys} < \tau_{\rm dec} < \tau_{\rm diss}$. The appropriate theoretical treatment is thus based on Fermi's Golden Rule or, equivalently, Markovian master equations.

I illustrate golden rule type decoherence for the damped harmonic oscillator, using the simple master equation familiar from quantum optics. The important prediction is $\tau_{\rm dec}/\tau_{\rm diss} = (\lambda/d)^2$. The underlying perturbative treatment of the system-environment interaction requires the self-consistency condition $\tau_{\rm sys} \ll \tau_{\rm dec}$.

Emergence of classical behavior in the macroworld: meso- and macroscopic superpositions

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The golden-rule prediction breaks down once the distance d between the superposed wave packets is so large, compared to the quantum scale of reference λ, that $\tau_{\rm dec} \lesssim \tau_{\rm sys}$. The golden rule can therefore not be invoked to explain the notorious absence of quantum interference effects from the macroworld. In the limit $\tau_{\rm dec} < \tau_{\rm sys}$ decoherence obviously no longer is a weak-damping phenomenon. A simple solution of the system $\oplus$ environment Schrödinger equation becomes possible when $\tau_{\rm dec} \ll \tau_{\rm sys}$, the limit of relevance for superpositions of macroscopically distinct wave packets. The simplicity of that limit rests on the fact that in the full Hamiltonian $H = H_{\rm sys} + H_{\rm bath} + H_{\rm int}$ the free-system part $H_{\rm sys}$ becomes an effectively small perturbation. The decoherence time scale is then found to obey the power law $\tau_{\rm dec} \propto \hbar^{\mu}/d^{\nu}$ with positive exponents μ, ν. That law is a universal one, independent of the character of the system and the environment. It is only based on the interaction Hamiltonian $H_{\rm int}$ additively involving a large number of degrees of freedom such that the central limit theorem holds for the reservoir means met with.

After treating the universal asymptotics of the limit $\tau_{\rm dec} \ll \tau_{\rm sys}$ I shall briefly discuss the crossover from that interaction dominated regime to golden-rule type decoherence. The crossover is system specific; I shall rely on an exactly solvable model system, an harmonic oscillator coupled to a reservoir which itself consists of harmonic oscillators.

Quantum measurement

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Textbook wisdom has it that superpositions $|\psi\rangle = \sum_i c_i|\psi_i\rangle$ of eigenstates ψ_i of some observable of a microscopic object "collapse", under measurement of that observable, to a mixture, $|\varphi|\rangle\langle\varphi| \xrightarrow{\text{collapse}} \sum_i |c_i|^2|\varphi_i\rangle\langle\varphi_i|$, with probabilities $|c_i|^2$ as in the original pure state but all coherences $c_i^*\ c_j$ for $i \neq j$ gone. Such collapse can be understood as due to unitary time evolution of a tripartite system comprising, besides the micro-object, a macroscopic pointer (idealizable as having a single degree of freedom), and a many-freedom environment. The Hamiltonian must allow micro-object and pointer to become entangled as

$$\begin{aligned} \varphi^{\mathrm{obj}}(0)\psi^{\mathrm{point}}(0) &= \left(\sum_i c_i\varphi_i\right)\psi^{\mathrm{point}}(0) \\ &\to \textstyle\sum_i c_i\varphi_i\psi_i^{\mathrm{point}} \qquad , \end{aligned}$$

with the various pointer states ψ_i^{point} corresponding to macroscopically distinct pointer displacements. Concomitantly, the many-freedom environment will decohere the superposition to the mixture $\sum_i |c_i|^2|\varphi_i\ \rangle\langle\varphi_i| \otimes |\psi_i^{\mathrm{point}}\rangle\langle\psi_i^{\mathrm{point}}|$; inasmuch as the pointer states $|\psi_i^{\mathrm{point}}\rangle$ are macroscopically distinct, the decay of the coherences $c_i^*\ c_j$ will occur with $\tau_{\mathrm{dec}} \ll \tau_{\mathrm{syst}}, \tau_{\mathrm{diss}}$, i.e. might appear as practically instantaneous. By disregarding all information about (i.e. tracing over) the pointer, one has the textbook-wisdom mixture for the micro-object.

I shall present exactly solvable models both for both processes involved, entanglement and decoherence.

Brain dynamics

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Lecture 1: Some basic facts about the brain

The human brain is a highly complex system that is composed of about hundred billion neurons each of which may interact with up to ten thousand other neurons. The brain serves many purposes as recognition of our surrounding, steering of movement, emotions, etc. Roughly speaking, the brain consists of individual areas that serve specific purposes. But these areas are again strongly interconnected. The neurons can fulfill the tasks only by a high degree of cooperation. But who or what steers the neurons? Since two decades I have been propagating the idea that the brain acts by self-organization; a point of view that is accepted more and more in the scientific community. In particular, concepts of synergetics can be applied to brain function. Some typical aspects will be illustrated by examples from visual perception, such as recognition of faces and facial expressions, perception of ambiguous figures, hysteresis, etc.

Lecture 2: Some forms of integrate and fire models

I first present some basic features about the structure and dynamics of individual neurons including cell body, axons, dendrites, and synapses. The signal processing via spikes will be discussed. Then I will study the interplay between dendritic currents and axonal spikes (pulses) via the light-house model. The spikes are described by a phase angle, increasing in the course of time in analogy to a rotating light beam from a light-house through which the intervals between spikes are determined. The rotation speed depends on the inputs from other neurons. After elimination of the dendritic currents, we find a special form of an integrate and fire model, whereupon I will discuss a number of different forms that take into account arbitrary strengths of synapses and time-lags.

Lecture 3: Phase locking

I analytically study the solutions of a network of integrate and fire neurons of a general form. In particular, I study the circumstances under which phase locking becomes possible and discuss cases of coexistence between phase-locked neurons and other neurons.

Lecture 4: Associative memory

The general equations of the integrate and fire neurons under different sensory inputs are time-averaged over short intervals. It is shown how by suitable choices of the synaptic connections these models give rise to associative memory. Associative memory means that a set of incomplete data is completed by the system to a well-defined set depending on the partly given data. A connection with Kaniza figures is established, as well as with other models of associative memory.

Lecture 5: Activity patterns

Again using suitable time-averages instead of the equations for spikes, rate equations for spike sequences as well as for dendritic currents are established. Such equations are related to the Nunez equations in a simplified form and have been derived along different lines by Jirsa and Haken. When the dendritic currents are eliminated, we reobtain the Wilson- Cowan equations that in turn allow the derivation of spatio-temporal patterns of brain activity in terms of firing rates. If, on the other hand, the axonal pulses or pulse-rates are eliminated, we obtain a new type of equation first derived by Jirsa and Haken for the dendritic currents, which, in turn, are responsible for the electric and magnetic fields measured in EEGs and MEGs.

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http://www.theo1.physik.uni-stuttgart.de/en/mitarbeiter/haken/

Intermediate quantum-level statistics by means of quaternion representation of distributions

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There has been a long-standing subject in quantum-chaos studies to achieve a framework of energy-level statistics which interplolates between Poisson for integrable systems and those of Gaussian random matrix ensembles (GOE, GUE and GSE) for fully chaotic systems by means of a parameter [1]. In 1985, Yukawa [2] gave an excellent idea to construct such a one-parameter family of N-level distributions, based on the so-called level dynamics [3]: one considers an $N \times N$ Hermitian matrix of the form $H = H_0 + tV$ (a perturbation of H_0 by another V with perturbation strength t which is regarded as the"time"). His result can be summarized by a special form

$$P_{N,\beta}(x_1, x_2, .., x_N) = C_{N,\beta} \prod_{j<k} \left(\frac{(x_j - x_k)^2}{a^2 + (x_j - x_k)^2} \right)^{\beta/2} \qquad \beta = 1 \text{ GOE},\ 2 \text{ GUE},\ 4 \text{ GSE}. \tag{1}$$

Here, the parameter that appears is a which interpolates the two limits as

$$a \to 0 \quad \text{Poisson limit}, \qquad a \to \infty \quad \text{Gaussian limit (Wigner-Dyson limit)}. \tag{2}$$

A further treatment of the distribution (1) to deduce concrete statistics (the spacing distribution, long-range 2-level correlations, etc.) is a hard problem, and have so far produced little fruits. However, for the unitary symmetry class $\beta = 2$, a fully analytic method was devised by Forrester [4]: he showed that for this class expression (1) can be rewritten as a determinant

$$W_N^{(\beta=2)}(x_1, .., x_N) \equiv a^{-N} \prod_{1 \le j < k \le N} \left(\frac{(x_j - x_k)^2}{a^2 + (x_j - x_k)^2} \right) = \det \left[\frac{1}{a - i(x_j - x_k)} \right]_{j,k=1,..,N}, \tag{3}$$

where the right-hand side allows an infinite sum over N with coefficient ζ, called the grand canonical series with fugacity ζ. In the present talk, it will be extended to the other two classes i.e. orthogonal and symplectic classes by using *quaternion representation* [5] which replaces the two opposite limits in (2) by $\zeta \to 0$ and $\zeta \to \infty$, respectively. It is based on our recent work [6].

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Dynamics of nonlocally coupled oscillators I

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The dynamics of large assemblies or extended fields of coupled nonlinear elements depends crucially on the interaction range involved. Most foregoing studies have been confined to the limiting cases of either local coupling or global coupling, while systematic studies of more general nonlocally coupled systems remain few. In this series of lectures, I will discuss some remarkable features in the dynamics which can arise peculiarly to nonlocally coupled systems, working mainly with oscillatory dynamics. The first lecture will be devoted to general discussions on what are essentially new with nonlocally coupled systems as contrasted with locally coupled systems. Some points to be discussed include the following:

1. Effective nonlocality in coupling arising from locally coupled systems (typically reaction-diffusion systems) as a result of elimination of some variables.
2. Proposal of a 3-component reaction-diffusion model as a canonical model covering the local, global and nonlocal regimes of the effective coupling, thus providing an ideal model for the study of the effects of nonlocality on the dynamics.
3. Pointing out the fact that nonlocal coupling has its own asymptotic regeme, not being merely something intermediate between the local and global.
4. Applicability of center-manifold reduction and phase reduction leading to some new forms of universal equations.
5. General conditions under which the effects of nonlocality become overt.
6. Mean-field picture applicable to nonlocally coupled systems.
7. Onset of spatial discontinuity of patterns as a general feature of nonlocally coupled systems.

Dynamics of nonlocally coupled oscillators II

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In this second lecture, the impact of the nonlocality in coupling will be demonstrated by showing a variety of curious dynamics exhibited by the canonical model introduced in the first lecture or by its reduced forms. Our emphasis will be placed upon how the types of behavior observed are peculiar to the coupling nonlocality and how they can naturally be understood in terms of the general notions developed in the first lecture. In particular, the mean-field picture valid for nonlocally coupled elements turns out quite useful for interpreting certain features of behavior. Some problems involving stochasticity are also discussed for which a theory can be formulated by virtue of the same mean-field picture. The types of behavior discussed are:

A. Self-sustained pacemakers in monostable excitable media whose origin is completely different from those proposed in the past.

B. Two-dimensional spiral waves with a strongly turbulent core which is initiated by a breakdonw of synchronization of a small group of central oscillators to the periodic internal forcing.

C. Coexistence of coherent and incoherent domains. This is a generalization of case B.

D. Spatio-temporal chaos with multi-scaling properties for which the pattern is fractalized with its fractal dimension changing continuously with the system parameter.

E. *Soft-mode turbulence* which occurs right at the Turing instability in the presence of a Goldstone mode.

F. *Sawtooth turbulence* characterized by persistent creations and annihilations of wave sources and sinks.

G. Transmission of waves in random media for which a statistical theory can be formulated in terms of a Boltzmann type kinetic equation.

Frequency Map Analysis, theory and practice

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Introduction. Frequency Map Analysis is a numerical method based on refined Fourier techniques which provides a clear representation of the global dynamics of many multi-dimensional systems, and which is particularly adapted for systems of 3-degrees of freedom and more. This method relies heavily on the possibility of making accurate quasiperiodic approximations of quasiperiodic signal given in a numerical way.

In these lectures, we will describe the basis of the frequency analysis method. This is intended to be of practical use for any researcher who is willing to explore and use the frequency map analysis methods for the understanding of the dynamics of Hamiltonian systems.

Applications to several examples will be provided, in Solar System, or Particle Accelerator Dynamics.

1. Quasi periodic approximation

2. Convergence of the quasi periodic approximation, asymptotic expansion

3. Numerical examples

4. Frequency Map Analysis

5. Application of Frequency Map Analysis in Solar System and Particle Accelerator dynamics

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Dynamics and structure of granular flow through a vertical pipe

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Static, dynamic and statistical properties of granular materials are one of the most important topics in current science and its application to technology. In some situations granules behave like ordinary fluid or ordinary solid. On the other hand, a variety of unusual motions peculiar to granules can be observed, such as size segregation, bubbling, standing waves and localized excitations under vertical vibrations, avalanche and other unusual motions in a rotating mill, chute flow down a slope and a fluidized bed due to air injected inside a box containing granules. Emergence of density waves of granules flowing through a vertical pipe is also a typical and the simplest example of unusual features of granular motion.

We have shown how density waves of granular particles (ordinary sand) emerge, while they flow through a vertical glass pipe, by controlling air flow out of a flask attached to the bottom-end of the pipe. When the cock attached to the flask is fully open, air is dragged by falling granules and flows together with them. No density waves are observed for this situation. As the cock is gradually closed, however, the pressure gradient of air inside the pipe becomes gradually large, inducing the velocity difference between granules and air. As a result, density waves emerge from the lower part of the pipe. The smaller the rate of air flow, i.e., the more the cock is closed, the higher the onset point (along the pipe) of density waves. The onset of density waves is characterized by the growth of the lower frequency part of the power spectra of time-series signals of density waves. The power spectra of density waves display a clear power-law form $P(f) \sim f^{-\alpha}$ with the value of the exponent $\alpha = 1.33 \pm 0.06$, which is very close to $4/3$. The value of α is robust even under the medium flow or variation of the pipe diameter, as far as density waves can be seen.

Very recently we have also controlled the flow rate of granules under the condition that the cock is completely closed, i.e., medium air does not flow with granules. When the flow rate of granules is small, they flow homogeneously with no density waves. The power spectra of the flow exhibit white-noise-like behavior. This is an expected result, just as raindrops fall homogeneously. What we observed rather unexpectedly is the following. If you gradually increase the flow rate of granules, density waves emerge suddenly at some threshold value of the flow rate. Above this threshold the power spectra exhibit clear power-law form with the same exponent $\alpha = 4/3$ robustly.

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Pattern formation in bacterial colonies

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We have studied the growth mechanism and morphological change in colony formation of bacteria from the viewpoint of physics of pattern formation. Even very small number of bacterial cells, once they are inoculated on the surface of an appropriate medium such as semi-solid and nutrient-rich agar plate and incubated for a while, repeat the growth and cell division many times. Eventually the cell number of the progeny bacteria becomes huge, and they swarm on the medium to form a visible colony. The colony changes its form sensitively with the variation of environmental conditions. This implies that although usual bacteria such as *Escherichia coli* are regarded as single cell organisms, they never make their colony independently and randomly but somehow collaborate multicellularly. We have thus tried to extract some simple and universal behavior in growth from such complex bacterial systems.

Here we varied only two parameters to investigate the colony growth; concentrations of nutrient C_n and agar C_a in a thin agar plate as the incubation medium. Other parameters specifying experimental conditions such as temperature were kept constant. We mainly used a typical bacterial species *Bacillus subtilis.* Otherwise the experimental procedures are standard. It was found that colonies show characteristic patterns in the specific regions of values of C_n and C_a in the morphological diagram and the patterns change drastically from one region to another. They were classified into five types; fractal DLA-like, compact Eden-like, concentric ring-like, simple disk-like and densely branched DBM-like. We have experimentally elaborated characteristic properties for each of these colony patterns.

We have also examined colony formation of a species *Proteus mirabilis*, which forms concentric-ring-like colonies that look much more regular than those produced by *Bacillus subtilis.* The colony grows cyclically with the interface repeating an advance (migration) and a stop (consolidation) alternately. Our experimental results suggest that macroscopically the most important factor for its repetitive growth is the cell population density, i.e., that there seem to be higher threshold of the cell population density to start migrating and lower one to stop migrating.

We have tried to construct a phenomenological model which produces characteristic colony patterns observed in our experiments. The basic idea is that the main features of individual biological organisms are, if focusing on their population behavior, reproduction and active motion. Our modeling is, therefore, based on the reaction-diffusion-type approach for the population density of bacterial cells and the concentration of nutrient.

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Optimal fluctuations and the control of chaos

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Large fluctuations are responsible for many important physical phenomena, including e.g. stochastic resonance and transport in Brownian ratchets. They usually proceed along *optimal paths.* Starting from Boltzmann (1904), a huge body of theory was developed during the last century; the modern understanding dates from Onsager and Machlup (1953). The introduction of the prehistory probability distribution established optimal paths as physical observables (Dykman et al, 1992), and the corresponding optimal force driving the fluctuations was measured for the first time by Luchinsky (1997). Recent developments, centered on nonequilibrium systems, will be discussed, including extensions of the work has to encompass escape from chaotic attractors (Khovanov et al, 2000; Luchinsky et al, 2002). In particular, it has been established that fluctuational escape from a chaotic attractor involves the system passing between unstable saddle cycles – thus paving the way for an analytic theory. Measurements of the optimal force can be used to determine the energy-optimal control function needed to effect escape in the deterministic system in the absence of fluctuations.

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Quantized turbulence

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Turbulence in superfluids – e.g. the superfluid states of liquid ^{4}He and ^{3}He, the electron gas in superconductors, the nucleonic fluids in neutron stars, and Bose-Einstein condensates in laser-cooled gases – is quantized. It consists of a tangle of vortex lines, each element of which is identical to every other in any given system. Apart from its intrinsic scientific interest it is of importance because (a) being in some ways a very simple form of turbulence one can hope to understand in considerable detail, and (b) it is the state believed to be created during a fast passage through a second order phase transition. Two ongoing research programmes on superfluid turbulence will be reviewed and discussed. First, the initial experiments (Davis et al, 2000) on the decay of turbulence in superfluid ^{4}He at mK temperatures will be considered. The vortices are created with a electrostatically-driven vibrating grid, and detected by the use of negative ions travelling near the Landau critical velocity in isotopically pure ^{4}He. Preliminary results indicate that the vortex decay rate apparently becomes temperature-independent below about 70 mK. It is believed (Vinen, 2000) that the corresponding decay mechanism may involve a Kolmogorov cascade, Kelvin waves and, ultimately, phonon creation. Secondly, the status of superfluid helium experiments modelling the GUT transition in the early universe 10^{-35} s after the Big Bang (Dodd et al, 1998) will be reviewed.

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Synchronization phenomena in the kidney

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The pressure and flow regulation in the individual functional unit of the kidney (the nephron) tends to operate in an unstable regime. For normal rats, the regulation displays regular self-sustained oscillations, but for rats with high blood pressure the oscillations become chaotic. The lecture explains the mechanisms responsible for this behavior and discusses the involved bifurcations. Experimental data show that neighboring nephrons adjust their pressure and flow regulation in accordance with one another. For rats with normal blood pressure, in-phase as well as anti-phase synchronization can be observed. For spontaneously hypertensive rats, indications of chaotic phase synchronization are found. Accounting for a hermodynamics as well as for a vascular coupling between nephrons that share a common interlobular artery, the lecture presents a model of the interaction of the pressure and flow regulation between adjacent nephrons. It is shown that this model, with physiologically realistic parameter values, can reproduce the different types of experimentally observed synchronization.

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Mosekilde E, Maistrenko Yu and Postnov D 2002 *Chaotic Synchronization – Applications to Living Systems* (World Scientific, Singapore)

Chaotic synchronization of time-continuous oscillators

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Considering two coupled identical Rössler oscillators the lecture first discusses the necessary and sufficient conditions for stability of the synchronized chaotic state. The lecture continues to examine the transitions through which low periodic orbits embedded in the synchronized chaotic state lose their transverse stability and produce the characteristic picture of riddled basins of attraction. We also discuss the distinction between local and global riddling and illustrate the further development of the asynchronous periodic orbits.

A similar approach is applied to a model of two interacting biological cells. Considering a prototypic model of the bursting oscillations in insulin producing pancreatic cells, we first present one- and two-dimensional bifurcation diagrams of the individual cell. These diagrams reveal a squid-formed area of chaotic dynamics in parameter space with period-doubling bifurcations on one side and saddle-node bifurcations on the other. The transition from this structure to the so-called period-adding structure is found to involve a subcritical period-doubling and the emergence of type-III intermittency. Finally, the lecture addresses the issue of the robustness of the synchronized chaotic state to a mismatch of the parameters between the interacting oscillators.

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Quantum transport in chaotic quantum dots: orbit bifurcations, Arnold diffusion and fractals

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I talk about three interesting sub-themes bridging between nonlinear dynamics and quantum transport in mesoscopic billiards.

Firstly, triangular antidot lattices are investigated. We analyze the semiclassical conductivity of fully-chaotic triangular antidots in the low but intermediate magnetic field. Taking into account both a smooth classical part evaluated by the mean density of states and an oscillation part evaluated by periodic orbits, we find that the resistivity of the system yields a monotonic decrease with respect to the magnetic field. But when including the effect of orbit bifurcation due to the overlapping of a pair of periodic orbits, several distinguished peaks of resistivity appear. The theoretical results nicely explain both the locations and intensities of the anomalously large peaks observed in the experiment by NEC group (Phys. Rev. B51(1995)4649) [1].

Then, we shall proceed to investigation of open three-dimensional (3-d) quantum dots. Mixed phase-space structures of 3-d billiards show the Arnold diffusion that cannot be seen in 2-d billiards. A semiclassical conductance formula for ballistic 3-d billiards is derived. We find that, for partially- or completely-broken ergodic 3-d billiards such as SU(2) symmetric billiards, the dependence of the conductance on the Fermi wavenumber is dramatically changed by the lead orientation. As a symmetry-breaking weak magnetic field is applied, the conductance shows a tendency to grow. We conclude: In contrast to the 2-d case, the anomalous increment of the conductance should include a contribution arising from the (classical) Arnold diffusion as well as the (quantum) weak localization correction [2].

Finally, within a formalism of the semiclassical Kubo formula for conductivity, we give a periodic-orbits picture for the fractal magneto-conductance fluctuations recently observed in submicron-scale phase coherent ballistic billiards [3]. The self-similar conductance fluctuations are shown to be caused by the self-similar unstable periodic orbits which are generated through a sequence of isochronous pitchfork bifurcations of straight-line orbits oscillating towards harmonic saddles. The saddles are universally created right at the point of contact with the leads or at certain places in the cavity as a consequence of the softwall confinement. Our mechanism is able to explain all the fractal-like magneto-conductance fluctuations in general softwall billiards [3].

Many other interesting themes in this field will be described in [4].

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Some classes of self-similar planar fractals

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We present some well known classes of planar fractals, based on regular polygons, and introduce a new class of fractals, appearing from the construction of simple branching trees. Most of the studied objects are self-similar in a strong sense. Therefore, the self-similar dimension $d_S = \frac{\ln N}{\ln k}$, where N is the total number of congruent sub objects and k is the coefficient of similarity, is introduced and its properties are studied.

Besides the dimension d_S, various other characteristics of the presented fractals are examined, for instance, when the overlapping occurs, what is the equation of the boundary curve, and what is the density of the embedded object. We also explain the concept of an iterated function system and give the IFS-codes for some of the studied fractals. Most of the results can be generalized to the 3-dimensional space (starting, for instance, with regular solids) and into the higher dimensions.

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From independent particle towards collective motion in a few electron lattice model with Coulomb repulsion

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A basic question in quantum many body theory is to know how one goes from independent particle motion towards collective motion when one decreases the density n_s of a system of charged particles repelling each other via a U/r Coulomb repulsion. The motivation to re-visit nowadays this question comes from the possibility to create two dimensional gases of charges in high quality field effect devices and to decrease by a gate the carrier density n_s down to a very dilute limit. Conductance measurements for different densities as a function of the temperature, of the bias voltage, of a parallel magnetic field, etc, show the existence of an unexpected low temperature metallic behavior when one goes towards the dilute limit. This raises the question of the existence of an intermediate metallic phase between two insulating phases of different nature: the Fermi glass of Anderson localized states at large n_s and the pinned Wigner solid at low n_s. This question is numerically investigated using mesoscopic lattice models with and without disorder.

In **lecture 1**, the recent experimental results motivating to re-visit the Fermi-Wigner crossover will be reviewed.

In **lecture 2**, detailed exact numerical studies of a few electron mesoscopic lattice models will be presented, showing a specific intermediate regime between the weak and strong coupling limits.

In **lecture 3**, the conjecture of an intermediate phase between the Fermi liquid and the Wigner solid, first proposed by Andreev-Lifshitz, will be discussed in relation with the numerical results.

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Transport of strongly correlated electrons

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One of the central open theoretical problems in the solid state physics is the understanding of strongly correlated electrons, where the properties are dominated by strong electron-electron repulsion and Pauli exclusion principle. We discuss the quantum electronic transport in such systems [1], in particular electrical conductivity, spin diffusion and heat conductivity. The concept of charge stiffness is introduced which makes qualitative distinction between conductors and insulators in the quantum ground state, while at finite temperatures it leads to possibilities of usual resistors, but also of anomalous ideal conductors and ideal insulators [2,3]. It is shown that the singular transport appears in many integrable systems of interacting fermions, even when the current is not a conserved quantity. The evidence comes from the relation with level dynamics [3], the existence of conserved quantities [4], from exact results as well as from numerical studies of small correlated systems using exact diagonalization method and finite-temperature Lanczos method [5]. Several open theoretical problems in this connection will be addressed: a) necessary ingredients for the quantum dissipationless transport, b) transport in systems close to integrability, and c) the existence of ideal insulators at finite temperatures.

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Stability of quantum motion and correlation decay I & II

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In this two-hour lecture we will make a review of the theory and the (numerical) experiments on the behavior of *quantum fidelity* of classically chaotic and regular hamiltonian systems.

We will derive a simple and general relation between the fidelity of quantum motion, characterizing the stability of quantum dynamics with respect to arbitrary static perturbation of the unitary evolution propagator, and the integrated time auto-correlation function of the generator of perturbation. Quite surprisingly, this relation predicts the slower decay of fidelity the faster decay of correlations. In particular, for non-ergodic and non-mixing dynamics, where asymptotic decay of correlations is absent, a qualitatively different and faster decay of fidelity is predicted on a time scale $\propto 1/\delta$ as opposed to mixing dynamics where the fidelity is found to decay exponentially on a time-scale $\propto 1/\delta^2$, where δ is proportional to the strength of perturbation. A detailed discussion of a semi-classical regime of small effective values of Planck constant $\hbar$ is given where classical correlation functions can be used to predict quantum fidelity decay. Note that the correct and intuitively expected classical stability behavior is recovered in the classical limit $\hbar \to 0$, as the two limits $\delta \to 0$ and $\hbar \to 0$ do not commute. In addition we also discuss a non-trivial dependence on the number of degrees of freedom and the role of the thermodynamic limit.

The theoretical predictions will be demonstrated mainly in two families of models: (i) a quantized kicked top and a quantized pair of coupled kicked tops where the semiclassical regime is emphasized, and (ii) kicked Ising spin 1/2 chain where the thermodynamic regime is emphasized. We also need to stress that these results have important implications for the stability of quantum computation, and may be used in order to optimize the accuracy of quantum algorithms.

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Chaotic resonances in quantum many-body dynamics

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The questions about the mechanisms and the conditions for the relaxation to equilibrium in the thermodynamic limit of a generic isolated hamiltonian system constitute important open problems in statistical mechanics. In this talk we consider an established technique in describing relaxation of strongly chaotic single-particle classical systems, namely the concept of Perron-Frobenius-Ruelle resonance spectrum, and use it for the dynamical description of non-integrable quantum many-body systems in thermodynamic limit.

We define a quantum Perron-Frobenius master operator over a suitable normed space of translationally invariant states adjoint to the quasi-local C^* algebra of quantum lattice gasses (e.g. spin chains), whose spectrum determines the exponents of decay of time correlation functions. The gap between the leading eigenvalue and the unit circle signals the exponential mixing (universal asymptotic exponential decay of arbitrary time correlation functions), whereas closing the gap typically corresponds to a transition to non-ergodic dynamics, which may as a consequence, lead to important anomalous transport properties. In particular, the conservation laws of completely integrable quantum lattices represent degenerate eigenvalue 1 eigenvectors of the Perron-Frobenius operator.

Theoretical ideas are applied and validated in a generic example of kicked Ising spin 1/2 chains, namely a one dimensional spin 1/2 lattice with nearest neighbor Ising interaction kicked with periodic pulses of a tilted homogeneous magnetic field. We show that the 'chaotic eigenmodes' corresponding to leading Perron-Frobenius-Ruelle eigenvalue resonances have fractal structure in the basis of local operators.

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Introduction to quantum chaos of generic Hamiltonian systems

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We shall review the basic aspects of complete integrability and complete chaos (ergodicity) in classical Hamiltonian systems, as well as all the cases in between, the generic, mixed type systems, where KAM Theory is applicable, and shall illustrate it using the billiard model systems.

Then we shall proceed to the quantum chaos and its stationary properties, that is the structure and the morphology of the solutions of the underlying Schroedinger equation which in case of 2-dim billiards is just the 2-dim Helmholtz equation. We shall discuss the statistical properties of chaotic eigenfunctions, the statistical properties of the energy spectra, and show arguments and results in support of the so-called universality classes of spectral fluctuations, where in the fully chaotic case the Random Matrix Theory (RMT) is applicable.

First we discuss the universality classes of spectral fluctuations (GOE/GUE for ergodic systems, and Poissonian for integrable systems). We explain the problems in the calculation of the invariant (Liouville) measure of classically chaotic components, which has recently been studied by Robnik et al (1997) and by Prosen and Robnik (1998). Then we describe the Berry-Robnik (1984) picture, which is claimed to become exact in the strict semiclassical limit $\hbar \to 0$. However, at not sufficiently small values of $\hbar$ we see a crossover regime due to the localization properties of stationary quantum states where Brody-like behaviour with the fractional power law level repulsion is observed in the corresponding quantal energy spectra.

We shall mention the rich variety of applications in the domain of physics.

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Introduction to computational algebra

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Consider a system of polynomials

$$\begin{aligned} &f_1(x_1, x_2, \ldots, x_n) = 0, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &f_k(x_1, x_2, \ldots, x_n) = 0 \end{aligned} \tag{1}$$

where $f_1, \ldots, f_k$ are polynomials with coefficients from some field k (usually, k is the field of real or complex numbers). In algebraic geometry the solution space of system (1) is called *the variety.* There are many numerical algorithms for solving non-linear systems such as (1). These algorithms solve for one solution at a time, and find an approximation to the solution. They ignore the geometric properties of the solutions space (the variety), and do not take into consideration possible alternate descriptions of the variety (using a different system of polynomials). However recently efficient computational algorithms have been developed which enable us to get algebraic and geometric information about the *entire* solution space of system (1). They are based on the Gröbner bases theory worked out by B.Buchberger around the middle of 60th of last century. The idea of the methods is to find the "best" representation of the corresponding variety. To illustrate this recall that the Gauss-Jordan elimination method transforms a system of linear equations into the so-called row echelon form. The system thus obtained has exactly the same solutions (the variety) as the original system, but it is trivial to solve. The other example is the system (1) with f_i being polynomials in a single variable, $f_i = f_i(x)$. In this case there is a polynomial $f(x)$ such that the system (1) is equivalent to the equation $f(x) = 0$. The polynomial $f(x)$ is the greatest common divisor of $\{f_1, \ldots, f_k\}$ and it can be found using the Euclidean Algorithm. Although the Gauss-Jordan elimination method cannot be directly expanded to the case of non-linear polynomials and the Euclidean Algorithm – to the case of multivariable polynomials, it turns out that an expansions of these methods is possible, and, in a sense, the Gröbner bases theory is a generalization of the Gauss-Jordan method and the Euclidean Algorithm to the case of non-linear multivariable polynomials.

In the lecture we give an introduction to the Gröbner bases theory, discuss some algorithms implemented in computer algebra systems (e.g. in Mathematica) and consider a few applications to the theory of dynamics systems, in particular, to the investigation of the time-reversible systems of ODE.

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Synchronization of irregular oscillators: from theory to data analysis

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In the classical sense, synchronization of coupled oscillating systems means appearance of certain relations between their phases and frequencies due to weak coupling. After giving brief introduction into the classical theory we review its recent extension to the case of chaotic systems. We discuss how the phase and mean frequency can be determined and consider synchronization effects in two- or many-oscillator systems.

Next, we discuss how synchronization theory can be used in data analysis. In particular, we consider how the phases and frequencies can be estimated from time series and how the intensity and directionality of interaction can be estimated. The methods are illustrated by the results of the investigation of cardiorespiratory interaction in humans.

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Ladder operators and moment problems

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Ladder operator formalisms typically arise in factorization approaches to Schrödinger operators. There, they serve to an algebraic understanding of spectral problems like finding suitable eigenvalues of the linear operators under consideration. Also when it comes to describing supersymmetric Schrödinger operators in quantum mechanics, the concept of lowering and raising operators turns out to have an important meaning. So far, typical scenarios when ladder operators arise are briefly sketched. In recent contributions it has become apparent that methods involving ladder operators can also be used to deal with moment problems in context of special functions in analysis. We give several examples for this application. It remains a fascinating task and also a kind of challenge to investigate the interactions between related analytic and stochastic structures. For instance, the role of discrete Hermite polynomials and their connections with discrete martingale theory has to be understood in detail.

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Adaptivity or chaoticity of strongly driven natural multiphase systems?

Synergetics as a transdisciplinary set of paradigms explaining amphidynamic behaviour vacillating between procedural coherency ("order") and "Self-Organised Criticality" ("pseudochaotic coordination")

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Subsequent to Edward LORENZ'es, David RUELLE's and Florin TALKEN's secular discoveries of "pseudochaotic coordination" in hydro-aero-dynamic systems, the strange coexistence of indeterminate kinematics and apparently deterministic dynamics has become the topic of intensive, computer based research in the physical sciences. Using the mere phenomenological analogy between evolution of systems with sensitive dependency on the initiating conditions, positive LYAPUNOV exponents and fractal multi-dimensionality, sterile extrapolations from idealising physicalism to profound natural sciences became popular, inter alia by search for "chaoticity" in the time series of cardio-vascular, endocrinological and/or skeletomuscular reactions. All of these are long known in their microscopic details and behavioural traits of "re-exitable membrane channels", the most important element for "non-linear-non-equilibrium phase transitions". These were systematically studied since the days of SHERRINGTON, one of the founding fathers of physiological synergetics around the turn of the 19th century. Hermann HAKEN, here cooperating with Hans Peter KOEPCHEN, the leading authority of the autonomous nervous system in the 1990ies: the two authors postulated that neurodynamic behavioural traits are characterised by only transient, self-limiting phases of coherent performance. To paraphrase the latter in a intuitive manner, they proposed to use the term "quasi attractor" in describing such short lived emergence and subsequent submergence of procedural coherency: methods to display this "natural behaviour" have now been developed.

Using time series obtained in awake human subjects exposed to cold environment, in patients undergoing psychomotor relaxation, in patients and volunteers undergoing pain stimuli and in subjects undergoing a specific regimen of bicycle ergometry, multiple base-line time series (skeletomotor cardiovascular, respiratory, continuous skin galvanic response) were analysed by a comprehensive algorithm based on primary MORLET-wavelet analysis, ARMA-procedures (moving average) and were then plotted as time frequency plots ("prosodograms" depicting in intuitive manner the emergence and submergence of preferred attractors). The combined data clearly corroborated the basic assumptions of the HAKEN-KOEPCHEN paradigm: in addition, clear indicators of n:m synchronisation (see Lecture TASS) became evident in even short lived coherency separated by likewise clearly detectable transient. In proposing that a moratorium should be it placed on the future publication of dynamic portraits from "single base-line" recordings, it can be anticipated that the "adaptive nature" of normal physiological reactions will become evident to the educated (and soon the general public) ending a "short lived historical transient" where natural adaptivity was misconceived as "chaos".

Self-limiting passive discharge followed by transfer blockade:

On putative microscopic causes of "non-linear" reactions in multiphase systems operated under robustly sustained dysequilibrated boundary conditions (BERTALANFFian "flow equilibria")

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HAKEN's proposal that rapidly emerging and subsequently submerging "dominance" of preferred "attractor" behaviour were the cause of apparent "chaoticity" of biological systems is guiding the project of physiological synergetics: it can be put it into the proper perspective by postulating, that owing to well known behavioural traits of ensembles of membrane channels on the microscopic, and of ensembles of effector neurons and inhibitor neurons, rapid "phase synchronisation" on the one hand, and automatic re-inhibition on the other forms the basis of functional adaptivity.

These concepts also apply - under boundary conditions with steep potential gradients mimicking those prevailing in vivo - , to prebiotic systems capable of undergoing "drive dependent consensualisation of a priori independent movements" (our rheological definition for the vague term "self-organisation"). In studying a wide spectrum of prebiotic (putatively) chaotic prebiotic systems (ranging form sand pile kinematics and hour glass behaviour, over dripping faucets and water clocks, the holocoherent BENARD-MARANGONI hyper-stability and various new versions of the BELOUSOV-ZHABOTINSKI reaction as paradigmatic example of self-organised catalytic activity, the HAKEN-KOEPCHEN-quasi attractor concept could be verified. In the latter, the well known "periodicity" could be enhanced, blurred or even abolished completely by the appropriate choice of "setting" providing "sinks" for products (CO2, electrons). Lastly, calcium waves in isolated cells (beating myocardiocytes) and in suspension of sarcoplasmatic reticulum in agar were studied: they all showed identical behaviour, i.e. autowaves due to self-limiting discharge, refractorisation with restitution of the "kinetic threshold" allowing "critical slowing" and "enthalpy peaking" as basis for BRILLOIN's negentropy principle of information.

Using disarmingly simple cellular automata simulating eruptive, self-limiting discharge and variable length of refractory periods, the above described (putatively universal) behavioural traits could be modelled, the resulting patterns displaying the very same "apparent kinematic indeterminancy" (due to spatio-temporal inhomogeneities) which can be easily corrected by choosing the proper combination (and homogeneity) of parameters reflecting the well known determinants for resonance prone behaviour, namely generalised inertance, generalised capacitive resilience and generalised inhibitance. We propose that, eruptivity abounds in "nature" (in highly "non-linear" reactions) due to the multiphase nature of natural materials, especially when systems are driven into the strongly dysequilibrated modes of operation first identified by BERTALANFFY as "cause" of sustained transfer of energy and matter "feeding the negentropy" postulated by SCHRÖDINGER for living systems.

Clustering of passive tracers in free-surface flows

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Experimental and numerical studies of turbulent fluid motion in a free surface are presented. The flow is realized experimentally on the surface of a tank filled with water stirred by a vertically oscillating grid positioned well below the surface (Goldburg et al 2001). The effect of surface waves appears to be negligible so that the flow can numerically be realized with a flat surface and stress-free boundary conditions above three-dimensional volume turbulence (Eckhardt and Schumacher 2001).

The two-dimensional free surface flow, $\mathbf{v}(x, y, t)$, is unconventional: it is not incompressible, i.e. $\partial_x v_x + \partial_y v_y \neq 0$, and neither kinetic energy, nor squared vorticity (enstrophy) are conserved in the limit of zero fluid viscosity and of absence of external driving as it is the case for "usual" two-dimensional turbulent flows (Lesieur 1990). According to both experiment and numerical simulation, statistical properties of the surface flow are closer to those of three-dimensional turbulence.

The dynamics of passive Lagrangian tracers that are advected in such flows is dominated by rapidly changing patches of the surface flow divergence. Single particle and pair dispersion show different behavior for short and large times: on short times particles cluster exponentially rapidly until patches of the size of the divergence correlation function are depleted; on larger times the pair dispersion is dominated by subdiffusive hopping between clusters. We also find that the distribution of particle density is algebraic, and not lognormal as predicted for flows that are delta-correlated in time (Klyatskin and Saichev 1997). The latter so-called Kraichnan flows are rather synthetic but allow for making analytical progress. Our results can be traced back to the exponential distribution of the divergence field of the surface flow. Very recently, physical mechanisms for the formation of rain drops were discussed by Balkovsky et al (2001). They considered the motion of tracers that have inertia (Maxey and Riley 1983) but are advected in an incompressible turbulent Kraichnan flow. The relation of our findings to this problem is discussed.

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Chaos, integrability, entanglement and decoherence

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We shall give an unexpected view of the relation of decoherence and entanglement to the integrability or chaoticity of the underlying classical systems.

This view stems from the fact, that we shall consider separately the dependence on the dynamics from the dependence of the initial state, and not limit ourselves to the usual coherent states, as initial conditions. This has two reasons: First we believe that the phenomenon can be better understood in this fashion and second in the context of quantum computing we are certainly more interested in the evolution of a random initial state, than in the one of a Gaussian packet. We shall use both random matrix methods and correlation function techniques, as presented in the lectures of Dr. Prosen, to illuminate the problem at hand. We shall give close attention to the question, when decoherence follows the trend of the corresponding autocorrelation function (including fidelity in echo situations), and when not. We shall also inquire, if other correlation functions become relevant in situations where the autocorrelation function does not explain the behaviour of decoherence or entanglement.

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Quantum chaos in the mixed phase space and the Julia set

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Phase space of multi-dimensional Hamiltonian systems is generally composed of infinitely many invariant components. Chaotic trajectories have the largest dimension as an invariant set, while the periodic orbits have the lowest. Coexistence of qualitatively different ergodic components, which are usually intermingled in a self-similar way in the phase space, characterizes a generic situation which is so complicated that our understanding is far from accomplished. The orbits in classical mechanics are always confined on the corresponding invariant set by definition, in particular, except in case of ideal chaotic systems, there are orbits with positive measure that move only on the limited subspace whose dimension is less than that of full phase space.

On the other hand, the wavepacket of quantum mechanics is not forced to stay on a certain limited classical manifold, but spreads over or shares different invariant subsets simultaneously. The spreading is a consequence of the wave effect which is the most marked difference between classical and quantum mechanics. There is not any obstacle in principle preventing the transition between arbitrary two points in the phase space and the quantum wavepacket can penetrate into any kinds of barriers. Such a classically forbidden process does not have classical counterparts. The penetration into the energy barrier is especially called *tunneling*, which is understood as the most typical quantum effect and plays important roles in many physical and chemical phenomena. The existence of chaos in the phase space crucially affects the nature of tunneling (Shudo & Ikeda 1995, 1998).

In this lecture, after introducing recent developments of the theory of multi-dimensional complex dynamical systems, with some technical tools necessary to construct the theory (Bedford & Smillie 1991a, 1991b 1992), we will give numerical and mathematical evidences which show that the orbits on the Julia set in the complex phase space can just be regarded as the *classical counterparts* of the quantum wave effects (Shudo, Ishii & Ikeda 2001). More precisely, arbitrary two regions in the phase space are necessarily connected via the orbits on the Julia set *even in the mixed system.* This remarkable property, which is absent in the real classical dynamics, comes from the transitivity of the Julia set, which has rigorously been proved for the complex Hénon map, and conjectured for the standard and semi-standard map. The measure whose support gives the Julia set is a unique ergodic measure, and unstable saddles are dense on it. Our arguments are based on the complex semiclassical description of quantum forbidden processes.

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Complication of linear spatial socio-economies

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The purpose of this lecture is to provide an explanation of the process of complication, i.e. the deepening and evolution of complexity in evolving complex linear systems. The complication means the transfer from complex structure to much more complex structure in the evolution of complex systems. The main feature of the evolution of a complex system is the emergence of new properties which did not exist in previous trends and which add new information to the system. The growth of complexity means the appearance and increase of information and therefore the decrease of the (Shannon) entropy. Even the simplification of the system is the part of process of complication, since simplification is clearing the room for the further adoption of new information. This clearing presents the essential force acting against the modern information explosion and playing the important role in the process of self-organization. Spread of information within the complex system presents the essence of the process of complication. This spread shows itself through the partial adoption of new information and through the path dependent process of self-organization within socio-spatial complex system. In this study we will concentrate ourselves only on the forms of complication and self-organization in linear socio-economic systems, leaving behind the innovation diffusion and bifurcation analysis. The concept of complication is pointed out on the deficiency of purely economic considerations of socio-economic systems and stresses the necessity to widen the concept of "Homo Oeconomicus" to the concept of "Homo Socialis". Such a widening is radical in the study of complex socio-economic processes because of the important difference between the economic and socio-economic rationality: the traditional identification of economic rationality of "Homo Oeconomicus" as the optimization is complementary to socio-economic rationality of "Homo Socialis" as parsimony. In our lecture we will apply the paradigm of complexity and complication to several main branches spatially connected with the augmentation and development of flows, networks and superposition of their hierarchies in linear systems. We are using the concept of complication as the unifying frame for theories of linear spatial analysis of complex socio-economic systems: the Push-Pull theory of Migration Streams, the theory of Central Place hierarchies, the spatial production cycles and trade feedback loops, the Dynamic Input-Output Analysis and the theory of the Fields of Influence of changes in Input-Output systems, the classical Key Sector Analysis, the Structural Q-analysis and the Miyazawa model of income distribution within Input-Output systems and their "onion skins" extensions.

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Determinism and noise in cardiovascular dynamics

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Signals derived from the human cardiovascular system are well known to exhibit highly complex, nearly periodic, oscillatory behaviour whose nature is something of an enigma and still the subject of vigorous debate. The variation of cardiac frequency with time, known as heart rate variability (HRV), has been intensively investigated using both deterministic and stochastic methods. It has, for example, been variously described as chaotic, fractal, stochastic, and subject to 1/f fluctuations and it was proposed that the state of the system can be classified by the slope of its power spectrum on a log-log plot.

We illustrate some problems in characterising slow modes in real measurements and show that oscillatory dynamics under the influence of strong noise, coupled with a limited time of observation, can lead to a $1/f$-like behaviour. We review and describe some recent experiments that illuminate the problem and discuss a combination of almost periodic and stochastic frequency modulation as a signature of the system dynamics.

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Interactions in the cardiovascular system

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In a healthy subject in repose a volume of blood equivalent to the total amount in the body returns to the heart every minute. Several oscillatory processes characterise cardiovascular dynamics within this circulation time. Cardiac and respiratory activities act on higher frequency scales, with characteristic frequencies of 1 Hz and 0.2 Hz, respectively. The lowest frequency component, at around 0.01 Hz, has been associated with the activity of the layer of endothelial cells forming the inner surfaces of all blood vessels.

The characteristic frequencies of all the cardiovascular oscillations are found to vary in time, apparently because the oscillatory processes mutually interact. Hales and Ludwig independently described the modulation of cardiac frequency by respiration, in 1773 and 1847 respectively, a process that today is known as respiratory sinus arrhythmia. The occurrence of episodes of synchronization between the cardiac and respiratory rhythms has also been demonstrated.

Recent developments of methods based on dynamical and information theory are facilitating studies of synchronization and of the directionality of couplings between the cardiovascular oscillations. We review and discuss their characteristics in health and disease. The characteristics of the cardiac and respiratory interaction during paced respiration are used to illustrate the role of directionality of coupling in the interplay between synchronization and modulation.

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Microwave experiments in chaotic and disordered systems

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In a sequence of two talks microwave experiments on spectra, line widths, and field distributions in various closed and open microwave resonators are presented with special emphasis on universal features common to all chaotic systems.

1. The random-superposition-of-plane-waves approach

According to a conjecture of Berry (1977) at any point in a chaotic billiard the wave function may be described by a random superposition of plane waves,

$$\psi(r) = \sum_n a_n e^{\imath \mathbf{k}_n \mathbf{r}_n} ,$$

where the modulus $k = |\mathbf{k}_n|$ of the incoming waves is fixed, but directions $\mathbf{k}_n/k$ and amplitudes a_n are considered as random. As an consequence of the central-limit theorem the approach predicts Gaussian distributions for the wave function amplitudes, or, equivalently, Porter-Thomas distributions for their squares. Such distributions have been observed for the first time for wave functions of chaotic billiards (McDonald and Kaufman 1988), and subsequently in numerous simulations and experiments on chaotic and disordered systems.

In this lecture microwave experiments are presented, exploiting further consequences of the Berry conjecture. Results for field distributions and spatial correlation functions in three-dimensional Sinai resonators (Dörr *et al* 1998) are presented. For spectral level dynamics in a disordered system with the position of one impurity as the parameter the approach allows to calculate velocity distributions and velocity autocorrelation functions which are in complete agreement with the experiment (Barth *et al* 1999). In open billiards and billiards with broken time-reversal symmetry the distributions of currents and vortices, as well as the vortex distance distribution are measured and compared with the prediction from the Berry conjecture (Barth and Stöckmann, Vraničar *et al*).

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2. Open microwave billiards as scattering systems

Whenever a microwave experiment is performed, the system has to be opened either by attaching wave guides or introducing antennas. This has the unavoidable consequence that the system is perturbed, and the measurement always yields an unwanted combination of properties of the system and the apparatus. A tailor-made approach to cope with this situation is provided by scattering theory. For the case of isolated resonances an expression for the matrix elements of the scattering matrix is obtained,

$$S_{ij} = \delta_{ij} - 2\imath\gamma \sum_n \frac{\psi_n(r_i)\psi_n(r_j)}{k^2 - k_n^2 + \frac{\imath}{2}\Gamma_n},$$

which is a direct equivalent of the Breit-Wigner formula known from nuclear physics for many years (Stein *et al* 1995, see chapter 6 of Stöckmann 1990 for details). In the equation $\psi_n(r_i)$ is the value of the wave function of the billiard (with Dirichlet boundary conditions at the wall, and Neumann ones at the opening) at the coupling position r_i. γ is a parameter describing the coupling to the wave guide or the antenna.

This correspondence of microwave billiards with atomic nuclei can be used to check predictions from theory which are unaccessible in nuclear physics. As an example the first unambiguous demonstration of resonance trapping is presented, namely the phenomenon that with increasing coupling strength the widths of the resonances do not increase unlimited but finally decrease again (Persson *et al* 2000). If the transmission through a cavity with a number of incoming and outgoing channels is measured as a function of frequency, irregular fluctuations are observed, an equivalent to the Ericson fluctuations observed in nuclear scattering processes. The distribution of these fluctuations was studied in an open microwave billiard in dependence of the number of channels, both for systems with and without time-reversal symmetry, and the results were compared with random matrix predictions (Schanze *et al* 2001). A new parameter comes into play if absorption is involved, which is unavoidable in experiments anyway, but has been considered by theory only recently (Beenakker and Brouwer 2001). Again the experiment is able to verify the theoretical predictions perfectly (Méndez *et al*).

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Quantum chaos on graphs

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Quantum graphs have recently been introduced as model systems to study quantum problems with chaotic classical limit (Kottos and Smilansky 1997). The most fascinating features of quantum graphs are that they can be constructed easily and almost at will covering a variety of classical limits such as chaotic dynamics, scattering or diffusive behaviour. Yet, the quantum mechanics can be formulated in terms of unitary propagators on finite Hilbert spaces. In addition, quantum graphs show many of the phenomena observed in more general quantum systems such as universality of the spectral statistics or Anderson localisation.

I will review recent developments on quantum graphs and generalise these concepts to quantum propagation on arbitrary, directed graphs (Tanner 2000). In its simplest version, the wave dynamics on the graph is solely determined by the topology of the graph given by the adjacency matrix, metric properties, that is, the length of the edges, and dynamical properties entering as (complex) transition amplitudes describing transitions between edges at the vertices. Necessary and sufficient conditions for a graph to be 'quantisable' can be given (Pakoński et al 2002).

A specific quantum graph can in a natural way be associated with an ensemble of unitary matrices (Tanner 2001). The 'classical' dynamics on the graph can be interpreted as a Markov chain defined on the graph with stochastic transition matrix $\mathbf{T}$ obtained from the unitary propagator on the graph (Kottos and Smilansky 1997, Pakoński et al 2001). I will formulate a conjecture linking universality of the statistical properties of the unitary matrix ensemble after ensemble average to the spectral gap of the stochastic transition matrix. More precisely, it is expected that the matrix ensemble follows random matrix statistics in the limit of large network size, if the spectral gap Δ, that is, the minimal distance of eigenvalues of $\mathbf{T}$ from the unit circle, scales like (Tanner 2001)

$$\lim_{N\to\infty} \frac{\log N}{\Delta(N)N} = 0$$

where N is the number of edges in the graph. Some examples will be presented. Results on correlation functions for eigenvalues and spectral determinants will be discussed (Tanner 2002).

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Classical and quantum spectra of intermittent dynamics

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Intermittency is a typical phenomenon on the transition from order to chaos. The existence of intermittent behaviour in dynamical systems can in general be traced back to the presence of marginal stability either at a single periodic orbit or along the boundary of stable islands. This leads to nearly regular together with strongly chaotic dynamics in connected components of the phase space. Intermittency is typically accompanied by algebraic decay of correlation and long tail memory effects. In this talk, I will present techniques to calculate the spectra of Frobenius-Perron operators for intermittent systems in terms of periodic orbits. Implications for a periodic orbit quantisation of intermittent dynamics using Gutzwiller's trace formula will be discussed briefly.

Periodic orbits, which approach the marginal stable regime in phase space are characterised by a vanishing Lyapunov exponent

$$\lambda_p = \frac{\log \Lambda_p}{T_p} \to 0, \qquad \text{for} \quad T_p \to \infty$$

with Λ_p the largest eigenvalue of the Monodromy matrix along the orbit. The algebraic decay in the eigenvalues Λ_p induces divergences in trace formula, which need to be removed systematically (in addition to the usual problem of overcoming convergence problems due to the exponential proliferation of periodic orbits). This leads to periodic orbit expansions of the trace of the Frobenius Perron operator in terms of families of periodic orbits converging towards the marginal stable region.

The talk is mainly based on the chapter on intermittency in the web-book by Cvitanović *et al.* I will introduce the method for a specific 1d-piecewise linear map for which the algebraic decay behaviour can be calculated explicitly. Generalisations to arbitrary uni-model maps will be given. The stadium billiard will serve as an example to discuss modifications of the method for semiclassical periodic orbit formulas (Tanner 1997).

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Development of demand-controlled deep brain stimulation-techniques based on stochastic phase resetting

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Pathological cerebral synchronization may severely perturb brain function as observed in several neurological diseases like Parkinson's disease and essential tremor. In patients that do no longer respond well to drug therapy, depth electrodes are chronically implanted in target areas located in the thalamus or the basal ganglia. To suppress the pathologically synchronized firing, a permanent high-frequency ($>$ 100 Hz) stimulation is performed (Benabid et al. 1991, Blond et al. 1992). Although the therapeutic effects are impressive, there are nevertheless significant drawbacks: (i) The energy consumption of the permanent stimulation is quite high. Thus, the generator (plus battery) has to be exchanged after 1-3 years by means of an operation. (ii) Even more important is the fact that the permanent high-frequency input is an unphysiological type of stimulation which causes the stimulated target areas to adapt. In a number of patients the amplitude of the stimulation has to be increased over the years, in order to maintain the tremor suppressive effect. With increasing stimulation amplitude, however, the probability of the occurence of severe side effects (like dysarthria, dysaesthesia, cerebellar ataxia, psychotic symptoms) increases.

With methods from synergetics (Haken 1983) and statistical physics (Kuramoto 1984) the concept of phase resetting (Winfree 1984) was extended to populations of interacting oscillators subjected to random forces (Tass 1999). This stochastic phase resetting approach has lead to the development of demand-controlled deep brain stimulation techniques (Tass 2001a-2001c, 2002a, 2002b). The latter work in a completely different way compared to the standard high-frequency stimulation: While the standard technique probably simply suppresses the neuronal firing in the target area (Wielepp et al. 2001), the novel techniques only desynchronize the firing whenever it gets pathologically synchronized. In this way, the novel methods intent to bring the neurons' dynamics as close to the physiological state (i.e. to the uncorrelated firing) as possible. The talk is about both theory and first experimental results.

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Synchronization tomography and phase resetting tomography: Three-dimensional anatomical localization of spontaneous and stimulus-locked synchronization in the human brain

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Cerebral synchronization processes play an essential role under both physiological (Freeman 1975) and pathological (Llinás and Jahnsen 1982, Bergman et al. 1994) conditions. To detect and localize phase synchronization and stochastic phase resetting dynamics in the human brain non-invasively with magnetoencephalography novel methods have been developed:

1. *Synchronization tomography* (Tass et al. 2002): First, the cerebral current source density is reconstructed in each cerebral voxel (i.e. volume element) for each time t by means of the magnetic field tomography (MFT) (Ioannides et al. 1990). Next, the phase synchronization analysis (Tass et al. 1998) is applied to each voxel and to external reference signals such as muscular activity. In this way brain/brain- and brain/muscle phase synchronization are determined. It turns out that phase synchronization is a fundamental coordination principle in cerebral motor control (Tass et al. 2002).

2. *Phase resetting tomography:* The cerebral current source density is reconstructed with MFT. Next, a stochastic phase resetting analysis (Tass 1999, 2002a, 2002b) is applied to each voxel as well as to all pairs of voxels and external signals. This enables the detection of transient stimulus-locked response clustering and transient stimulus-locked synchronization and desynchronization. In contrast, standard techniques like cross-trial averaging or cross-trial cross-correlation are not able to detect such processes and may even produce artifacts.

The talk is about the theoretical background of the novel methods, their application to experimental data, and their diagnostic relevance.

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Clustering in granular gas I: Birth and sudden death of a cluster

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The key feature of a granular gas, making it fundamentally different from any ordinary molecular gas, is its tendency to form clusters (Goldhirsch and Zanetti, 1993; Kudrolli et al., 1997). This can be traced back to the inelasticity of the collisions between the particles. In applications such as conveyor belts and sorting machines, the clustering is an unwanted and very costly effect. Here we study the phenomenon in the setting of the so-called Maxwel Demon experiment (Eggers, 1999).

Granular material in N connected compartments is brought into a gaseous state through vertical shaking. For sufficiently strong shaking the particles are uniformly distributed over the compartments, but if the shaking intensity is lowered this uniform distribution gives way to a clustered state. The clustering transition is experimentally shown to be of 2nd order for $N = 2$ and of 1st order for $N \geq 3$. In particular, the latter is *hysteretic*, involves long-lived transient states, and exhibits a striking lack of time reversibility (Van der Weele et al., 2001; Van der Meer et al., 2001).

In the strong shaking regime, a cluster breaks down very abruptly (sudden death) and in its further decay shows anomalous diffusion, with the length scale going as $t^{1/3}$ rather than the standard $t^{1/2}$ (Van der Meer et al., 2002). We focus upon the self-similar nature of this process. The observed phenomena are all accounted for within a dynamical flux model.

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Clustering in granular gas II: David vs. Goliath and other competitive effects

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There are many extensions of the Maxwell Demon experiment. First we consider a bi-disperse mixture consisting of large and small particles (Mikkelsen et al., 2002). This is done with an eye to practical applications, where granular material is rarely mono-disperse. For moderate shaking the material clusters into the compartment which initially contained most of the large particles: Goliath wins. For very mild shaking, however, the cluster goes into the compartment originally dominated by small particles: David wins. These experimental observations are quantitatively explained within a bi-disperse version of the flux model.

Second, we study a system in which the compartments are arranged in the form of a staircase, resembling an industrial conveyor belt. The central topic here is the competition between the clustering effect and the natural tendency of the particles to stream downwards. When a cluster is formed, one can get rid of it by shaking sufficiently hard. The ensuing transition to the desired uniform flow is found to be a self-similar process involving Burgers-like shockwaves (Kloosterman et al., 2002).

Finally, we discuss two related clustering phenomena from other fields: the traffic jam problem (Helbing, 2001) and the formation of sand ripples at the beach (Andersen et al., 2001). Both turn out to be well described by flux models markedly similar to our own.

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Dissipation

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Dissipation is the irreversible transfer of energy into a reservoir, having a large number of degrees of freedom. I will consider the case where the reservoir is a system of nearly independent fermions. This includes dissipation by electrical conduction and dissipation in the dynamics of the nucleus. The lectures will demonstrate that this is a rich subject for investigation, with significant problems outstanding.

Lecture 1: Physical applications, and the standard approach (the Kubo formula). Energy diffusion as an alternative approach to dissipation. Classical-quantum correspondence. Limitations of the Kubo formula approach.

Complex quantum systems. Random matrix theory, and universality hypotheses. Dimensionless parameters describing response of complex systems. Parametric random matrix models. Matrix element sum-rules, and semiclassical estimates. Some parametric statistics.

Lecture 2: Estimates for energy diffusion constant in Landau-Zener and Kubo formula regimes. Predictions of various anomalous effects. Numerical experiments testing these predictions. Theoretical arguments reconciling random matrix and semiclassical predictions.

Much of the material is covered in *Parametric Random Matrices: Static and Dynamic Applications* , M. Wilkinson, in '*Supersymmetry and Trace Formulae*, eds. I. V. Lerner, J. P. Keating and D. E. Khmelnitskii, New York: Plenum, p.369-399, (1999). Several new results will be discussed.

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Abstracts of Short Reports

A method to enhance noise reduction for data generated from a known differential equation

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There are several papers which show the use of principle component analysis to reduce noise from a discretely sampled data. This method consists of embedding the data in higher dimensions and then using the singular value decomposition of the resulting matrix, retaining only the dominant components in the data. This method does involve throwing away some information and is not very effective if the sampling interval is not very small. If we know the underlying differential equation, we can find expressions for the neglected components and restore them. We first develop a theoretical basis for the procedure and then illustrate it in the case of some data generated using the Duffing's equation.

Breathers and homoclinic orbits

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Homoclinic orbits are presented as a useful tool for obtaining breather solutions (time periodic, spatially localized) of one dimensional nonlinear lattices (Klein Gordon, FPU and mixed). We present how the relationship between homoclinic orbits and breathers is formed, and advantages of using this method for obtaining breathers over the traditional method of continuation.

Stochastic resonance: Some new results

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Stochastic resonance is a mechanism which was originally proposed to explain the close-to-periodic appearance of Ice Ages. It has been observed in a large number of physical and biological systems. Its main feature is that small additive noise may considerably enhance a system's response to a weak, periodic signal. I will report on joint work with Barbara Gentz (WIAS, Berlin). We introduce a new approach, which aims at describing the behaviour of "typical" paths, instead of merely computing the probability density. This allows us to give precise results on the dependence of transition probabilities on noise intensity, and provides an alternative measure of the output's "periodicity" to the usual signal-to-noise ratio. Various important quantities have a nontrivial power-law dependence on the small parameters. I will also mention some related results on dynamical hysteresis.

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On periodic orbits picture of fractal magnetoconductance in quantum billiards

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Starting from evaluating a simple soft-wall multichannel billiard and using the semiclassical Kubo formula for conductivity, we gave a periodic-orbits picture for the exact self-similar magnetoconductance fluctuations obtained in the phase coherent ballistic quantum billiards experiments. The applicability of Kubo formula to these single devices are justified, since the dots are coupled to large source and drain so that due to level broadening, the energy level inside the dots can be considered as effectively continuum. In our picture the soft-wall boundary is experimentally essential. We claimed that the exact self-similar magnetoconductance fluctuations is due to the self-similar periodic orbits generated through a sequence of isochronous pitchfork bifurcations of straight-line liberating orbits oscillating toward harmonic saddles. The saddles are naturally created right at the point of contact with the leads or at certain places in the cavity as a consequence of the sofwall confinement. We will show that the function we obtained, approximately satisfies a scaling relation, which generates its self-similar properties. In fact, its spectrum can be approximated by the famous Weierstrass spectrum with an upper cut. In addition, we would like to show that the Hurst exponent of the fractal is independent of the detailed geometrical shapes of the cavity and determined only by the local information on the saddles. On this issue we would like to confirm the recent experimental findings that the fractal dimension of the fluctuations are determined only by the quantumness of the system. We choose to discuss the Hurst exponent which is defined as the ratio of the logarithmic values of the scaling constants in the y-direction and x-direction, instead of the fractal dimensions, since they are easy to calculate and to be confirmed experimentally for an exact self-affine fractal. Finally, using the same reasoning and again using the semiclassical Kubo formula for conductivity, we would like to show that even in the situation when the conductance fluctuations versus the applied weak magnetic field show fractal behaviour, the conductance fluctuations as a function of the Fermi energy are not fractal-like.

Experimental test of a trace formula for chaotic 3D microwave cavities

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We have measured resonance spectra in a superconducting microwave cavity with the shape of a three-dimensional generalized Bunimovich stadium billiard and analyzed their spectral fluctuation properties. The experimental length spectrum exhibits contributions from periodic orbits of non-generic modes and from unstable periodic orbits of the underlying classical system. It is well reproduced by our theoretical calculations based on the trace formula derived by Balian and Duplantier for chaotic electromagnetic cavities. Analysis of the spectral statistics reveals that there is a partial decoupling between TE- and TM-modes which decreases with increasing frequency.

Optical wave chaos in a macroscopic open resonator

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In recent years, microwave resonators have lead to tremendous experimental progress in the field of wave chaos. Most notably, stadium type billiards have been spectacularly successful. *Optical* wave chaos has not benefited from this, and progress here has been modest. The primary reason for this is the inherent difficulty in producing and controlling stadium-type resonators in the optical range. Recently, we have put forward a radically new approach to optical wave chaos, using a system that is highly macroscopic, therefore offers unsurpassed control, and allows a wealth of different experimental techniques. We have demonstrated that an open, three-mirror, folded optical cavity can show chaotic dynamics [1]. The key ingredient in this resonator is a *curved* folding mirror, introducing, even at very modest numerical aperture, considerable aberrations through its use at non-normal incidence. These aberrations cause the paraxial approximation to be violated, making the wave equation describing the intra-cavity field non-separable. The *strength* of these aberrations crucially influences the chaos in the system. These aberrations, as introduced by the folding mirror, may be modified in two distinct ways: by varying the folding angle of the resonator (changing the angle of incidence on the curved folding mirror), and by varying the effective numerical aperture of the system. In line with expectations, increasing the folding angle of the resonator from $0°$ to $90°$ shows a smooth transition from a non-chaotic to a chaotic resonator. Also, increasing the effective numerical aperture, thereby preferentially exciting modes that have appreciable amplitude far from the optical axis of the resonator, leads to increased chaotic behaviour. We expect that this highly versatile and promising approach will boost research into optical wave chaos. At the same time, it may serve as an excellent model system to study many intriguing phenomena in wave chaos in general.

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Towards chaos criterion in quantum field theory

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Chaos criterion for quantum field theory in terms of functional integral is proposed. Its correspondence with known criteria in semi-classical regime of finite dimension quantum mechanics is shown. It is demonstrated for real scalar field forth power interaction Lagrangian that chaos corresponds to spontaneous symmetry breakdown, degeneration of vacuum state and confinement for lattice formulation.

Modelling of chaos enforced instanton tunnelling

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Chaos assisted tunnelling has been attracting much attention. Instanton technique mainly used in quantum field theory describes quantum tunnelling between vacua. It seems to be important to investigate influence of chaos on properties of instanton transitions. In this work on the example of model system we demonstrate that chaos enforces the rate of instanton tunnelling and increases density of instanton gas in dilute instanton gas approximation.

Chaotic wavefunction statistics in quantum map and billiard

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In the chaotic quantum map, the overlap distributions between the eigenstates $|n>$ and a probe state $|k>$ are well described by the random matrix theory (RMT) in most cases. However, if the probe state is localized on a fixed point, the overlap distribution shows a strong deviation from the RMT expectation, which is a result of the **scarring phenomena** in chaotic eigenstates [1]. In our recent paper [2], a joint-probability distribution has been developed for the overlaps with the eigenstates $|k>$ of harmonic oscillator which act as the probe states. This is a kind of scarred RMT, and has explained successfully the scarring effect in tunneling rate distributions. This joint-probability distribution contains all information of wavefunction statistics near the fixed point, and has an interesting structure which is directly related to the corresponding classical dynamics. We find that one linear combination of $|k>$'s is an optimized probe state. Following Vergini *et al* [3] we call this the **scar state**, and the statistics of its overlap with chaotic states shows maximum deviation from RMT. This scar state shows that the scarring appears along the stable and unstable manifolds. We show that this joint distribution works very well not only in the perturbed cat map, but also in the boundary function of billiard. This means that the chaotic wavefunction statistics are characterized by the Lyapunov exponent and the angle between the stable and unstable manifolds of the fixed point. In the billiard case, we investigate the relation between the statistics of the boundary functions and those of the real eigenfunctions. This study gives the way to see which eigenstate should be scarred or not in a statistical way.

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Spatial pattern formation in Turing systems

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In the 1950s British mathematician Alan Turing proposed a simple reaction-diffusion system describing chemical reactions and diffusion to account for morphogenesis [1]. Due to the complexity of nature, researchers have not yet succeeded in developing a Turing system based model that would describe morphogenesis in essence, although specific examples such as the skin coloring of animals have been modeled using Turing systems.

Turing systems show a very rich behavior from the pattern formation point of view, which means that by numerically solving these mathematically defined systems we obtain a variety of spatial patterns in two dimensions and structures in three dimensions, varying from spots to stripes and from lamella to chaotic structures. In this study we have obtained results for a three-dimensional system, which, to the authors' knowledge, has never before been studied using numerical simulations [2]. We have also studied the basic characteristics of a three-dimensional Turing system and compared them with those of a two-dimensional one. In 3D the morphological development becomes more interesting and complex. We have also studied the transition between two and three dimensional morphologies. In addition, the robustness of Turing structures has been investigated against Gaussian random noise.

Our motivation for studying the Turing systems is biological. We have shown how connections between certain points can be grown by using a Turing system with sources of chemicals. The resulting connected network has many interesting properties, and in the future work we will concentrate on a Turing system and an active random walker model combined to explain some of the features of neural patterning, i.e., how neurons establish connections to other neurons.

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Spectral statistics of R-R intervals in ECG

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The statistical distribution of R-R intervals and Delta-3 statistics provide useful information on the structure of fluctuation-patterns in heart-rate dynamics. We compare the spectral analysis of R-R intervals with the prediction of the random matrix theory.

Pseudointegrable systems with growing surface roughness

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We study numerically the level statistics of pseudointegrable billards with different degrees of surface roughness, expressed by the different genus numbers g and calculate the eigenvalues and the eigenfunctions under Dirichlet and Neumann boundary conditions.

We study the distance distribution $P(s)$ of the eigenvalues and related quantities such as half of the second moment $I_0 = (1/2)\int s^2 P(s)ds$, the spectral rigidity Δ_3 and the number variance Σ^2. The considered systems possess genus numbers between $g = 2$ and $g \approx 700$. Their shapes range from very simple two-step systems to systems with many small teeth at the boundary and fractal drums with complicated scaling surface roughness. The eigenfunctions are analyzed according to their localization volume $V_{\rm loc}$ (inverse participation ratio), their amplitude distribution $P(\psi)$ and the momentum space.

For lower energies we can distinguish several energy windows with different behavior. In some windows, the values of I_0 come close to the value of the Poisson distribution $I_0 = 1$. The eigenfunctions in these regimes are either localized (with $V_{\rm loc} \leq 0.2\,V_0$, where V_0 is the surface of the system) or relatively regular functions with $V_{\rm loc} \approx 4/9V_0$, the value of a sine or cosine function. In other energy windows, the values of I_0 come closer to the Wigner limit of $I_0 \approx 0.637$ and the eigenfunctions approach chaotic functions with a Gaussian distribution $P(\psi)$.

Except for some unusual nearly-symmetric system shapes, we find an asymptotic energy regime above some non-universal limiting energy value. In the asymptotic regime, the genus number g seems to be the determining parameter that governs the level statistics. With increasing g, the behavior of the systems in the asymptotic regime changes from Poisson-like to Wigner-like behavior.

3D BEC solitons under transverse confinement

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The Bose-Einstein condensate (BEC) of a dilute gas of bosons is well described by the 3D Gross-Pitaevskii equation (3D GPE), that is a nonlinear Schrodinger equation. By imposing a transverse confinement the BEC can travel only in the cylindrical axial direction. We show that in this case the BEC with attractive interaction admits a 3D bright soliton (shape invariant) solution which generalizes the text-book one, that is valid in the 1D limit (1D GPE). We investigate stability and collective oscillations of this Bose-condensed bright soliton. By comparing the results of the 3D GPE with those of the 1D GPE, we find a good agreement only for weakly-interacting bright solitons. In particular, contrary to the 1D case, the 3D bright soliton exists only below a critical attractive interaction which depends on the extent of confinement. Analyzing the macroscopic quantum tunneling of the bright soliton on a Gaussian barrier we find that its interference in the tunneling region is strongly suppressed with respect to non-solitonic case, moreover the reflected and transmitted matter waves are not solitonic. Finally, we show that the collapse of the soliton is induced by the scattering on the barrier or by the collision with another matter wave when the density reaches a critical value, for which we derive an accurate analytical formula.

Influence of stochastic conditions on self-organization in irradiated materials

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The theoretical investigation for the role of nonlinear couplings between elements of damage structure of irradiated crystal in its self-organization and influence of external fluctuations of defect generation rate on this process is developed. It is shown that nonlinear couplings determine kind and parameters of the arising structure or terminal behavior namely concentration decomposition in alloys, oxygen impurity redistribution and its nonhomogeneous precipitation in silicon, breaks on curves of the creep-rate dependence on temperature and dose, creep and swelling rate oscillations. Parameters of the arising structures and conditions of its development are obtained. It is determined that the fluctuations of irradiation conditions are most strongly apparent in dynamic behavior of system. As a result new unstable regions and new dynamic behavior that do not have deterministic analog appear. The cause of it is strong interaction between fluctuations that is accompanied by their essential increase. Fluctuations in irradiated materials are substantially nonequilibrium, since the main cause for generation of defects, i.e. irradiation, is an external factor. Unlike the equilibrium internal fluctuations, they are not inversely proportional to a certain power of the characteristic size of the system. On the contrary, significant deviations of the density of point defects from its average in the process of irradiation are quite common. Especially great is the role of random external disturbances in systems with nonlinear links, in processes of radiation-induced transformations, which have a threshold nature and exhibit branching points, where qualitatively different ways of evolution are possible. On the one hand, the stochastic nature of defects is related to the fluctuations of the flux of impinging particles (neutrons, ions, etc.). On the other hand, the formation of defects in real crystal correlates with the disposition of various imperfections in it, such as dislocation, grain boundaries, precipitates, etc., which are distributed in the crystal randomly.

Determining the ordered or chaotic nature of orbits in Hamiltonian systems by the smaller alignment index (SALI) method

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We apply the method of the Smaller Alignment Index (SALI) for determining the ordered or chaotic nature of orbits, on some examples of Hamiltonian systems and symplectic maps. The computation of the SALI for a sample of initial conditions allows us to distinguish easily between regions in the phase space where ordered or chaotic motion occurs. The computation of SALI is performed rather easily: for a given orbit we follow the evolution in time of two different initial deviation vectors computing the norms of the difference (parallel alignment index) and the addition (antiparallel alignment index) of the two normalized vectors. The time evolution of the smaller alignment index reflects clearly the chaotic or ordered nature of the orbit. In general the SALI tends to zero for chaotic orbits, while it fluctuates around non-zero values for ordered orbits. The relationship of this index with the Lyapunov exponent is also discussed.

Periodic trace formulae for elastodynamic resonators

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We report on progress in Gutzwiller periodic orbit theory for elastodynamical resonators. The equations we solve are the conventional equations from continuum mechanics which are used to model phonon properties of mesoscopic structures as well as macroscopic bodies such as cars and bridges. The wave chaos is quantified in Gutzwiller theory which predicts the mode density using periodic classical orbits.

The complexity arises from the vectorial character of the elastic wave equation and the boundary conditions which for free boundaries leads to wave splitting and a multitude of surface waves. Nevertheless when these are incorporated a Gutzwiller description of the spectral density as a sum over periodic orbits arise [1]. Formally from this the diagonal approximation can be made giving random matrix behaviour as observed also in experiments.

The talk will concentrate on solid bodies with simple shapes as for example discs and rectangles. Appropriate trace formulae for the density of modes differ here from the Gutzwiller trace formula in that sums over isolated orbits are replaced by sums over families of orbits.

We shall pay particular attention to orbits with incidence angles beyond the critical as these turn out to be less attenuated than ordinary ray splitting orbits. Furthermore surface orbits will be included as some for elastodynamics are virtually undamped at high frequencies. Similar surface orbits are also seen for impedance boundary condition in electrodynamics. Such behaviour does not occur in conventional scalar quantum mechanics where surface orbits are only important at low frequencies.

The ray splitting itself can be thought to introduce mixing in otherwise regular systems or more precisely a much larger number of periodic orbits arise. As a consequence even for simple shapes spectral statistics from integrable to chaotic can be observed contrary to scalar quantum mechanics.

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Steady plane Stokes flow with long range correlation, fractal Fourier spectrum, and anomalous transport

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We demonstrate a class of two-dimensional steady viscous flows which have singular continuous (fractal) Fourier spectra. Such flows represent a novel intermediate stage between order and Lagrangian chaos: the motion of individual fluid particles in them is neither entirely correlated nor completely disordered. We describe qualitatively and numerically the solutions of the two-dimensional Stokes equations for a flow of a viscous incompressible fluid past a periodic array of solid cylinders. The equations are solved on a square with periodic boundary conditions; the central part of the domain is occupied by the solid body with no-slip condition on its border. For the observable associated with the moving fluid particle (e.g. instantaneous velocity of the particle), the temporal autocorrelation function decays; the decay rate is prescribed by the power law. Estimation of the Fourier spectrum indicates that the spectral measure is supported by the multifractal set. Furthermore, this simple steady two-dimensional flow pattern exhibits anomalous transport properties. Existence of this unusual state is caused by the power-like singularities of passage times which develop along the particle paths near the surface of the solid body.

Pulses in high-order dissipative models of active systems

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Many important problems of applied mathematics and physics may be reduced to analyses of single evolution PDE with dissipation (like the Kuramoto-Sivashinsky equation in chemical physics and fluid mechanics). In order to balance internal energy influx such equations include high-order (> 2) spatial derivatives responsible for the energy dissipation. We consider two high-order dissipative models of that kind, one of which describes spinning combustion front, and the other describes extended elementary particle. The equations have similar properties. They are invariant with respect to constant shift of function of interest and contain nonlinear internal source of energy. As a result of a balance between the source and dissipation, nonlinear dissipative structures emerge in the form of pulses. The first equation in question, which is fourth-order, leads to stable standing pulse representing extended elementary particle, that is particle with finite size (not just physical point as in classical quantum mechanics). This model develops the work of Sivashinsky (Nuovo Cimento, 77A, 1983, 21) where the classical Hamilton-Jacobi (HJ) equation was extended using fourth-order dissipation. That approach, however, led to unstable pulse. We show that the Sivashinsky equation can be modified further to include nonlinear terms in order to ensure stability of the pulse. Remarkably, the original aim of his model was to put forward possible explanation of quantum randomness by deterministic law. This aim was achieved, because the extended HJ equation had global chaotic solutions. Whether or not chaos exists within our model is not clear yet. The second equation in question, which is sixth-order, models spinning combustion waves. It has fundamental solution in the form of travelling pulse. We present and discuss the results of numerical analyses of the models.

Spectral statistics of a spin-1/2 coupled quartic oscillators

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The quantum spectral statistical properties, i.e. the nearest neighbor level spacing distribution (NNSD), the mode fluctuation distribution (MFD), the spectral rigidity etc. of a spin-1/2 particle in two- and three-dimensional coupled quartic oscillator potentials are numerically studied. Changing coupling parameters the system is continuously transformed from an integrable one to chaotic ones. In the chaotic regime, selecting non-zero parameters, various kinds of the Gaussian ensembles(GEs): GOE, GUE and GSE can be achieved [1]. Especially, it is a rare example for the GSE. We have been interested in the Gaussian ensembles in the aspect of the intermediate system between the integrable one and the GEs. In order to have reliable statistics of quantum levels, it is necessary to evaluate thousands of energy levels from the ground state without missing any. We compute the quantum energy levels by numerical diagonalization of the truncated matrix of the Hamiltonian in the basis of harmonic oscillators. Most of our calculations start with the system of the superposition of two or three truncated harmonic oscillators [2,3]. The method with the spherical Bessel functions and the surface harmonics has been also developed. Several types of the interpolation formula of the Poissonian and the Wigner-like distribution of each GEs for the NNSD are examined. It is found that the MFD is more sensitive to the integrability of the system than the chaoticity as in the case of a spin-less particle. The distribution of $n(> 1)$ nearest neighbors is also examined [4].

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Neutrally bouyant particles and bailout embeddings in three-dimensional flows

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We use the bailout embeddings of three-dimensional volume-preserving maps to study qualitatively the dynamics of small spherical neutrally buoyant impurities suspended in a time-periodic incompressible fluid flow. The accumulation of impurities in tubular vortical structures, the detachment of particles from fluid trajectories near hyperbolic invariant lines, and the formation of nontrivial three-dimensional structures in the distribution of particles are predicted.

Semiclassical analysis of Wigner functions

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The Wigner functions are quantum analogues of the classical phase space density. They are functions of both momenta and coordinates and are typically obtained from wavefunctions that are usually given as functions of only either coordinates or momenta. The motivation for this work was the question whether using the Wigner functions one can perform the formalism of quantum mechanics in the full phase space without referring to the actual wavefunction. While the formal equations for such a procedure are well known in the form of Moyal brackets, these can hardly be used directly as, in the general case, they represent a differential equation of an infinite order. While a naive semiclassical expansion of the Wigner function does make the equations in the Moyal approach to become of finite order for any order of the Planck's constant, such an approach fails immediately after the lowest order solution, which itself is the delta distribution on the invariant component of the classical phase space of the system. The Moyal bracket can, however, be rewritten in the form of an integral equation. In analogy to the usual WKB semiclassical approach for the wavefunction we expand the phase of the Wigner function in terms of the powers of the Planck's constant. If we now perform the analysis of the integral equation around the stationary point as given by the lowest order contribution to the phase of the Wigner function, we obtain a set of recursive equations that, at each order of the Planck's constant, contain only a finite number of terms. Unlike in the previously mentioned naive Moyal approach this set of equations allows us to find a solution to all orders of the Planck's constant. One should also note that such a separation does not happen if one inserts the phase expansion directly into the Moyal bracket. We apply this approach to the harmonic oscillator. We find the energy eigenvalues to all orders of the Planck's constant and hence obtain them exactly, while for the Wigner functions themselves we show that even the low order solutions match the exact ones very nicely and that they also agree with certain analytical approximations for Legendre polynomials.

‘Persistent currents’ and eigenfunctions in thin microwave resonators with broken time reversal symmetry

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A magnetic flux through a mesoscopic metallic ring gives rise to a persistent current which can be detected via characteristic oscillations of the magnetization depending on of the applied field [Lévy et al., Phys. Rev. Lett. **64**, 2074 (1990)]. In this paper a direct visualization of such persistent currents in a microwave analogue experiment is reported, making use of the analogy between the probability density current in the quantum-mechanical system and the Poynting vector in the corresponding electromagnetic one. To break time-reversal symmetry, a small ring of a ferrite material in a static external magnetic field was introduced into the resonator. In our analysis of the experimental data we employ the off-diagonal elements of the scattering matrix. Due to the small size of the ferrite compared to the resonator size the symmetry was partially broken in the sense that the real and the imaginary parts of the wave functions are not equally large on average. The statistical properties of the wave function's real and imaginary parts $\mathrm{Re}(\psi)$, $\mathrm{Im}(\psi)$, respectively, as well as the distributions of the total wave function's amplitudes $|\psi|$ and the statistical properties of the probability density currents were also examined and compared to the theoretical predictions.

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Can chaos improve stability of quantum algorithms?

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Stability of quantum computers to imperfections is of crucial importance for their proper working. Indeed, the power of a quantum computer relies on an interference and entanglement and can be as such very sensitive to perturbations, whether they result from the coupling with the environment or from the faulty algorithm itself. For successful realization of even modest-size quantum computer, understanding sensitivity to the perturbation is very important. As a measure of stability we choose to study fidelity. First, we expand fidelity in terms of correlation functions of a generator of perturbation. Decay of fidelity is related to the integral of a correlation function. Contrary to common intuition, we argue that for "chaotic" quantum systems, where correlation functions decay fast, the fidelity falls exponentially with time. In turn, for "integrable" systems with a non-decaying correlation function, the dependence is Gaussian. Although simple, this novel way of looking at fidelity greatly facilitates understanding of its behavior under perturbation. To demonstrate the above theory we apply it on a Quantum algorithm for discrete Fourier Transformation (QFT) algorithm. For a constant static random matrix perturbation on the whole Hilbert space of a computer, the fidelity falls with the number of qubits n as $\exp(-cn^3)$. By a simple change of an algorithm, so that the correlation function decays faster, we change this behavior into $\exp(-cn^2)$. For instance, this would mean improving fidelity in the case of $n = 100$ qubits from 0.1 for plain FFT algorithm to 0.9 for our improved one.

Experiments on chaotic systems with a mixed phase space

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Experimental and theoretical studies devoted to investigation of chaotic systems with a mixed phase space such as H atoms interacting with strong microwave fields and quantum billiards are reported. We describe combined experimental-theoretical study to realize and control the behavior of nonlinear common (joint) resonances that occur when excited H atoms are driven by linearly polarized microwave field consisting of two commensurate frequencies. We show how the relative phase of the field can be used to affect the resonances in the mixed phase space and, by this means, control the ionization probability. We also study the regime of quantum cantori in quarter-stadium billiards. Experimentally, the stadium billiard is simulated by a thin quarter-stadium microwave cavity. The eigenfunctions of the quarter-stadium billiard with the parameter $\varepsilon = 0.1$ are reconstructed in the cantori regime $N = 7 - 63$ using a field perturbation technique and a circular wave expansion method. We show that in the quantum cantori regime the rescaled localization length of the eigenfunctions does not depend on average on N. Support by KBN grant No. 2 P03B 023 17 is acknowledged.

Abstracts of Posters

Directed transport in a spatially periodic magnetic billiard

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As an example of a Hamiltonian ratchet in two-dimensional space but without driving, we study a spatially periodic billiard immersed in a uniform stationary magnetic field. Our billiard consists of a straight waveguide with equidistant walls attached perpendicularly to one side. The magnetic field is sufficient to break all symmetries relevant for transport, an external driving therefore is unnecessary. In the low- and high-field limits, the motion becomes respectively pseudo-integrable and integrable; in both cases no transport is possible. In the medium-field regime, surfaces of section reveal mixed phase-space structures. Chaotic components support diffusive transport only in one direction, while regular components can support ballistic transport in both directions. We analyze the dynamical mechanisms underlying directed transport in terms of stable periodic orbits.

Nonlinear dynamics of pathogenesis and prognosis factor

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A mathematical model showing the interaction of normal cells, cancer cells and food supply is shown to yield interesting results. This model is extended to a more general case of pathogenesis. From these models a prognosis factor is computed that relates to the eventual outcome of the disease.

Sparks like coral reefs (the press said...)

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The streamers of glowing, ionized gas in a lightning strike have an inevitable tendency to break up at their tips, creating a familiar tree-like pattern. This tip-splitting is similar to the same kind of process that gives rise to branching in snowflakes, river channels, tumor growth and other different natural phenomena. In the 1980s, researchers identified a process called Laplacian growth which explains why branching occurs at the boundary between two substances. We have shown that a Laplacian instability also occurs at the edge of a lightning streamer. We analyse the minimal continuum model for a negative streamer in a non-ionized and non-attaching gas with impact ionization reaction in local field approximation. Using computer simulations, we have found that a streamer undergoes spontaneous branching at the tip. The mathematical analysis of the model identifies the instabilities as Laplacian ones. Despite their complicated structure, the basic model for streamers is simple. A single free electron traveling in a strong, uniform electric field ionizes the gaseous molecules around it, generating more electrons and a chain reaction of ionization. The ionized gas creates its own electric field, and a streamer is born. Eventually its charge and field concentrate in a thin, dome-shaped layer at the tip of the growing streamer. In this state, any tiny protrusions in the charged front focus the electric field around them. Because the speed of ionization depends on the surrounding field, these bumps grow faster than the rest of the front, and branching results. A similar mechanism can operate in a coral reef, when one part pulls ahead of the rest, it gets more food and so grows even faster.

Deterministic diffusion in a gravitational billiard

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We study numerically the motion of a classical particle in a homogeneous gravitational field bouncing elastically from a boundary limiting the motion from below. The boundary is piecewise linear and has wedge-like shape which is periodically extended over the entire horizontal axis. The angle characterizing the symmetric wedge discriminates between the region with only chaotic motion, and the region with periodic, limited or extended quasiperiodic, and chaotic trajectories.

It is observed that for a random set of initial conditions on the constant energy surface, the mean-square displacement asymptotically becomes linear function of time. The diffusion coefficient is examined as a function of energy and wedge angle. For large wedge angles and intermediate values of time t, the mean-square displacement scales as a power function of time with degree which can be larger then 2.

Examination of higher order moments gives further numerical evidence for tendency toward Gaussian distribution.

Homoclinic orbits of invertible maps

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We present a numerical method for finding all homoclinic orbits of an invertible map in any dimension. The method has completely controllable accuracy and is capable of uniquely identifying and naming each homoclinic orbit.

Breathers and homoclinic orbits

Homoclinic orbits are presented as a useful tool for obtaining breather solutions (time periodic, spatially localized) of one dimensional nonlinear lattices (Klein Gordon, FPU and mixed). We present how the relationship between homoclinic orbits and breathers is formed, and advantages of using this method for obtaining breathers over the traditional method of continuation.

Physics of love

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Groups of individuals will be discussed by the proposed report. Both are described as very abstract things. The theory is applicable to a lot of individual kinds, which tend to organise themselves to groups. Some examples are cells, animals, humans, economies, nations. The property all of them have to share is the possibility of information processing, although the minimum can be one single method of comparison. Quantum physical results and a fermionic explanation of gender are issued. The theory uses mainly quasi-static description, but has at least revealed a standard thermodynamic law: dH=TdS+Vdp The two terms at the right side display the two main parts of the reports content. The properties derived from the axioms show similarity to real thermodynamic gases. Publications of social science in the past had left well-known problems unsolved. One of them is feedback of experience another are unselfish individuals, a third one is diversity. The three named problems and probably others are vanished in the models of the theory. The founding book content (3MB, 260 pages) can be downloaded at: http://www.celebran.de. One target of application for the conference is searching for a doctor father, scientific acceptation of the theory and/or controversy discussion. Learning about other contributions and stimulation of new ideas is another target.

A pedagogical survey on some aspects of martingales

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We give a pedagogical introduction to the concept of martingales: The contribution addresses especially students participating in the International Conference "Let's Face Chaos Through Nonlinear Dynamics" in Maribor 2002. We deal in particular with Doob's Upcrossing Lemma and present some applications. A recently established connection between the structure of martingales and Hermite functions by Fitzsimmons is presented. Basic properties of the classical Hermite functions are revised. There remains the question whether one is able to generalize the obtained results in context of a discrete scenario, i.e. when replacing the used continuous Hermite functions by suitable discrete ones.

Quantum manifestation of classical stochasticity in the mixed state

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A Hamilton system with a few local minima in the potential energy surface represents a model in frame of which one can describe the dynamics of transitions between different equilibrium states, including such important transitions as chemical reactions, nuclear fission and others. Systems of such type allow existence of several critical energy values even for a fixed set of potential parameters. That leads to a possibility of existence of so-called mixed state [1] for such potentials: different dynamic regimes (regular or stochastic) are realised in the same energy interval in different local minima. It gives new possibilities for studies of the quantum manifestations of classical stochasticity (QMCS), using the following objects: statistical properties of energy spectrum (nearest-neighbour spacing distribution), structure of the wave functions (nodal curves, probability density) and wave packet dynamics. We consider such possibilities in application to quadrupole surface oscillations of nuclei, described by the lowest terms of the deformation potential decomposition in deformation parameters. In that potential the mixed state is observed in that part of the parameter space, where the equilibrium shape of the nucleus can be either spherically symmetric or deformed, i.e. for the potentials with a few local minima. In the case of the potential energy surface of complicated topology numerical calculation based on matrix diagonalization becomes ineffective, but the so-called spectral method [2] can become an inspiring alternative. Since the spectral method is fundamentally based on numerical solutions of a time-dependent differential equation, its implementation is always straightforward. Neither special ad hoc selection of basis function is required, nor is it necessary for the potential to have a special analytic form. The spectral method is in principle applicable to problems involving any number of dimensions.

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b-cell repolarization in the bursting Chay-87 model

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The pancreatic b-cell is the only insulin-producing type of cell in the body and is thus crucial in the regulation of the plasma glucose concentration. Failure of the glucose regulation leads to diabetes mellitus, a medical condition of almost epidemic proportions: According to the WHO, there are presently in the order of 150 million people suffering from diabetes mellitus. A number which is believed to double by the year 2025. The insulin secretion of the b-cell is known to be associated to a distinct pattern of oscillation of the cell membrane potential called bursting. The bursting pattern contains a silent phase which alternates with bursts of rapid oscillations (spikes). Since the mid-80s, a significant number of models of the b-cell electrophysiology have been constructed. In 1987 T. R. Chay proposed a b-cell model, which is based on five ionic currents through the cell membrane: Voltage gated calcium and potassium currents generate the rapid spikes of the active phase, while ATP- and calcium-dependent potassium currents are responsible for the level of activity and the repolarizing of the membrane, respectively. Finally, a leak current stabilizes the dynamics. The model contains three variables. The time constants of the membrane potential, V, and the opening probability of the voltage gated potassium channels, n, are small compared to that of the slow variable, the intracellular free calcium concentration. This difference inspires the separation of the three-dimensional phase space into a plane of rapid dynamics and a slow direction. Using the slow variable as a parameter for the fast subsystem yields the so-called reduced system. It has been shown that the reduced system contains a stable limit cycle which disappears in a homoclinic bifurcation as the system leaves the active phase. The poster will show the homoclinic bifurcation of the reduced system and investigate the dynamics in the three-dimensional phase space in vicinity of the isoclines of the reduced system.

Quantum chaos experiments with superconducting microwave cavities

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In the last few decades, billiard systems have provided a most appropriate model for the understanding and discussion of classical chaotic Hamiltonian systems and their quantum counterparts. In two dimensions the Schrödinger equation for quantum billiards coincides with the Helmholtz equation for flat cylindrical microwave cavities of corresponding shape. This correspondence provides the basis for experimental studies of quantum manifestations of classical chaos. We present results of experiments with normal and superconducting microwave cavities, where the former are used for the measurement of wavefunctions, the latter for the measurement of energy eigenvalues.

Power laws in a noisy finite-time-singularity model

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We discuss the influence of white noise on a generic dynamical finite-time-singularity model for a single degree of freedom. We find that the noise effectively resolves the finite-time-singularity and replaces it by a first-passage-time or absorbing state distribution with a peak at the singularity and a long time tail exhibiting power law or stretched exponential behavior. The study might be of relevance in the context of hydrodynamics on a nanometer scale, in material physics, and in biophysics.

Wigner function statistics of quantum maps

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We studied statistical properties of Wigner functions $W(x)$ of 1D quantum maps on compact phase space of finite area V. For this purpose we defined a Wigner function probability distribution $P(w) = 1/V \int \delta(w - W(x))dx$, which has, by definition, fixed first and second moment. In particular, we concentrate on relaxation of time evolving quantum state in terms of $W(x)$, starting from a coherent state.

We have shown that for a classically chaotic quantum counterpart the distribution $P(w)$ becomes a Gaussian distribution that is determinated by the first two moments. The numerical studies were done on the quantum sawtooth map and the quantized kicked top. In a quantum system with Hilbert space dimension $N(\sim 1/\hbar)$ the transition of $P(w)$ to a Gaussian distribution was observed at times $t \propto \log N$. In addition, it has been shown, that the statistics of Wigner functions of propagator eigenstates is a Gaussian as well in classically fully chaotic regime.

Estimation of effective vertical diffusivity and viscosity in turbulent exchange flows by numerical experiments

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Goitsche lake complex in Central Germany comprises two sub-basins, namely Muehlbeck and Niemegk, with distinct water compositions, which are linked to one another by a channel. The channel owes its nearly constant cross section and flat design of the bottom to the mining activity, which left a void after closure of the mine in the 1990s. The importance of exchange flows through such channels or straits, which are interconnecting water bodies of different densities, lies in the fact that water mass characteristics in sub-basins are determined by these flows, extend of which depends on the exchange rate between the interconnected water bodies. The knowledge of kinematic viscosity, k, and vertical diffusivity, v, along a channel/strait is therefore crucial as this gives us an idea about the amount of heat, mass and dissolved solids transported across it and provide parameterisations for circulation models. By conducting a number of numerical experiments with varying Rayleigh number and aspect ratio, we attempted to give an estimation of effective values for k and v in this channel. From comparison with numerical results of an exchange through a channel at various values for Ra, Pr and aspect ratio, we infer the effective value for kv as it appears in the Rayleigh number.

Rule and density behavior in rule-dynamical systems

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'Rule-dynamics' is a framework for simple models of rule-changing systems using cellular automata rules. Due to the coupling between the several density-range and rules, the correlation between rule and density generates, and some rule-dynamical systems have complete correspondence of rule sequence to the density-value. We propose one of possible methods to capture the rule and density behavior in terms of the correspondence of rule sequence to small density-range using typical example system.

Wavefunction scarring in open quantum dots: A microwave-billiard analog study

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We study the wavefunctions and transmission properties of an open microwave cavity in the frequency range of 1 - 17 GHz. The transmission of microwaves between the input and output antennas of cavity is found to exhibit large fluctuations as a function of frequency, while the cavity wavefunctions are found to reveal the presence of periodically-recurring scars, in close correspondence with specific peaks in the Fourier transform of the transmission fluctuations. This work therefore provides strong support for the results of recent studies of open quantum dots [1], which have revealed the presence of measurable transport effects associated with specific wavefunction scars.

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Universal transmission fluctuations in open microwave billiards with and without time-reversal symmetry

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Random matrix theory predicts characteristic distributions for the transmission amplitude T in dependence of the number of attached wave guides, and of time-reversal symmetry as well [1,2]. In quantum dots the predictions could be verified only qualitatively [3,4]. In the present experiment the transmission of microwave billiards was measured with and without [5] reflection symmetry for different numbers of channels. Time-reversal symmetry was broken via introduced ferrite cylinders. The obtained transmission intensity distributions are compared with predictions from random matrix theory. Because of the strong absorption caused by the ferrites, the existing statistical scattering theories had to be modified by incorporating a number of additional absorbing scattering channels. A better characteristic for the systems can be obtained from the distribution of dT/dE.

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Periodic phase synchronization in coupled chaotic oscillators

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Recently synchronization behavior in mutually coupled chaotic oscillators with slight parameter mismatch has been extensively studied because of its potentiality in application to many different disciplines of science and technology. What has been known about it that non-synchronization state transits to lag synchronization through phase jumps and next phase synchronization (PS) as the coupling strength increases. One of the most intriguing subjects among those is PS which is characterized by the coincidence of the phases of two chaotic oscillators within 2π, while their amplitudes remain chaos and uncorrelated each other. A typical behavior here is that one of the vanishing Lyapunov exponents becomes negative. However, before phase synchronization, one can usually observe a negative dip in the Lyapunov exponent. So the aim of our study is to investigate synchronization behavior in this dip and to manifest the full route from non-synchronization state to lag synchronization in coupled chaotic oscillators. When two Rössler oscillators with slight parameter mismatch are coupled with each other, the phase difference of them can be classified into two kinds of dynamics, fast and slow. When the fast dynamics is removed after being averaged over during the time segment of the half period of the slow dynamics, we can observe that the slow dynamic is synchronized periodically at the dip. We call this synchronization phenomenon periodic phase synchronization. In this region, the phase difference of the two oscillators jumps periodically, so that the derivative of it touches the zero line periodically while their phases are synchronized temporally. Before periodic phase synchronization, the two chaotic oscillators are de-synchronized. So what we clarify about the synchronization transition in coupled chaotic oscillators is that non-synchronization state develops to lag synchronization through periodic phase synchronization, next intermittent phase synchronization, and then PS.

Chaotic properties of some billiards with singular boundaries

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Several types of lemon-shaped billiards and some symmetrical billiards with discontinuities in the curvature radius are investigated classically and quantally. Special attention is paid to orbits in which the particle is reflected from the boundary at the singular points. The orbit stability is investigated in dependence on the shape parameter and compared for different boundary types. Further analysis of the presented classical billiards include the Poincare surface sections. It can be observed that for certain boundary shapes and for some ranges of the shape parameter the chaotic part of the phase plane separates into several distinct chaotic segments. In the quantal calculation the chaotic fraction of the phase space is computed. The obtained classical values are compared with the corresponding results obtained from the statistical analysis of the energy level densities. The method used is the Nearest Neighbour Spacing Distribution (NNSD), and in fitting the calculated histograms the Brody, Berry-Robnik and Berry-Robnik-Brody distributions are used. Some properties of the calculated wave functions will also be discussed.

Limiting level spacing distribution of integrable quantum systems

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We report our recent studies on the energy level statistics of integrable quantum systems. Based on the assumption proposed by Berry and Robnik, level spacing distribution of a system consisting of infinitely many independent components is derived as a weak limit and its deviations from the Poisson distribution is discussed. The limiting level spacing distribution is specified by a single monotonically increasing function $m(S)$ of the level spacing S. Three cases are distinguished by using $m = m(\infty)$: (i) Poissonian if $m = 0$, (ii) Poissonian for large S, but possibly not for small S if $0 < m(\infty) < 1$, and (iii) sub-Poissonian if $m(\infty) = 1$. This implies that, even when energy level distributions of individual components are statistically independent, the Berry-Robnik approach would provide level spacing distributions other than the Poissonian distribution.

On Marcus approach to quadratic ODE and stability of origin

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A possible hypothesis for algebraic properties for stability of the origin of quadratic systems of ODE's is given. Marcus classification of all commutative algebras in the plane is used for proving the absolute validity of given hypothesis for quadratic systems in dimension two. For inducing the hypothesis a large family of quadratic systems in 3D was treated.

Anomalous diffusion in a Hamiltonian system

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We numerically study transport property inside the stochastic layer of 2 degrees of freedom Hamiltonian system where the trajectories are chaotic because of Homoclinic tangles. We found that the diffusion is anomalous and density profile is not Gaussian. Furthermore, we show that, with a random sampling method, this system exhibits Levy flight like diffusion. We discuss the relation between regular and random sampling in terms of scaling exponents.

Equivalence between isospectrality and iso-length spectrality for a certain class of planar billiard domains

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Isospectrality of the planar domains which are obtained by successive unfolding of a fundamental building block is studied in relation to iso-length spectrality of the corresponding domains. Although an explicit and exact trace formula such as Poisson's summation formula or Selberg's trace formula is not known to exist for such planar domains, equivalence between isospectrality and iso-length spectrality in a certain setting can be proved by employing the matrix representation of "transplantation of eigenfunctions" [1]. As an application of the equivalence, transplantable pairs of domains, which are all isospectral pair of planar domains and therefore counter examples of Kac's question "can one hear the shape of a drum?", are numerically enumerated and it is found at least up to the domain composed of 13 building blocks transplantable pairs coincide with those constructed by the method due to Sunada [1,2]

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Can one determine the shape of a drum through the spectrum of the interior Dirichlet problem and the cross sections of the exterior Neumann scattering?

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The quantum billiard problem, that is the Dirichlet problem for the Helmholtz equation, can be rewritten as a Fredholm integral equation of the second kind with the aid of the Green's theorem, and then the eigenenergies of the quantum billiard can be characterized as the zeros of the Fredholm determinant on the real axis [1,2]. From this view point, we pose a new question "can one determine the shape of billiard table through the Fredholm determinant?" instead of the famous Kac's question "can one determine the shape through the eigenenergies i.e. the zeros of the Fredholm determinant?" [3], which was solved negatively [4]. Our numerical tests reveal that the answer to our question addressed above is "yes", that is, the shapes of the isospectral pair of billiards are distinguishable from the eyes of the Fredholm determinants although they have exactly the same eigenenegies [5]. Via "the interior-exterior factorization of the Fredholm determinant" [2], the difference of the Fredholm determinants between the isospectral pair can be interpreted as the difference of the scattering phase shifts when the billiard table are regarded as a scatterer against the exterior wave function.

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Noise induced phase synchronization in diode lasers

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Chaos synchronization by external common noise is an intriguing phenomenon in chaotic dynamical systems in connection with application to secure communication and generalized synchronization. When common arbitrary noise is added to identical chaotic systems externally, the systems are synchronized at a certain region of the noise amplitude. In apart from these studies, phase synchronization is also extensively studied. When two slightly different chaotic systems are coupled with each other, their phases begin to be synchronized when the coupling strength exceeds the critical value. Here, we study phase synchronization in diode lasers by applying a common noise-like signal simultaneously. In the experiment, when two diode lasers generate chaotic outputs, we feed a common noise-like signal to the laser drivers simultaneously. Then, each laser pumped by dc current is modulated by the noise-like signal. We observe the transition to phase synchronization as the amplitude of the noise-like signal increases. When the amplitude exceeds a critical voltage, the phases of the two lasers are synchronized with a time delay. Before phase synchronization, the dynamics shows intermittent phase jumps, which is similar to that appearing in coupled chaotic oscillators. This transition due to the common noise-like signal is a new phenomenon that has not been reported yet. We study the characteristics of the transition to phase synchronization in the phase spaces. And phase synchronization and intermittent phase jumps are analyzed by the dynamics of the phase difference of the two lasers according to the amplitude of the noise-like signal.

Random matrix analysis of human EEG data

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The correllation analysis of the human EEG and NIRS data is performed. We show that the spectral statistics of the underlying correlation matrix ensemble can be linked to the medical diagnosis of the given patient. Similar analysis is done also for data with chanels shifted by varying time lag. In this case the the analysis of (nonsymmetric) correlation matrix uncovers certain causal processes in the brain that participate in the signal generation.

Stochastic auto-oscillation of temperature and defect density in crystals under fluctuating conditions of irradiation

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The influence of external noise (that is fluctuation of defect production rate and heating under irradiation) on the dynamics of defect density and irradiated-crystal temperature change is examined. Time-dependence of crystal temperature and defect density according to irradiation conditions and crystal properties is one of the various stochastic processes. This is a stationary stochastic process or process which looks like auto-oscillation and Wiener's process. It is shown that regular auto-oscillation of mean values of defect density and temperature intermit with periods of irregular behavior. Characteristics of the auto-oscillations are not dependent on selection of initial conditions. The oscillation period is situated in the range 20 - 50 s. Changes of the second moments look like positive pulses, their values increase from zero up to amplitude value and then decrease again. With increase of intensity of external noise, the amplitude value begin to grow slowly, behavior of mean values do not change. When dispersion reaches the critical value, that is comparable to corresponding mean values, breakdown of the periodic regime takes place. It is characterized by sharp irregular change of all variables and significant increase of dispersion. Then the regular periodic regime is restored, amplitudes of dispersion become small and the whole process repeats again. The duration of the irregular behavior in various cases can be different. It is equal to nearly ten periods of the regular movement that continues during hundreds and thousands of its own regular periods. Regular periodic change of mean values is near to auto-oscillations of deterministic model. On the basis of these results one can conclude that the role of fluctuations of irradiation conditions becomes apparent mainly in the dynamic behavior of a system.

Stability of low-dimensional solutions in the Fermi-Pasta-Ulam system

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The Fermi-Pasta-Ulam (FPU) system represents a one-dimensional array of equi-mass particles coupled by nonlinear springs. Since the pioneering work of FPU [1], a large amount of numerical experiments have been carried out with this system in order to elucidate how nonlinearity leads to ergodicity and the equipartition law of energy. One of the striking phenomena revealed in these experiments is the "induction phenomenon" in the energy exchange process among normal modes, i.e., initially excited modes retain energy over a certain period before violent energy exchange process sets in [2]. The FPU system possesses low-dimensional solutions whose dynamics can be completely described by reduced Hamiltonians. Since the growth of initially non-excited modes can be attributed to the instability of the low-dimensional solutions, identification of them is important for precise understanding of the induction phenomenon. One can systematically construct the reduced Hamiltonians by introducing in the mode number space the "type I subsets" defined by Poggi and Ruffo [3]. In this presentation, we first present general expressions for the type I subsets, which, as far as we numerically checked, cover all of the possible type I subsets in the FPU system [4]. Using these expressions, we next construct the reduced Hamiltonians and investigate the stability of their solutions under the full system's dynamics, where the linear stability of these solutions is described by coupled Hill's equations. We report the independence of the stability properties on the phase space structure of the reduced Hamiltonian systems. Finally, we discuss the conditions for the occurrence of the induction phenomenon in Hamiltonian systems.

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On some difference equations in context of q-Fourier analysis

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A discrete Fourier transform on a q-linear grid is presented. Several of its analytic properties are discussed and compared with the continuum situation. We recognize that a special invariant of the related Fourier operator is closely connected to discretizations of the Hermite functions. This result is similar to the continuum scenario and reveals a key role of the q-Fourier transform to the understanding of difference equations. The continuum limit $q \to 1$ in the sense of strong L2-convergence is investigated for the derived q-Fourier invariant generalization of the Gauss curve. Applications to Schrödinger difference equations are briefly mentioned.

Finding periodic orbits in area preserving maps by topological degree theory

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We consider methods based on the topological degree theory to compute periodic orbits of area preserving maps. Numerical approximations to the Kronecker integral give the number of fixed points of the map provided that the integration step is small "enough". Since in any neighborhood of a fixed point the map gets four different combinations of its algebraic signs we use points on a lattice to detect the candidate fixed points by selecting boxes whose corners show all combinations of sign. This method and the Kronecker integral can be applied to bounded continuous maps such as the beam-beam map. On the other hand they cannot be applied to maps defined on the torus, such as the standard map, which has discontinuity lines propagating by iteration, or unbounded maps such as the Henon map. However, the systematic use of the bisection method initialized on the lattice, even though unable to detect all fixed points of a given order, allows us to find a sufficient number of them to provide a clear picture of the dynamics, even for maps of the torus because the discontinuity lines have measure zero.

Constrained random matrices

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The study of random matrices [1] has provided insight into many physical problems both in the quantum as well as in the classical domain. For example, random matrices have been very successfully used to model statistical properties of disordered conductors and of highly excited classically chaotic quantum systems. In the classical domain, random matrices arise in the context of diffusion in random, directed environments (see [2] and references therein). In all of these cases the random matrix elements obey symmetry requirements where appropriate, and are otherwise taken to be independently distributed random variables.

There are many cases, however, where constrained matrices must be considered: examples are electron hopping in amorphous semiconductors (see [3] and references therein), random reactance networks [4], and random Master equations.

We calculate the density of eigenvalues of constrained random $N \times N$ matrices in the limit of large N using a perturbative expansion [5]. The results are compared to the results of numerical simulations for finite values of N.

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Bifurcations in sixth-order PDE modelling spinning reaction fronts

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Spinning solid flames, so named because reacting solid compounds produce solid reaction products, and spinning detonation waves may be phenomenologically described by the sixth-order evolution-type PDE (Strunin, IMA J. Appl. Math., 63, 1999, 163). The equation is formulated in terms of the location of combustion front defined as the surface separating cold fresh mixture from hot reaction products. The equation contains nonlinear energy source term, nonlinear term facilitating energy flux towards smaller scales, and linear dissipation term expressed by sixth-order derivative. The model yields stable kink-type solutions travelling along spiral trajectories on cylindrical sample. Only large initial distortions of plane front will develop into nontrivial regimes, namely the spinning waves; small distortions will decay. Qualitatively, such dynamics well addresses the real phenomena. In terms of derivative of the front profile with respect to transverse coordinate the kinks correspond to pulses, or auto-solitons. The auto-soliton manifests itself as fundamental solution of the equation. We demonstrate numerically that arbitrary initial condition eventually breaks up into a set of identical pulses. Stress that the travelling waves (pulses) here are produced by purely dissipative mechanism. We present and analyse new results of numerical simulations of the model, focusing on bifurcations of regimes with various wave numbers. Of particular interest is the question whether chaotic regimes occur in large domains. We give some preliminary evidences in favour of positive answer to the question. We also discuss mathematical links of the equation to the generalized nonlinear phase diffusion equation of Kuramoto.

Multiplicity of limit cycle attractors in a coupled heteroclinic cycle system

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We study a new type of coupled oscillators system in which each oscillator has a heteroclinic cycle attractor instead of a limit cycle or a phase oscillator. Oscillators are distributed on a two-dimensional square lattice and coupled with the nearest neighbors diffusively. If an orbit approaches a heteroclinic cycle attractor, the period of the oscillation gets exponentially slower for each oscillation and the system has no characteristic time scale. We employ a replicator system with 4 components as the heteroclinic oscillator.

A replicator system is a model for an ecological system or a chemical reaction network of self-catalyzing molecules. Therefore the situation of our system can occur in a spotted ecological system with diffusion (ex. a system on trees of an orchard) or a population dynamics of self-catalyzing proteins in cells.

In this poster, we report a novel class of patterns that are spatially disordered but periodic in time. These patterns are found in the range of the small diffusion constant. We investigate the patterns and show that they are limit cycle attractors in the ambient phase space (i.e. not chaotic) and many limit cycles exist dividing the phase space as their basins. In addition, these patterns are constructed with a local law of difference of phases among the oscillators. The number of patterns grows exponentially with increasing of the number of oscillators.

Self-similar magneto-conductance fluctuations in general-shaped soft-wall quantum billiards

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Recently, using semiclassical Kubo formula, Budiyono and Nakamura showed [1] that the self-similar conductance is caused by the self-similar periodic orbits generated by pitchfork bifurcations of a straight-line liberating orbit oscillating toward saddle point in Henon-Heiles potential. This potential has three saddle points that give rise to the bifurcations, and is suitable to model a real triangle billiard with three leads [2]. They argue that the existence of such harmonic saddles is essential and that the self-similar conductance phenomena should be observed in systems with non-Henon-Heiles potential. To support their idea, using their methods, we show theoretically the emergence of the self-similar magneto-conductance fluctuations in general-shaped soft-wall quantum billiards. First we calculate the monodromy matrices of the periodic orbits in the Barbanis potential with two harmonic saddles that give rise to the bifurcations. And using the above result, we show the self-similar conductance. Furthermore we will show the self-similar conductance in real systems with more general potentials. Finally we expect that our results confirm the experimental evidence observed by Micolich et al. [3], that the global shapes of billiards are not the relevant issue to discuss the dimension of the fractal fluctuation.

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Geometric and statistical properties induced by separatrix crossings in volume-preserving systems

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We consider a 3D volume preserving system inside the unit sphere. The system is a small perturbation of an integrable one. Almost all phase trajectories of the integrable system are closed curves. Under an arbitrarily small non-zero perturbation all the interior of the sphere, up to a residue of a small measure, is apparently a domain of chaotic motion. The phenomenon is described as a result of jumps in an adiabatic invariant of the system occurring when a phase trajectory crosses the 2D separatrix of the unperturbed (integrable) system. The dynamical properties can be understood as a consequence of splitting of invariant manifolds under the perturbation. A 2D return map generated by the system possesses strong stretching properties. This makes the dynamics of the system close to hyperbolic. Numerical investigation of the statistical properties of the system demonstrates a good agreement with theoretical predictions made under the assumption that the system has a big ergodic component whose measure tends to the measure of the sphere's interior as the perturbation tends to zero. However, the system is not ergodic, and stable periodic solutions are found, surrounded by stability islands of measure exponentially small with the perturbation.

Parametrically driven dark solitons

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It is well known that bright parametrically driven, damped nonlinear Schrödinger solitons are prone to oscillatory instabilities which set in above a certain driving threshold. In this communication I shall demonstrate that the parametrically driven, damped dark solitons are immune from instabilities throughout their existence region. Furthermore, I shall show that for the undamped case there are two different stable solutions, which are also capable of forming stable complexes. I shall mention some noteworthy characteristics of the interaction of these different solutions.

Chaotic outputs of an operational amplifier

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Electronic circuits exhibiting chaotic behaviors have attracted much attention for their availability to simple experimental verification of chaos theory. Until now, such nonlinear electronic circuits have been developed as an inductor-resistor-diode circuit, a Chua circuit, and a double scroll circuit, and many nonlinear phenomena studied in theory have been investigated in these circuits. So to find out a new circuit that exhibits rich chaotic behavior is very important not only for experimental verification of chaos theory but also for application of chaos to practical fields like secure communication. For this, we introduce an element, operational amplifier (OP Amp), which generates chaos for itself. An OP Amp is used as one of the most important electronic elements that performs a wide variety of linear functions. In an OP amp, when the inverting terminal is connected with the output terminal through a resistor, and positive input terminal is grounded, the output signal has a phase shift due to the resistance-capacitance lag network within the amplifier. At this time when the total lag network phase shift equals or exceeds 180 degrees, the system is unstable and can generate a periodic signal. Here, when the absolute voltages of the negative and the positive power supplies are much less than 15 Volts and the resistor is zero, a fast OP Amp generates chaotic behaviors. We carry out an experiment by using LF357, and observe various chaotic behaviors according to the power supplies voltages. The bifurcation diagram and the phase diagram exhibit that the system develops to chaos through period doubling bifurcation. And the return maps show the origins of the chaotic behaviors that are type- I and II intermittencies, periodic windows, crisis, etc. All the experimental results well agree with the chaotic behaviors of the logistic map.

Wave propagation in random media with nonlocal coupling

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The purpose of this poster is to elucidate how noise intensity effects wave propagation in nonlocally coupled oscillators. It was shown that in the case of globally coupled oscillators, wave cannot propagate if the intensity of white Gaussian noise becomes above a critical point and that this transition occurs via a Hopf Bifurcation. We expect from this that the same fact is true for nonlocally coupled oscillators and show it by both a theoretical approach and computer simulation. The phase of nonlocally coupled oscillators obeys a stochastic equation. We can lead the Fokker-Planck equation equivalent to it. Near a point of the critical point, the Fokker-Planck equation can be reduced to the Ginzburg-Landau equation. The coefficients of the GL equation depend on a coupling function. When it is the pulse coupling function, the GL equation has an instability. We investigate the chaotic behavior. This model is thought to represent the real neurons and has an important meaning.

Many-body symbolic dynamics of a classical oscillator chain

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We study a certain type of the celebrated Fermi-Pasta-Ulam particle chain, namely the inverted FPU model, where the inter-particle potential has a form of a quartic double well. Numerical evidence is given in support of a simple symbolic description of dynamics (in the regime of sufficiently high potential barrier between the wells) in terms of an (approximate) Markov process. The corresponding transition matrix is formally identical to a ferromagnetic Heisenberg quantum spin-1/2 chain with long range coupling, whose diagonalization yields accurate estimates for a class of time correlation functions of the model.