CAMTP

LET'S FACE CHAOS
through
NONLINEAR DYNAMICS

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4th International Summer School/Conference
at the University of Maribor
27 June - 11 July 1999

1st Week	Mon 28 June	Tue 29 June	Wed 30 June	Thu 1 July	Fri 2 July	Sat 3 July
Chairman	Robnik	Stöckmann	Bountis	Prosen	Hasegawa	Casati
09:00-10:00	HAKEN	HAKEN	HAKEN	HAKEN	09:00-09:50 AIZAWA	09:00-09:20 ALONSO
10:00-11:00	CVITANOVIĆ	CASATI	CVITANOVIĆ	CASATI	09:50-10:40 PROSEN	09:20-09:40 CLERC
11:00-11:30	— COFFEE & TEA —				10:40-11:10 – COFFEE –	09:40-10:00 DAMGOV
11:30-12:30	BOUNTIS	AIZAWA	GROSSMANN	GROSSMANN	11:10-12:00 KURAMOTO	10:00-10:20 GARCIA- RUIZ
12:30-13:30	KURAMOTO	HASEGAWA	STÖCKMANN	GALLAVOTTI	12:00-12:50 ROBNIK	10:20-10:40 GORIN
13:30-15:00	— LUNCH —				12:50-15:00 – LUNCH –	10:40-11:00 GYERGYEK
Chairman	Cvitanović	Aizawa	Grossmann	Stefanovska	Robnik	11:00-11:30 – COFFEE –
15:00-16:00	MATSUSHITA	KURAMOTO	GALLAVOTTI	HAKEN	15:00-15:40 HASEGAWA	Chairman: Cvitanović
16:00-16:30	— COFFEE & TEA —				15:40-16:30 GRAVNER	11:30-11:50 HARAYAMA
16:30-17:30	McCLINTOCK	McCLINTOCK	TASS	STÖCKMANN	16:30-17:00 – COFFEE –	11:50-12:10 KRYLOV
17:30-18:30	NAKAMURA	NAKAMURA	GRAVNER	MATSUSHITA	17:00-17:50 MATSUSHITA	12:10-12:30 LAULICHT
	18:30-18:50 ZYCKOWSKI	18:30-19:30 DINNER	18:30-18:50 SALASNICH	18:30-19:30 JIRSA	17:50-18:40 STEFANOVSKA	12:30-12:50 RINKEVICH
	18:50-19:10 LOPAC	19:30-21:00 CONCERT	18:50-19:10 BERGLUND	19:30-21:00 DINNER	18:40-20:10 DINNER	12:50-13:10 RUFFING
	19:10-20:40 DINNER	21:00-23:00 RECEPTION	19:10-20:40 DINNER			13:10-13:30 VEBLE
						13:30-15:00 – LUNCH –

2nd Week	Mon 5 July	Tue 6 July	Wed 7 July	Thu 8 July	Fri 9 July	Sat 10 July
Chairman	Fujisaka	Eckhardt	Coullet	Mosekilde	Sonis	Stefanovska
09:00-10:00	GROSSMANN	CVITANOVIĆ	CVITANOVIĆ	COULLET	CVITANOVIĆ	MOSEKILDE
10:00-11:00	BOUNTIS	BISHOP	BISHOP	ECKHARDT	COULLET	PROSEN
11:00-11:30	— COFFEE & TEA —					
11:30-12:30	BOUNTIS	GEISEL	FUJISAKA	JIRSA	SONIS	SELIGMAN
12:30-13:30	AIZAWA	SELIGMAN	KRINSKY	SHUDO	RAPISARDA	ECKHARDT
13:30-15:00	— LUNCH —					
Chairman	Shudo	Mosekilde	Krinsky	Richter	Pichard	Kuvshinov
15:00-16:00	GLASS	GEISEL	GLASS	GLASS	MOSEKILDE	ROBNIK
16:00-16:30	— COFFEE & TEA —					
16:30-17:30	TASS	STARMER	SHUDO	PICHARD	RICHTER	SONIS
17:30-18:30	FUJISAKA	PICHARD	STARMER	PICHARD	STEFANOVSKA	MOSEKILDE
	18:30-18:50 ROMANOVSKY	18:30-19:15 CHIAN	18:30-19:15 PRELOVŠEK	18:30-19:15 KUVSHINOV	18:30-18:50 MARCQ	18:30-20:00 DINNER
	18:50-19:10 LEYVRAZ	19:15-20:45 DINNER	20:30-23:00 FESTIVE DINNER 'SLOW FOOD'	19:15-20:45 DINNER	18:50-20:20 DINNER	
	19:10-20:40 DINNER					

CAMTP

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The Supporting Scientific Institution of

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"Let's Face Chaos through Nonlinear Dynamics"
University of Maribor, Maribor, Slovenia
27 June - 11 July 1999
e-mail ss99 at uni-mb.si
http://www.uni-mb.si/~ucamtpssn/
The mailing and contact address otherwise as for CAMTP above

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THE SCIENTIFIC PROGRAMME - THE INVITED LECTURES

The following are extended abstracts of all invited lectures, each one 60 minutes long. The ordering is alphabetical according to the surname of the lecturer, whilst his or her lectures are ordered temporaly in accordance with appearance in the programme. We expect only a few minor updates which will be done in due course.

On Friday, 2 July 1999, the whole daily programme will be devoted to the **Japan-Slovenia Seminar on Chaos Science**, which is fully and separately displayed on the next page. **The Organizing Committee thanks the Japan Society for the Promotion of Science** for the full financial support of this Seminar, which is a part of the scientific programme of the School/Conference.

JAPAN-SLOVENIA SEMINAR ON CHAOS SCIENCE

(A Special Session at Maribor, Slovenia, 2 July 1999)

Main Talks (50 minutes for each)

1. **Yoji Aizawa** *Ergodic Properties of Non-stationary Chaos*
2. **Tomaž Prosen** *Quantum Poincaré Mapping*
3. **Yoshiki Kuramoto** *Turbulence with Multiscaling in Large Assemblies of Oscillators*
4. **Marko Robnik** *Topics in Quantum Chaos of Generic Systems*

Intermission for Lunch

5. **Hiroshi Hasegawa** *Information Theoretical Basis of Random Matrix Distributions*
6. **Janko Gravner** *Growth Phenomena in Cellular Automata*
7. **Mitsugu Matsushita** *Formation of Colony Patterns by a Bacterial Cell Population*
8. **Aneta Stefanovska** *Topics in Nonlinear Dynamics in the Human Cardiovascular System*

The Organizing Committee thanks the Japan Society for the Promotion of Science for the full financial support of this Seminar, which is a special programme embedded into the scientific programme of the School/Conference.

Long time tails in N-body Hamiltonian systems

Yoji Aizawa

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One of the most striking phenomena in chaotic dynamics is the appearance of the long time tails such as the $1/f$ fluctuations [1,2]. In the nearly integrable Hamiltonian systems, the long time tails are universally generated due to the stagnant motions near the invariant KAM tori;

$$H(I,\theta) = H_0(I) + \varepsilon H_1(I,\theta) \tag{1}$$

Here the parameter ε stands for the perturbation to the integrable Hamiltonian H_0. An important theorem (Nekhoroshev,1977) explained that the residence time T in the stagnant layer obeys [3],

$$T \simeq \frac{1}{\varepsilon}\exp[\varepsilon^{-b}]\ ,\ \ (\varepsilon \ll 1) \tag{2}$$

where b is a positive constant determined by the unperturbed Hamiltonian H_0 in Eq.(1).
The significant point in Eq.(2) is that the divergence of T does not obey the inverse power law but exhibits an essential singularity when ε goes to zero. In 1980's, the origin of such singularity was studied in terms of the scaling theory for the stagnant layers mentioned above, where the hierarchical structure of resonant tori (islands around island) plays an essential role to induce the long time tails in dynamical quantities. The stagnant layer theory (Aizawa , 1989) demonstrated that the distribution of the residence time, say P(T), obeys a universal law [2],

$$P(T) \simeq \frac{1}{T[\log T]^c}\ ,\ \ (T \gg 1) \tag{3}$$

where c is a positive constant larger than unity. Equation(3) has been confirmed by simulations (Aizawa *et al*, 1989). The essential singularity in Eq.(2) reflects the onset of $1/T$ divergence in Eq.(3). The point is that the distribution is not normalizable,i.e., a typical infinite measure.
Firstly, my lecture will be directed to the review of the stagnant layer theory and some numerical evidence in many body systems. Secondly, the onset of a new type of long time tails will be discussed carrying out with the clustering motions in N-body systems with short range attractive forces, where the distribution of the trapping time T obeys another universal law,

$$P(T) \simeq T^{-\beta-1}\exp[-aT^{-\beta}] \tag{4}$$

where a is a positive constant which depends on the size of cluster. The regularly varying part of Eq.(4) denotes the tail with $T^{-\beta-1}(T \gg 1)$, and the parameter β depends on the dimension of the cluster under consideration. The stability of the clustering motions will be explained based on the long time tail of Eq.(4).

References

[1] Aizawa Y 1989a *Prog.Theor.Phys.Suppl.* **99**149; Aizawa Y 1999 *Chaos, Soliton and Fractals* (in press); Tanaka K and Aizawa Y 1993 *Prog.Theor.Phys.* **90(3)**
[2] Aizawa Y 1989b *Prog.Theor.Phys.* **81(2)** 249; Aizawa Y *et al* 1989c *Prog.Theor.Phys.Suppl.* **98** 37; Aizawa Y 1995 *J.Korean Phys.Soc.* **28** 310; Aizawa Y 1991 *Dynamical Theory of $f^{-\nu}$ spectral chaos*, eds. Musha.T *et al*,pp483-487
[3] Nekhoroshev N N 1977 *Russ.Math.Surveys* **32** 1

Ergodic properties of non-stationary chaos

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Non-stationary chaos is a universal phenomenon in non-hyperbolic dynamical systems. Basic problems regarding the non-stationarity are discussed from ergodic-theoretical viewpoints. By use of a simple system, it is shown that "the law of large number" as well as "the law of small number" break down in the non-stationary regime. The non-stationarity in dynamical systems proposes a crucial problem underlying in the transitional region between chance and necessity(onceness and recurrence), where non-observable processes behind reality interplay with observable ones. The incompleteness of statistical ensembles is discussed from the Karamata's theory. Finally, the significance of the stationary/non-stationary interface is emphasized in relation to the universality of $1/f$ fluctuations.

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Universal aspects in level statistics of oval billiards

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We studied the energy level statistics for one parameter family of oval billiards whose classical phase space consists of some regular and some irregular components. As the parameter is varied, a transition from an integrable system to a strongly chaotic one occurs with successive bifurcations. We applied the Berry-Robnik formulae to the level-spacing distributions for a variety of shapes of quantum oval billiards and found some striking effects of bifurcations in the classical mechanical systems on the level-spacing distributions. The validity of the Berry-Robnik formula is also checked for those shapes of the oval billiard for which there exist two separated chaotic components in the phase space [1]. However, the Berry-Robnik formula is not available for the entire energy range, especially in the lower energy region the Brody-like behaviors have been often observed until now [2,3]. In the latter part of our talk, we will discuss that the transition from the Brody distribution to the Berry-Robnik formula universally occurs for all parameter regime of our model, and the detailed process of the transition near the bifurcation point will be reported.

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Complex dynamics of a "simple" mechanical system: the parametrically excited pendulum

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A planar pendulum is perhaps the simplest and most quoted example of a dynamical system, yet when driven its simplicity of description belies a range of complex dynamical motions. If driven vertically at the pivot, a pendulum which can freely move in the plane exhibits equilibrium states, periodic solutions, as well as chaotic motions, all of which can easily be seen in a mechanical experiment. Miles (1989) produced an excellent overview of the basic, qualitative dynamics but research interest is not yet exhausted with some recent results investigating the topological structure of phase space (Clifford and Bishop 1994) and the stability of the inverted state (Clifford and Bishop 1996).

The dynamics of the pendulum can be envisaged a particle moving in a cosinusoidal potential energy function. Small oscillations correspond to periodic motions within the well while rotations lead to escape from the local potential well between $(+\pi, -\pi)$. For small driving amplitudes the downward hanging state forms a stable equilibrium of the model system which becomes unstable in a series of zones. Numerically we can follow the stable solutions and follow bifurcations to plot the zones in parameter space in which the various solutions exist where typically the boundary between solutions is fractal and as a consequence the resulting dynamics is complicated even for this 'simple' system.

In addition, combining a mixture of methods to locate unstable orbits, and developing robust methods for control even in the presence of noise, means that now we are able to select a desired solution from a large selection of unstable motions onto which the system can be controlled (Bishop *et.al.* 1996) without globally changing the system parameters.

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The nonlinear dynamics of ship roll and capsize

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Ensuring that a ship can resist capsize when confronted with steep waves is an important consideration during design. Unfortunately, current ship design practices, due their reliance on ad hoc, empirical and static-type criteria of stability, are grossly inadequate to address the dynamic effects incurred on the ship due to wave action where the capsize process is both nonlinear and transient. The need for suitable dynamic capsize criteria is accentuated by the proximity of typical ocean wave frequencies to the natural roll frequency of many ships, which makes the possibility of beam-sea resonance a key feature of any worst-case scenario.

Due to the nonlinearity of the accompanying restoring force multiple steady-state roll responses can emerge, which can undergo further complicated qualitative changes of their character sometimes even becoming chaotic. More complex still can be the transient motions near to capsize limits. Recent applications of the global geometrical techniques of nonlinear dynamics and the associated problem of escape of a driven oscillator from a potential well, have provided new possibilities for advancing the state-of-the-art by achieving deeper understanding of the phenomena that precede capsize. An interesting characteristic of these phenomena is their remarkable robustness against gross changes in the forms of stiffness and damping functions, which can yield simple and useful design criteria against transient capsize.

We review here novel studies on ship capsize in beam seas, which have taken place at the Centre for Nonlinear Dynamics and its Applications of University College London. Large-amplitude ship rolling has been investigated both theoretically and experimentally, under steady-state as well as under transient conditions.

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Bifurcations, symmetry-breaking and pattern formation in nonlinear systems

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Lecture 1 : Bifurcations and Symmetry-Breaking in Nature

In these 3 lectures we shall review the basic concepts of stability analysis and bifurcation theory for nonlinear systems, with emphasis on the applications of these concepts to physics, chemistry and biology. We shall begin with a discussion of the meaning of **bifurcation** and its connection with **symmetry-breaking** in finite dimensional and **spatially extended** (infinite dimensional) nonlinear systems. As an example of the former, we examine chemical oscillations in closed, continuously stirred reactors, while for the latter, we use the Rayleigh-Benard experiment and reaction-diffusion systems from chemistry and biology.

Concentrating then on the topic of bifurcations in finite dimensional systems we will list the **normal forms** of the most commonly encountered types of bifurcations : saddle-node, transcritical, pitchfork and Hopf. Introducing the notions of **codimension** and **transversality** we will consider which of these types are **structurally stable**, when additional parameters and lower order terms are included in the dynamics. The importance of nonlinear stability analysis will be emphasized in connection with the study of **center manifolds** at bifurcation points.

Lecture 2 : Linear Stability of Spatially Extended Systems

In this lecture we review the main features of linear stability analysis of steady states of spatially distributed systems, occurring in problems of hydrodynamics and reaction-diffusion models of chemical and biological systems. More specifically, we consider the Rayleigh-Benard problem of a thin fluid layer heated from below and describe how one can estimate the critical Rayleigh parameter R_c, at which the symmetry of the spatially uniform state is broken by the formation of cylindrical rolls.

Next, we discuss the onset of diffusion-driven or **Turing instability** in reaction-diffusion systems and compare it with the onset of other types of instability (caused e.g. by a Hopf bifurcation) on the example of the Brusselator model. Symmetry-breaking and **pattern formation** will be examined on diffusive, 2-variable models of competing species. The dependence of the critical parameter value λ_c on a characteristic length scale in such systems will be compared to the absence of such a dependence in hydrodynamics.

Lecture 3 : Wave Propagation in Excitable Media

In the last lecture, we turn our attention to the study of reaction-diffusion equations describing **excitable media**. These are systems composed of active units, like cells, possessing a certain threshold of excitability, beyond which they can "fire" and transmit their action potential to neighboring cells. They are described by Fitzhugh-Nagumo (FHN) 2-variable partial differential equations in one or more space dimensions.

Depending on the parameters of the FHN equations, action potentials can travel in the form of localized waves, delivering their "message" along the medium. At other parameters (and places) however, on the medium, **stationary pulses** can either collapse or grow, forming a possible "obstacle", hindering the propagation of action potential waves. We derive analytical expressions for these stationary pulses and examine ways by which their effect on the travelling waves can be controlled. The connection of these phenomena with the occurrence of **cardiac arrhythmia** is discussed, when the excitable medium under consideration is the myocardium.

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Quantum localization and cantori in the stadium billiard

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We discuss the quantum behaviour of chaotic billiards which exhibit classically diffusive behaviour. More precisely we consider the stadium billiard and discuss how the interplay between quantum localization and the rich structure of the classical phase space influences the quantum dynamics. In particular we show how the presence of cantori in the classical phase space affects the structure of the eigenfunctions and the statistical properties of the eigenvalues. The analysis of this model leads to new insight in the understanding of quantum properties of classically chaotic systems.

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Quantum Poincaré recurrences

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We study quantum chaos in open dynamical systems and show that it is characterized by quantum fractal eigenstates located on the underlying classical strange repeller. The states with longest life times typically reveal a scar structure on the classical fractal set.We also show that quantum effects modify the decay rate of Poincaré recurrences $P(t)$ in classical chaotic systems with mixed hierarchical structure of phase space. It is shown that $P(t)$ has an algebraic decay with a universal power law $p = 1$, due to tunnelling and localization effects. Experimental evidence of such decay should be observable in mesoscopic systems and cold atoms.

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Nonlinear dynamics and chaos in space plasmas

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We discuss the theory and observation of nonlinear phonomena in solar-system plasmas. Plasma dynamics is governed by a variety of complex wave motions resulting from the collective electrodynamic interactions involving plasma field and plasma particles. Nonlinear wave-wave coupling in plasmas can occur due to the growth of parametric instabilities. Rocket and satellite observations have provided evidence of nonlinear wave interactions in the solar system, in particular, in relation to the generation and propagation of radio waves in solar corona, interplanetary medium and Earth's magnetosphere. These radio waves represent the electromagnetic signature of solar-terrestrial coupling and are useful for monitoring the space weather.

Nonlinear wave-wave coupling in plasmas can be modelled as a dynamical system of coupled oscillators involving one, two or higher-order wave triplets. For a three-wave dissipative system, the transition from order to chaos may evolve via period doubling or intermittence. For a two coupled three-wave Hamiltonian system, the transition may evolve via the route of quasiperiodicity. We show that chaos in a dissipative three-wave system can be controlled by applying a small sinusoidal wave to the system.

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Robust spatiotemporal phenomena in one space dimension (I)

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We review some of the robust instabilities which are observed in one dimensional dissipative systems. More particularly, this lecture is devoted to the instabilities which are induced by the continuous spatial coupling of local dynamical systems.

The first of these instabilities was discovered by A.Turing in 1952. It occurs already in two components reaction diffusion models when the diffusion of the two species are very different. The singular nature of this instability will be discussed. We will show in particular that close to a particular bifurcation (Bogdanov-Takens codimension two bifurcation) of the homogeneous reaction system, the instability can occur with almost equal diffusion constant.

The Benjamin-Feir-Kuramoto-Newell instability will then be considered. This "phase" instability of spatially coupled limit cycles will be described geometrically.

We eventually discuss the question of the synchronization of coupled limit cycles close to a homoclinic bifurcation. It will be shown that, generically an instability always occurs in the vicinity of a homoclinic bifurcation destroying the synchronization of the individual oscillators. The nature of the instability depends on the nature of the local dynamical system and the coupling considered. It can be either a Benjamin-Feir-Kuramoto-Newell instability or a new amplitude instability characterized by a period doubling which occurs at a finite wave-number. A complete study of this new instability will be given.

The aim of this lecture is to give a qualitative approach to the phenomena by using their normal form description. Interactive simulations will be used during the lecture in order to illustrate these phenomena. Examples from mechanics, chemistry, optics will be considered.

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Robust spatiotemporal phenomena in one space dimension (II)

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This second lecture is devoted to the description of localised structures which occur in one-dimensional dissipative systems.

In the first part of the talk we study the propagative localised structures known as excitable waves. Using the language of dynamical system, we will describe the collision of two pulses. Depending on the parameters the pulses either cross each other or coalesce. We show that this phenomenon is actually a global bifurcation for the partial differential equation. This bifurcation will be analyzed in the frame of the geometrical theory of dynamical systems. This example provides a unique case where the qualitative dynamical system theory have been used to understand a phenomenon which cannot be understood in the frame of a low dimensional dynamical system.

Static localised structures which can be observed in dissipative one-dimensional systems have received recently a great deal of attention. In the context of optics in particular they have been considered as good candidates for building storage devices. The question of the existence, the stability and the bifurcations of these static localized structures will be related to the bistability between a stable homogeneous state and a spatially periodic one. We will show that the existence of a persistent front between a homogeneous solution and a static periodic pattern is actually a consequence of the transversality of heteroclinic orbits connecting a fixed point and a limit cycle in reversible dynamical systems. This transversality property implies the existence and the stability of localised solutions. It also gives indications on the nature of the bifurcations of those objects.

Examples from Optics and Chemistry will be considered. Interactive numerical simulations will be presented during the talk in order to give an intuition of the phenomena described.

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Chaos and what to do about it: An overview

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That deterministic dynamics leads to chaos is no surprise to anyone who has tried pool, billiards or snooker - that is what the game is about - so we start our course about what is chaos and what to do about it by a game of pinball. This might seem a trifle trivial, but a pinball is to chaotic dynamics what a pendulum is to integrable systems: thinking clearly about what is "chaos" in a pinball will help us tackle more difficult problems, such as computing diffusion constants in deterministic gases, or computing the Helium spectrum.

We all have an intuitive feeling for what a pinball does as it bounces between the pinball machine disks, and only highschool level Euclidean geometry is needed to describe the trajectory. Turning this intuition into calculation will lead us, in clear physically motivated steps, to almost everything one needs to know about deterministic chaos: from unstable dynamical flows, Poincaré sections, Smale horseshoes, symbolic dynamics, pruning, discrete symmetries, periodic orbits, averaging over chaotic sets, evolution operators, dynamical zeta functions, Fredholm determinants, cycle expansions, quantum trace formulas and zeta functions, and to the semiclassical quantization of helium.

Reference

Read chapter 1 and appendix A of P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, `http://www.nbi.dk/ChaosBook/`.

Dynamics, qualitative

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Confronted with a potentially chaotic dynamical system, we analyze it through a sequence of three distinct stages; diagnose, count, measure. First, we determine the intrinsic *dimension* of the system - the minimum number of degrees of freedom necessary to capture its essential dynamics. If the system is very turbulent (its attractor is of high dimension) we are, at present, out of luck. We know only how to deal with the transitional regime between regular motions and weak turbulence. In this regime the chaotic dynamics is restricted to a space of low dimension, the number of relevant parameters is small, and we can proceed to the second step; we *count* and *classify* all possible topologically distinct trajectories of the system. If successful, we can proceed with the third step: investigate the *weights* of the different pieces of the system.

In this lecture qualitative dynamics of simple stretching and mixing flows is used to introduce Smale horseshoes and symbolic dynamics, and the topological dynamics is encoded by means of transition matrices/Markov graphs.

We learn how to count and describe itineraries. While computing the topological entropy from transition matrices/Markov graphs, we encounter our first zeta function.

By now we have covered for the first time the whole distance from diagnosing chaotic dynamics to computing zeta functions. Historically, these topological zeta functions were the inspiration for injecting statistical mechanics into computation of dynamical averages; Ruelle's zeta functions are a weighted generalization of the counting zeta functions.

Reference

Read chapters 2 and 3 of P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, `http://www.nbi.dk/ChaosBook/`.

Global dynamics

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This lecture is the core of the course: we discuss the necessity of studying the averages of observables in chaotic dynamics, and cast the formulas for averages in a multiplicative form that motivates the introduction of evolution operators.

In chaotic dynamics detailed prediction is impossible, as any finitely specified initial condition, no matter how precise, will fill out the entire accessible phase space (similarly finitely grained) in finite time. Hence for chaotic dynamics one does not attempt to follow individual trajectories to asymptotic times; what is possible (and sensible) is description of the geometry of the set of possible outcomes, and evaluation of the asymptotic time averages. Examples of such averages are transport coefficients for chaotic dynamical flows, such as the escape rate, mean drift and the diffusion rate; power spectra; and a host of mathematical constructs such as the generalized dimensions, Lyapunov exponents and the Kolmogorov entropy. We shall now set up the formalism for evaluating such averages within the framework of the periodic orbit theory. The key idea is to replace the expectation values of observables by the expectation values of generating functionals. This associates an evolution operator with a given observable, and leads to formulas for its dynamical averages.

If there is one idea that you should learn about dynamics, it happens in this lecture(s) and it is this: there is a fundamental local - global duality which says that (global) eigenstates are dual to the (local) periodic geodesics. For dynamics on the circle, this is called Fourier analysis; for dynamics on well-tiled manifolds this is called Selberg trace formulas and zeta functions; and for generic nonlinear dynamical systems the duality is embodied in trace formulas, zeta functions and spectral determinants that we will now introduce. These objects are to dynamics what partition functions are to statistical mechanics. The bold claim is that once you understand this, classical ergodicity, wave mechanics and stochastic mechanics are but special cases, to be worked out at your leisure.

The strategy is this: Global averages such as escape rates can be extracted from the eigenvalues of evolution operators. The eigenvalues are given by the zeros of appropriate determinants. One way to evaluate determinants is to expand them in terms of traces, log det = tr log. The traces are evaluated as integrals over Dirac delta functions, and in this way the spectra of evolution operators become related to periodic orbits.

The rest of the course is making sense out of this objects and learning how to apply them to evaluation of physically measurable properties of chaotic dynamical systems.

Reference
Read chapters 7, 8 and 9 of P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, `http://www.nbi.dk/ChaosBook/`.

Cycle expansions: Semiclassical quantum mechanics

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In last lecture we have derived a plethora of periodic orbit trace formulas, spectral determinants and zeta functions. Now we learn how to expand these as cycle expansions, series ordered by increasing topological cycle length, and evaluate average quantities like escape rates. These formulas are exact, and, when the winds are kind, highly convergent. The pleasant surprise is that the terms in such expansions fall off exponentially or even faster, so that a handful of shortest orbits suffices for rather accurate estimates of asymptotic averages.

The course now shifts gear to recent advances in the periodic orbit theory of chaotic, non-integrable systems, and the modern generalization of the De Broglie - Bohr quantization of hydrogen atom.

Instead of quantizing by suspending standing-wave configurations on stable Keplerian orbits, one suspends the standing-wave configurations on the infinity of unstable orbits. Such unstable periodic orbits are observed experimentally in the helium atom, the hydrogen in strong external fields, and other systems.

This is what could have been done with the old quantum mechanics if physicists of 1910's were as familiar with chaos as you by now are. The Gutzwiller trace formula together with the corresponding spectral determinant, the central results of the semiclassical periodic orbit theory, are derived.

The helium atom spectrum can then be computed via spectral determinants.

Reference
Read chapters 9, 15 and 17 of P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, `http://www.nbi.dk/ChaosBook/`.

Trace formulas for stochastic evolution operators

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Intuitively, the noise inherent in any realistic system washes out fine details and makes chaotic averages more robust. Quantum mechanical $\hbar$ resolution of phase space implies that in semiclassical approaches no orbits longer than the Heisenberg time need be taken into account. We explore these ideas in some detail by casting stochastic dynamics into path integral form and developing perturbative and nonperturbative methods for evaluating such integrals. In the weak noise case the standard perturbation theory is expansion in terms of Feynman diagrams. Now the surprise; we can compute the same corrections faster and to a higher order in perturbation theory by integrating over the neighborhood of a given saddlepoint exactly by means of a nonlinear change of field variables. The new perturbative expansion appears more compact than the standard Feynman diagram perturbation theory; whether it is better than traditional loop expansions for computing field-theoretic saddlepoint expansions remains to be seen, but for a simple system we study the result is a stochastic analog of the Gutzwiller trace formula with the $\hbar$ corrections so far computed to five orders higher than what has been attainable in the quantum-mechanical applications.

Resume

A motion on a strange attractor can be approximated by shadowing the orbit by a sequence of nearby periodic orbits of finite length. This notion is here made precise by approximating orbits by primitive cycles, and evaluating associated curvatures. A curvature measures the deviation of a longer cycle from its approximation by shorter cycles; the smoothness of the dynamical system implies exponential (or faster) fall-off for (almost) all curvatures. The technical prerequisite for implementing this shadowing is a good understanding of the symbolic dynamics of the classical dynamical system. The resulting cycle expansions offer an efficient method for evaluating classical and quantum periodic orbit sums; accurate estimates can be obtained by using as input the lengths and eigenvalues of a few prime cycles.

To keep exposition simple we have here illustrated the utility of cycles and their curvatures by a pinball game. Glancing back, we see that the formalism is very general, and should work for any average over any chaotic set which satisfies two conditions: 1. the weight associated with the observable under consideration is multiplicative along the trajectory; 2. the set is organized in such a way that the nearby points in the symbolic dynamics have nearby weights.

Reference

Read chapter 22 and the take-home problem set for the next millennium in P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, `http://www.nbi.dk/ChaosBook/`.

Semiclassical matrix elements and cross sections

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The semiclassical trace formula of Gutzwiller can be extended to include diagonal matrix elements. With this it becomes possible to describe cross sections, e.g. for excitations of molecules. In the lectures I will discuss how this can be used to describe both universal features which otherwise could be discussed within random matrix theory and non-universal features connected with classical periodic orbits and their bifurcations.

Lecture 1:
Transitions in molecules are described by Franck-Condon factors, which are matrix elements between eigenstates of an operator and projectors on the initial state. In order to calculate them within periodic orbit theory the Gutzwiller trace formula has to be extended. For operators which are smooth in the classical limit this is easy, but for singular ones like projection operators higher order corrections have to be taken into account. This extension is particularly easy to achieve in the time domain, both for the calculation of the smooth average part and for the contributions from periodic orbits.
Given this expression for the matrix element weighted density of states one can now turn to a calculation of the correlation function or, more generally, the two point form factor. One of the aims of the lecture will be to describe how the random matrix results of Alhassid and Fyodorov can be obtained.

Lecture 2:
One of the distinguishing features of semiclassical periodic orbit theory is its potential to describe non-universal properties, specific to the system in question and beyond the reach of random matrix theory. The most prominent effects arise in the neighborhood of bifurcations where the semiclassical amplitudes diverge. Catastrophe theory can be used to calculate uniformized amplitudes (as shown in a number of recent works). The lecture will focus on two aspects of our work:
i) In order to analyze the collective effects of many bifurcations we have studied the statistical behaviour in the standard map as a function of the control parameter k. This contributes to our understanding of the importance of bifurcations in long periodic orbits.
ii) The divergent classical weights have a pronounced effect also on wave functions. One measure of localization in wave functions is the inverse participation ratio. In particular, for saddle node bifurcations, we find prelocalized states below the bifurcation and oscillations in the inverse participation ratio above the bifurcation. The oscillations can be described by Airy functions, which can be motivated (but not fully justified) by the semiclassical matrix element theory discussed in lecture 1.

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Chaotic Huygens phenomenon and on-off intermittency

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Intermittency phenomena is quite ubiquitous in nonlinear dynamics. The most famous one is observed in fluid turbulence. Nowadays it is believed that the intermittency is one of key concepts for characterization or analyzing various phenomena in nonlinear dynamical systems. The most familiar one is the Pomeau-Manneville intermittency, which is observed when a periodic motion is either destroyed or undergoes the instability when an external control parameter is changed. On the other hand, when a particular chaotic motion undergoes an instability, an intermittency with statistical characteristics different from that of PM comes to appear. This is called the modulational intermittency or on-off intermittency. In particular, under certain condition two identical chaotic oscillators synchronize, (chaotic Huygens phenomenon). When the synchronization becomes unstable, we typically observe the on-off intermittency, (Fujisaka-Yamada 1985).

On-off intermittency is one of typical evolutions in nonlinear dynamics. In the present lecture, I will give several different mathematical models with both small and large number of degrees of freedom and also discuss physical system. Then I will review general criteria on the observability of on-off intermittency and give statistical laws. Introducing a stochastic model, I will show that these statistical laws can be derived. Furthermore the possibility of the construction of solvable models of on-off intermittency will be addressed.

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“Thermodynamics” approach to on-off intermittency

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Large deviation theory (LDT) in the probability theory is the straightforward extension of the central limit theorem which has played a very important role in statistical mechanics. Since LDT addresses the large fluctuations observed in nonlinear dynamics, typically for intermittency dynamics, it is natural to expect that LDT can also be a powerful theoretical framework to analyze chaotic dynamics.

In the lecture I first review LDT from the physicist point of view, and discuss how the statistical quantities characterizing LDT are determined both experimentally and theoretically. Then I will apply the LDT approach to examples in both chaotic and stochastic systems including the stochastic model of on-off intermittency. It is shown that the LDT analysis yields many different aspects of fluctuations which cannot be captured by the traditional statistical analysis based on two-time correction functions.

The short time dynamics of on-off intermittency shows a characteristic quite different from the long time dynamics which can be discussed within LDT. To analyze it, we introduce the concept of a self-similar dynamics for on-off fluctuations. Although the conventional LDT cannot be applied for such short time dynamics, I will show that LDT is “again” applicable.

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The chaotic hypothesis: a review and some applications

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The chaotic hypothesis is a reformulation of a principle stated by Ruelle in the early '70s (1973). It is a principle of very ambitious nature as it is a proposal for the extension of the ergodic hypothesis to nonequilibrium statistical mechanics systems. The entropy creation rate in a thermostated system by thermostats of mechanical nature, in which the energy cannot grow in spite of nonconservative external forces acting on the system, is identified with the divergence of the equations of motion. The fluctuations of the latter obey a universal relation: the fluctuation theorem. Further more heuristic developments of the chaotic hypothesis suggest introducing the notion of nonequilibrium ensembles: they offer the new idea that in systems out of equilibrium it can happen that the equations of motion themselves may become "parameters" of the nonequilibrium ensembles. And systems obeying different equations of motions may be equivalent in a sense analogous to the equivalence of the canonical and microcanonical ensembles, for instance. Similar ideas can be discussed in the theory of developed turbulence, which is the field in which the original ideas of Ruelle were developed.

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Quantum diffusion and fractal spectra

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While in the past the influence of classical chaos on quantum spectra was investigated intensely in the case of discrete spectra, the cases of uncountable and in particular fractal spectra have received attention only recently. We show that the latter are associated with specific dynamical properties:
(i) They generate unbounded quantum mechanical diffusion processes in contrast to the dynamical localization known from the kicked rotator and other systems, and
(ii) cause an algebraic decay of correlations as a function of time. We show that these properties are governed by generalized dimensions D_q of the spectrum and eigenfunctions.
In particular, the asymptotic decay of correlations is determined by the dimension D_2 of the spectral measure, while the diffusive spreading of wave packets is related to the ratio of the dimensions D_2 of the spectral measure and of the eigenfunctions, respectively.

Such fractal spectra show up for Bloch electrons in magnetic fields, in particular in artificial superlattices of quantum dots and antidots on semiconductor heterojunctions (electronic Sinai billiards). Quantum mechanical descriptions based on Harper's equation lead to a fractal spectrum known as Hofstadter's butterfly. This model has an integrable classical limit, however, and thus fails in these superlattices, where chaotic trajectories prevail. More realistic models that allow for chaos in the classical limit exhibit peculiar metal-insulator transitions and transitions between absolutely continuous and pure point spectra induced by classical chaos. We show that they can be understood in terms of avoided band crossings.

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Pattern formation in the developing visual cortex

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Nonlinear dynamics appears to play a role not only in chaos and faces but also in the information processing in brains and the development of brains. Visual and other external information is often represented in the cortex in the form of neural maps (e.g. retinal space or ocularity into cortical space), which exhibit characteristic patterns varying from species to species. These maps are known in great detail from neurobiological studies which employ high resolution techniques e.g. like optical imaging. Some of the best studied examples are the orientation preference maps, cortical neurons exhibiting different preferred orientations of visual stimuli.

It has long been hypothesized that the orientation selectivity of single neurons and the spatial pattern of orientation preferences in primary visual cortex arise due to activity-dependent self-organization of neuronal circuitry during early life. We demonstrate the relevance of nonlinear dynamics for the theoretical analysis of this self-organization process.

The talk will present theoretical work applied to recent experiments on strabismic and normal cats and will address the following questions:
Which mechanisms control the formation of neural maps?
How does experience influence the patterns?
Does the cortex undergo a symmetry breaking bifurcation during development?
Can we explain the different pattern layout in different species from common principles?

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Resetting and entraining biological oscillators

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Biological oscillators are ubiquitous and underly many important biological functions including the heartbeat, respiration, and reproduction. Each of these rhythms is generated and regulated by complex mechanisms and feedback. Despite the different mechanisms, from a mathematical perspective, they might all be associated with a stable limit cycle oscillations in an appropriate nonlinear equation. Based on this assumption, it is possible to make a number of different predictions concerning the effects of single and periodic stimuli delivered to the oscillator (Winfree, 1980; Glass and Mackey, 1988). The main results are: single stimuli might either advance or delay the onset of the next oscillation depending on the amplitude and phase of the stimulus; plots of the resetting functions as a function of the phase of the stimuli fall into distinct classes which can be classified based on continuity considerations; and if the limit cycle is strongly attracting, then based on the analysis of the effects of single stimuli on the oscillations it is possible to predict the effects of periodic stimuli (Guevara and Glass, 1982; Keener and Glass, 1984). These theoretical predictions have succeeded in helping to identify chaotic dynamics during periodic stimulation delivered to spontaneously beating aggregates of heart cells (Guevara, Glass, Shrier, 1981). I also discuss the limitations of the above theory. In particular, if relaxation to the limit cycle is not instantaneous, then the analysis becomes much more difficult (Glass and Sun, 1994). Also the effects of stimulation can change the properties of the limit cycle oscillator (Kunysz, Glass, Shrier, 1995). Although the geometric approach I describe in this and subsequent lectures is conceptually simple, there are many subtleties. It is hard to appreciate these without doing computations. Students who are particularly motivated by this topic, would benefit by reading Chapters 6 and 7 in Glass and Mackey (1988) prior to the summer school and attempting Problem 14(B) on p. 208. I will treat anyone who makes a decent effort to a free beer (or coffee)!

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Dynamics of cardiac arrhythmias

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Cardiac arrhythmias are abnormal cardiac rhythms in which there is abnormal impulse generation, abnormal conduction of excitation, or both. Since the heart is a nonlinear system, the classifications of cardiac arrhythmias (by cardiologists) should correspond to classifications of bifurcations in nonlinear systems (by mathematicians). This lecture introduces basic cardiac anatomy and electrophysiology (Goldberger and Goldberger, 1994; Glass, 1996). Two arrhythmias that have successfully been analyzed mathematically are heart block, in which some of the excitations in the upper chambers of the heart do not conduct successfully to the lower chambers of the heart (Shrier et al., 1987) and parasystole, in which there is a competition for control of the heart by two competing pacemakers (Glass, Goldberger, Bélair, 1984; Bub and Glass, 1995). These two arrhythmias are not particularly dangerous. However, there has also been mathematical analysis of more dangerous arrhythmias such as tachycardias, in which the heartbeat is abnormally fast. One conceptual model of tachycardias is that the excitation is associated with a pulse that circulates on a one-dimensional ring. Surprisingly, this simple model leads to the following predictions: if the circumference of the ring is too small, either reentry will be impossible or there will be quasiperiodic fluctuations of cycle time (Courtemanche, Glass, Keener, 1993); a single stimulus will either annihilate or reset the rhythm (Glass and Josephson, 1995); the effects of periodic stimulation can be computed based on the analysis of the resetting induced by a single stimulus (Nomura and Glass, 1996). I will describe recent efforts to test these predictions experimentally. Anyone wishing to prepare for this lecture should try to analyze the following not so hypothetical situation. An electrode is inserted into the right atrium of a patient's heart. The electrode is controlled by a computer under your guidance. The electrode delivers brief shocks that are capable of exciting the heart (this is the principle of an artificial pacemaker). What different rhythms could you elicit from the heart as you change the frequency and amplitude of the stimulation?

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Controlling cardiac arrhythmias

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A variety of different approaches have been devised to help control cardiac arrhythmias. Therapeutic approaches to controlling cardiac arrhythmias include drug therapy, electrical therapy such as pacemakers or implantable defibrillators, and ablation in which a portion of the heart is destroyed. To the best of my knowledge, none of the therapies currently used have been developed based on a mathematical analysis of nonlinear dynamics of cardiac arrhythmias. This lecture further analyzes dynamics of arrhythmias and demonstrates approaches to controlling arrhythmias in model systems using mathematical analyses. The first part of the talk describes repetitive paroxysmal tachycardias, in which there is an incessant starting and stopping of arrhythmia (Parkinson and Papp, 1967), and experimental models displaying similar dynamics. The experimental models are generated by: stimulation of spontaneously oscillating cardiac cells with a fixed delay after an activation (Kunysz, Shrier, Glass, 1997); stimulation of the atria of rabbit hearts at a fixed delay after the activation of the ventricles (Sun et al., 1995); and culturing heart cells in a monolayer (Bub, Glass, Shrier, 1998). In all three examples, the interactions between pacemakers, conduction and "fatigue" leads to bursting rhythms. I then describe mathematical methods being developed to control cardiac arrhythmias. An early example adopted methods of "chaos" control to an experimental model system (Garfinkel et al., 1992) but see also (Glass and Zeng, 1994). More recent work tunes the adjustable delay in a reentrant circuit to control an alternating rhythm (Hall et al., 1997). Finally, analysis of wave propagation might help cardiologists successfully locate areas to target for ablation (Hall and Glass, 1999). In view of the nonlinear nature of the heart, and the complex dynamics of cardiac arrhythmias it seems inevitable that theoretical analyses based on nonlinear dynamics will lead to successful new therapies, but we are still awaiting the first demonstration.

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Growth phenomena in cellular automata

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A cellular automaton (CA) is a spatially distributed dynamical system which evolves via local, homogeneous, parallel updating, which may be deterministic or random. Our focus will be on self–organization: a tendency toward large–scale, coherent structure from disordered initial states. The examples will mostly feature irreversible dynamics outside the scope of traditional statistical mechanics, which we loosely call *growth models*. We will mainly review development of mathematical theory developed for several classes of CA, but will also present insights gathered from simulations and open problems.

In the first talk, we will start with the basic definitions and a menagerie of illustrative examples. We will present an analysis of *additive* CA, discussing particularly replication and generation of fractals. Another basic example is *oriented percolation*, a prototype for a phase transition and the fundamental building block in rescaling arguments. A model for an excitable medium will provide an example of locally periodic CA. We will then briefly touch on measures of complexity of CA rules and on the most famous of all CA, the *Game of Life*.

The second talk will go into more depth on the subject of nucleation and growth properties of CA. The first subject will be shape theory for monotone growth, reviewing Wulff construction for deterministic and random CA. By contrast, non-monotone cases can only be rigorously handled when a recursive analysis is available. We will present some examples of recursive solidification which lead to fractal growth and are extremely sensitive to random perturbations. Next, we will mention some growth models based on random walks that generate isotropic shapes, and other, much less tractable, which yield random fractals. Finally, we will review nucleation theory in both supercritical and critical monotone CA. Specifically, we will review power laws, density decays and last holes problems, emergent shapes, and effects of polluted environments. Time permitting, some percolation effects in excitable media will be discussed.

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The onset of turbulence despite stable laminarity

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This lecture is devoted to the recent explanation of the mechanism which determines the onset of turbulence in laminar shear flows. This is different from the routes to turbulence characterized by a sequence of instabilities or bifurcations cf. Schuster (1988) or Bergé et al (1986), showing quasi-periodicity, period-doubling, or temporal intermittency. In fluid flow these routes can be observed in the onset range of heat convection in Rayleigh-Bénard systems (fluid heated from below) or in Taylor-Couette systems (concentric cylinders, the inner one rotating), see e.g. Koschmieder (1993). Laminar shear flows between plane boundaries or through pipes stay linearly stable, instead, but nevertheless become turbulent, though at larger Reynolds numbers $\mathrm{Re} = O(1\,000)$. The mechanism is not an eigenvalue instability but a subtle interplay between **nonnormal** bunching of the eigenstates and their **nonlinear** interaction, see Boberg and Brosa (1988), Gebhardt and Grossmann (1993, 1994), Trefethen et al (1993), Grossmann (1996, 1999). The most characteristic features are the double threshold and a large phase space Hausdorff dimension at onset. The threshold is fractal under a given type of perturbations and in phase space a chaotic repeller (arising from unstable stationary states) generates the complex, chaotic dynamics of turbulent flow, Eckhardt et al (1998).

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Turbulent heat transport: global performance–global scaling

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A temperature difference Δ between the bottom and the top plates enclosing a gas or a liquid at rest imply molecular heat transport. The heat current J_κ is determined by, besides Δ, the thermal conductivity κ and the height L of the fluid layer, J_κ being $\kappa\Delta/L$. If the nondimensionalized temperature difference $Ra = \beta g L^3 \Delta/\kappa\nu$, the Rayleigh number of this Rayleigh-Bénard system, becomes larger, the molecular heat transport is supported by convective transport. Here ν is the kinematic viscosity, g the gravitational acceleration, and β the thermal volume expansion coefficient. At medium Ra rolls of fluid flow with increasing velocity U are developed. For large Ra the flow in the bulk of the rolls becomes turbulent. The heat current J then can be carried by the turbulent convection. This is very effective, so J increases with Ra. The heat current, called the Nusselt number Nu when measured in multiples of the molecular current J_κ, i.e., $Nu = J/J_\kappa$, depends algebraically on Ra. It also scales algebraically with the Prandtl number Pr, which characterizes the importance of the kinematic viscosity relative to the thermal conductivity, i.e., $Pr = \nu/\kappa$. The scaling exponents of the heat current Nu as well as of the roll velocity, measured by the roll Reynolds number $Re = UL/\nu$ as functions of Ra and Pr, the control parameters, are derived and discussed in this lecture.

Since heat current Nu and roll velocity Re are global responses of the fluid to the driving temperature difference only global arguments turn out to determine the scaling behaviors with Ra and with Pr. It is suggested that the only relevant physical quantities are the kinematic and the thermal dissipation rates. These can be determined in the bulk as well as in the boundary layers. Utilizing exact relations between the dissipation rates and the currents Nu and Re allows to derive the various scaling laws valid in the different regions of the parameter space $Ra - Pr$. We also give the crossover corrections between regions, which jeopardize pure scaling. Comparison with several previous and recent experiments is very encouraging for this unifying theory of scaling in thermal convection. The reference for this unifying theory is Grossmann and Lohse (1998).

Introductory references to thermal convection are Koschmieder (1993), Castaing et al. (1989); for information about experiments see Wu and Libchaber (1991), Chavanne et al. (1997), Cioni et al. (1997); theoretical overviews are given by Shraiman and Siggia (1990), Siggia (1994); an extended list of references can be found in the cited work by Grossmann and Lohse (1998).

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Energy dissipation in turbulent flow: rigorous and less rigorous insights

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To maintain fluid flowing one needs stirring by some external force. There is no turbulence without ongoing energy input, which compensates the permanent energy loss by viscous damping. Therefore the most relevant quantity to characterize turbulence is the mean energy flow rate per mass, $\varepsilon = \nu\langle(\partial_j u_i)(\partial_j u_i)\rangle$, and its dependence on the Reynolds number $Re = UL/\nu$. From dimensional argument the energy flow ε must be $\propto U^3L^{-1}$. The dimensionless prefactor, denoted as the dissipation coefficient $c_\varepsilon(Re)$, is Re^{-1} for laminar flow. Its behavior for turbulent flow and in particular asymptotically for large Re is of high interest.
In this lecture, after introductory estimates of the dissipation coefficient $c_\varepsilon(Re)$, a variational principle is derived, leading to a rigorous upper bound of the dissipation rate's coefficient (Nicodemus, Grossmann and Holthaus 1997). The variational principle is formulated in terms of a static auxiliary field which is optimized to carry the dissipation of the real turbulent flow field. It is one of the few exact results on the nonlinear Navier-Stokes dynamics. This principle is a generalization of Doering and Constantin's work (1992, 1994). Hopf (1941) was the first to study bounds on energy dissipation with this idea. Another line of approach is the Busse (1970) and Howard (1972) variational method; in that the flow field is decomposed into its mean and the fluctuating deviations. Using our results it could be shown (Kerswell 1997) that both methods lead to the same bounds on energy dissipation. Asymptotically $c_\varepsilon(Re \to \infty) = 0.01087(1)$. From the volume mean one has to distinguish the bulk dissipation, Sreenivasan (1984), Lohse (1994), Grossmann(1995). Recently we identified for the first time a mechanism that leads to a *decrease* of c_ε with $Re \to \infty$, Nicodemus, Grossmann and Holthaus (1999). The lecture finishes with a comparison of the kinetic dissipation with the thermal dissipation in heat convection (Nicodemus, Grossmann and Holthaus, unpublished). More details about the mathematical and the numerical methods for d-dimensional bounds, also for the Re-asymptotics, can be found in Nicodemus, Grossmann and Holthaus (1997, 1998, 1999).

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Brain dynamics - Part I

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The human brain is the most mysterious organ we know of. In it perception, speech control, thinking, feelings, emotions, and many other things exist. Seen from the point of view of a physicist, it is the most complex system in the world. It consists of about a hundred billion of neurons that are interconnected in a highly complicated fashion.

In my lectures I will address two main topics:

1. The explicit example of a neural net (the lighthouse model)

2. How do macroscopic features, such as perception or motor-control, arise from microscopic elements.

The vehicle I shall use to study the second question is synergetics. In my first lecture I wish to remind the audience of some physical methods to study brain functions, such as EEG, MEG, PET and MRI. In order to understand the macroscopic activity patterns of the brain, basic concepts of synergetics will be used and a reminder will be given of basic concepts, such as control parameters, instability, order parameters, slaving principle, critical fluctuations, and symmetry breaking in open systems far from thermal equilibrium. These concepts are exemplified by phenomena of visual perception, such as bistability, hysteresis, and oscillations. The synergetic computer will serve as a model for pattern recognition and links with Gestalt theory will be established.

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Brain dynamics - Part II: The lighthouse model of a neural net

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I present some salient features of neurons and their connections, and I discuss the role of soma, axon, dendrites and synapses. I briefly sketch previous models, such as that of McCulloch and Pitts, Wilson and Cowan, Jirsa and Haken, which neglect phase relations, and models that deal with phase couplings (Kuramoto, Tass), with sinusoidal couplings, and integrate and fire models (Strogatz et al., Geisel et al.).

I then present the basic concepts of the lighthouse model of a neural net as developed by the present author, where attention is focused on phase-locking by means of pulse trains. Basic variables are dendritic currents that are generated by axonal pulses and phases describing the timing of the axonal pulses. The latter are generated by dendritic currents, processed by the soma of a neuron. While below a threshold, the neuron is, as usually assumed, quiescent, according to my model above that threshold its firing rate increases linearly with the inputs.

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Brain dynamics - Part III: Steady states, mutual interaction and associative memory of the lighthouse model

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Though the model is highly nonlinear, because of the occurrence of pulses, it can be treated exactly. Especially the case of small damping of the dendritic currents leads to very elegant results. The elimination of the variables describing the dendritic currents leads to phase-equations. By means of the example of two neurons, I demonstrate how the steady state can be calculated. This leads to a mutual activation of neurons so that neurons become active even if they have been below threshold because of a too weak sensory input.

I then treat the case of an arbitrary number of neurons that are arbitrarily coupled with each other and are subject to arbitrary sensory inputs. The activation of patterns shows up in specific pulse rates, and the action of the whole net can be interpreted as that of a associative memory. I distinguish between weak and strong associative memory and explain the application of this concept to Kaniza figures.

Brain dynamics - Part IV: Phase-locking of two and many neurons, impact of noise and delayed interactions

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In this part it is assumed that all sensory inputs are equal, but the coupling between the neurons may still be arbitrary. I start from a phase-locked state and derive equations for deviations from that state. The deviations may have different causes, namely resetting of the individual phases externally, or internal noise. In this way I demonstrate the stability of the phase-locked state against these perturbations. On the other hand, if a pulse is lacking due to failure of a vesicle, or an additional pulse is emitted due to spontaneous opening of a vesicle, a phase lag is caused. The results can be extended to an arbitrary net and delayed interactions, provided the delay-time is the same for all neurons.

Brain dynamics - Part V: Phase averaged equations

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I show how the lighthouse model and phase averaged equations, such as the Nunez and Jirsa-Haken equations, are related to each other. I will discuss a number of open problems.

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Random matrix theories and the problem of Anderson localization

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The problem "Anderson localization" is one of major subjects in Condensed Matter Physics, and in recent years it has been explored intensively from a viewpoint of *quantum level statistics*[1]. Historically, this emphasis has occurred from a technical development in the decade of 80's about Gaussian mean over matrix ensembles by the method of *supersymmetry* that implemented studies of correlation functions[2]. An interest in the subject is the intermediate nature of the statistics between the standard Wigner-Dyson (in the metallic phase) and the Poisson (in the localized phase): it would be a level-statistical evidence of the phase transition between these two phases. Among numerous works on this subject in the past decade, the results provided by Al'tshuler and coworkers should not be overlooked[4-6][8-11]. In the present lecture, therefore, we focus our attention on the most interesting result due to them, namely the *compressibility of level gas at the mobility edge.*

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Random Matrix Theories and the Problem of Anderson Localization II

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There exists a framework of computing *two-point correlation function* between a pair of levels of a random eigenvalue sequence that is supposed to arise from a perturbed hermitian $H = H_0 + \lambda V$, where H_0 belongs to a set of fully uncorrelated Hermitians (a Poisson ensemble) and V to a set of Hermitians in the Wigner-Dyson class (one of the three standard RMT ensembles)[12-15]: it has been regarded as adapted for level statistics with Anderson transition[15]. This correlation function $R(r)$ has two remarkable characteristics. (1) its long-range behavior ($r > r_c$) reflects an attractiveness between the pair of levels ($R(r) > 1$ [12,13]), and (2) $\int_{-\infty}^{\infty}(R(r)-1)dr = 0$ [14]. We argue that (1) may be regarded as the characteristic of level statistics in a metallic phase[16] that could undergo *Anderson transition*, whereas (2) contradicts the existence of an intermediate compressibility[10,11]. Here, we present a new framework adapted for Anderson transition based on equilibrium statistical mechanics of level gas[18-22] (a Hamiltonian level dynamics for $H = H_0 + \lambda V$) satisfying (1) and, furthermore, capable for the intermediate compressibility.

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From small to large scale activity of spatiotemporal neural patterns

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The neuron is thought to be the central microscopic information-processing unit in the brain. We review its morphology (see Braitenberg and Schüz (1991)) and dynamics and discuss mathematical models of neuronal temporal activity (Hodgkin-Huxley, integrate-and-fire and spike-response model; see e.g. Kistler et al (1997)). Interconnectivity among neurons requires to traverse to a description of neural networks. In many areas of the brain neurons are organized in populations of units with similar properties which allows to describe the mean activity of a neuronal population rather than the temporal behavior of individual neurons. Spatiotemporal neural activity in terms of neural ensembles has been described by Wilson and Cowan (1972,1973) and Nunez (1974). Later approaches by Jirsa and Haken (1996) and others unified and generalized these models to a neural field theory on large spatiotemporal scales accessible to electroencephalography (EEG) and magnetoencephalography (MEG). The introduction of the notion of "Functional units" by Jirsa and Haken (1996) provides a means to correlate behavioral patterns to EEG and MEG patterns (for the specific realization of Bimanual Coordination see Jirsa et al (1998), for reviews of large scale brain and behavioral dynamics see Kelso (1995), Nunez (1995), Haken (1996)).

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Spatiotemporal pattern formation in systems with heterogeneous connection topology

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We briefly review spatiotemporal pattern formation via self-organization in open non-equilibrium systems of different nature, e.g. in physical systems as in hydrodynamics or the laser, in chemical systems as in the Belousov Zhabotinsky reaction (see Haken 1975, Cross and Hohenberg 1993 for reviews). In these systems the underlying connection topology is typically homogeneous, i.e. translationally invariant, which provides the basis for modelling their spatiotemporal dynamics by nonlinear partial differential equations as in the Ginzburg-Landau theory (see Haken (1983)). This approach has been applied successfully to a class of biological systems for which the connection topology may be assumed to be regular such as coat markings of mammals or fish (see Murray (1993) for review). However, many biological systems such as the human cortex show homogeneous connectivity with additional strongly heterogeneous projections from one area to another (see Braitenberg and Schüz 1991, Felleman and Van Essen 1991), but still perform coherent spatiotemporal pattern formation (see Kelso (1995)). Such heterogeneous connectivity may undergo temporal changes via mechanisms of asymmetric Hebbian learning or synaptic plasticity (see Markram et al (1997)). Here we show mathematically and numerically how the global spatiotemporal dynamics of a heterogeneously connected system can be controlled via local changes in the connection topology and thus be guided through a series of non-equilibrium phase transitions. We provide a mathematical description of the system in terms of a nonlinear integral equation which is retarded by delays via propagation of signals along connecting pathways (see Jirsa and Haken (1996)). Our formulation allows to use the connectivity as a topological control parameter which could not be achieved by means of partial differential equations in a straight forward way.

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Control of chaos in the heart: New approaches, problems

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We review the main ideas and results in the mathematical theory of chaotic wave propagation in cardiac muscle. Mechanisms of chaos and approaches to control the chaos are understood for ordinary differential equations (d.e.), but for partial d.e, only isolated results are published. In cardiac muscle, the chaotic regime known as fibrillation is the main reason of mortality in industrial world. A very strong electrical discharge (defibrillation) eliminates the chaos. Mechanisms of defibrillation are poorly understood. Important is to study alternative approaches to eliminate the chaos.

For cardiac muscle (and excitable media in general), the initial stage in transition to chaos is formation of rotating vortices. Elimination of vortices at this stage effectively prevents chaos development. We study two different kind of vortices, free vortices, and vortices pinned to a defect. We found that a weak shock, with an amplitude an order of magnitude less than the defibrillating shock, may unpin the vortices rotating around the defects (obstacles). The unpinning results from a depolarization of the tissue near the obstacle, induced by an external electric field within a distance of order electrical space constant in cardiac muscle $\sim 1mm$ (Weidmann, 1970). Unpinning was observed both in the Fitz Hugh model of excitable tissue, and in a specific Beeler-Reuter model of cardiac tissue. This theoretical observation suggests that anatomical reentries can be transformed into functional reentries, an effect that can be tested in experiments with cardiac muscle.

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Phase dynamics — Its conceptual universality and different specific forms —

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Our understanding of the dynamics of many nonlinear dissipative systems relies crucially upon the possibility that under various conditions complicated nonlinear evolution equations could be reduced to far simpler forms. This lecture is devoted to clarifying the theoretical grounds for such dynamical reduction with particular emphasis on the phase dynamics.

We first argue that, explicit or implicit, virtually all theories of reduction involve commonly one universal feature, namely, a perturbative construction of an attracting invariant manifold together with a definition of an appropriate coordinate system on this manifold in such a way that the evolution law there may look as simple as possible. This assertion is then demonstrated for the two of the most fundamental reduction methods, i.e., the center-manifold reduction and the phase reduction. Here, the trivial invariant manifold to start with is given by the critical eigenspace in the first case, and the neutral space spanned by the Goldstone mode in the second case. We also discuss how the reduction idea developed for ODEs, where such perturbative methods presuppose low-dimensionality of the invariant manifold, can immediately be carried over to PDEs with large spatial extension for which the dimension of the invariant manifold should practically be infinite. From such a viewpoint, the Chapman-Enskog reduction, a monumental reduction theory of nonequilibrium statistical mechanics, turns out almost identical in structure with the phase dynamics.

As a separate topic of reduction, a different form of phase reduction particularly adapted to coupled oscillators systems will be presented, and we demonstrate its usefulness through the derivation of phase equations in various situations of practical interest.

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Collective dynamics of coupled oscillators

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Numerous examples of interacting rhythmic processes are found in various fields of science including physics, chemistry and biology, and their proper mathematical description can be provided by dynamical systems of coupled limit-cycle oscillators. In this lecture, we will discuss the dynamics of large populations of oscillators with mean-field coupling, and present an overview of various theoretical approaches developed up to this day since the pioneering work of Winfree (1967). We describe in some detail a prototypal phase oscillator model with distributed native frequencies (Kuramoto 1975) which admits an exact solution for the phase transition due to mutual synchronization. This model and its generalizations have been analyzed more deeply, and also found some important applications. Some of the important works among them will be surveyed. They include Strogatz et al's theory on the stability of the collective states, Crawford's center-manifold reduction of the model with external noise, Daido's generalization of the coupling function in terms of the order function, and Wiesenfeld et al's proof of the equivalence of the original model with circuit equations for certain series arrays of Josephson junctions. We also describe the possibility of nontrivial forms of collective dynamics, e.g., clustering, bursting and collective chaos, some of which may have implication in biological information processing.

Aside from collective dynamics, there are many applications of the phase reduction method itself, e.g, to the construction of the phase-coupling function for various realistic models, which will also be touched upon.

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Turbulence with multiscaling in large assemblies of oscillators

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We discuss a universal class of turbulent states discovered recently in a wide variety of nonequilibrium systems including nonlocally coupled chaotic and non-chaotic oscillators, arrays of uncoupled oscillators with long-wave random forcing, and certain types of 3-component reaction-diffusion systems. The turbulence here is characterized by multiscaling properties of the structure functions, which is reminiscent of fully developed Navier-Stokes turbulence. Specifically, various moments of the amplitude increments between two points exhibit a power-law dependence on the mutual distance. As a consequence, the amplitude profile is fractal with multi-affine properties. Remarkably, the fractal dimension changes continuously with the system parameter, and there is a critical condition beyond which the amplitude profile completely loses the spatial continuity, disintegrating into infinitesimal pieces. We develop a theory based on a multiplicative stochastic process obeyed by the amplitude increments, and discuss its approximate stationary solutions from which all multiscaling properties as well as the continuity-discontinuity transition can be explained.

It is also argued that the field defined by a coarse-grained spatial derivative of the amplitude is characterized by multifractals, which is again reminiscent of the multifractal nature of the energy dissipation field in fully developed turbulence. Such properties are, however, by no means direct consequences of the multiscaling of the original field, and some attempts will be made toward their understanding.

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Chaos and order in high energy physics and Yang-Mills theories

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Chaos and fractal signatures in high-energy physics processes are discussed. In particular intermittency phenomenon, fractal jets and clusters are analysed.

All four fundamental physical interactions have Yang-Mills gauge nature and all of them have chaotic classical solutions in specific cases. The talk reviews some situations when YM field theories have chaos–order transition (at corresponding controlling parameters): non-Abelian nature, monopoles, sphalerons, other topological solutions, Higgs fields, etc are discussed as sources of chaos and order. For example, it is known that Higgs fields—a very important element of gauge theories giving masses to the particles—in its vacuum state can suppress chaos. Here it is shown that non-vacuum states of Higgs fields can both suppress and induce chaos.

Numerical and analytical methods of study of chaos properties of YM theories are reviewed.

The problem of the description of quantum chaos in second quantized field HEP theories is mentioned and its physical role questioned.

Some unsolved problems of modern HEP models such as confinement of color quarks and gluons and their possible connections with presence of chaos as immanent property of YM-equations are discussed.

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Pattern formation in bacterial colonies

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We have studied the growth mechanism and morphological change in colony formation of bacteria from the viewpoint of physics of pattern formation. Even very small number of bacterial cells, once they are inoculated on the surface of an appropriate medium such as semi-solid and nutrient-rich agar plate and incubated for a while, repeat the growth and cell division many times. Eventually the cell number of the progeny bacteria becomes huge, and they swarm on the medium to form a visible colony. The colony differs in size, form, and color according to bacterial species. It also changes its form sensitively with the variation of environmental conditions. This implies that although usual bacteria such as *Escherichia coli* are regarded as single cell organisms, they never make their colony independently and randomly but somehow collaborate multicellularly. We have thus tried to extract some simple and universal behaviors in growth from such complex bacterial systems.

Here we varied only two parameters to investigate the colony growth; concentrations of nutrient C_n and agar C_a in a thin agar plate as the incubation medium. Other parameters specifying experimental conditions such as temperature were kept constant. We mainly used a typical bacterial species *Bacillus subtilis*. Otherwise the experimental procedures are standard. It was found that colonies show characteristic patterns in the specific regions of values of C_n and C_a in the morphological diagram and the patterns change drastically from one region to another. They were classified into five types; fractal DLA-like, compact Eden-like, concentric ring-like, simple disk-like and densely branched DBM-like. We have experimentally elaborated characteristic properties for each of these colony patterns.

We have also tried to construct a phenomenological but unified model which produces characteristic colony patterns observed in our experiments. It is a reaction-diffusion type model for the population density of bacterial cells and the concentration of nutrient. The essential assumption is that there exist two types of bacterial cells; *active* cells that move actively, grow and perform cell division, and *inactive* ones that do nothing at all. We have varied values of the initial nutrient concentration and the ratio of diffusion rate of bacterial cells to that of nutrients. They correspond to the specified nutrient concentration in agar plates and the substrate softness in our experiments, respectively. Our model is found to be able to globally reproduce all the colony patterns described above, and is phenomenologically quite satisfactory.

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Spatiotemporal patterns produced by bacteria

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The growth of bacterial colonies is a "Treasure Island" of pattern formation. Apparently, the pattern-forming members (bacterial cells) are strongly interacting, self-reproducible and self-driven "particles", very different from conventional physical systems. Some colony patterns grown on the surface of agar plates can be understood in terms of well-defined models of pattern formation such as DLA, but not in all cases.

Here we used a bacterial species *Proteus mirabilis.* This species has been well-known for more than a century to form macroscopically almost perfect concentric-ring like colonies with approximately equal spacings on the surface of semi-solid substrate such as an agar plate. However, when this *Proteus mirabilis* was inoculated and incubated on the surface of a very soft agar medium with rich nutrient, we observed that entirely different, spatiotemporal patterns emerge inside a colony after the colony spreads over the whole surface of the agar medium. These spatiotemporal patterns include target and rotating spiral patterns. In this sense the emerging patterns are very similar to those seen in dissipative systems such as chemical oscillators and liquid-crystalline electrohydrodynamic convective systems. They also seem to exhibit characteristic features of spatiotemporal chaos. Microscopic observations revealed that the two-dimensional collective motion of bacterial cells seems to be responsible for the formation of the observed spatiotemporal patterns.

Emphasis is also put on the construction of morphological diagram for obtaining some clue to elucidate the growth mechanism and sometimes finding new morphologies as well. Examples are shown for bacterial colony formation and crystal growth in gelatin gels.

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Formation of colony patterns by a bacterial cell population

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Bacterial species *Bacillus subtilis* is known to exhibit various colony patterns, such as diffusion-limited aggregation (DLA)-like, compact Eden-like, dense-branching-morphology (DBM)-like, concentric-ring-like and disk-like, depending on the substrate softness and nutrient concentration. We have established the morphological diagram of colony patterns, and examined and characterized both macroscopically and microscopically how they grow. For instance, we have found that there seem to be two kinds of bacterial cells; active and inactive cells, the former of which drive colony interfaces outward. The active cells are particularly distinguished from the inactive ones at the tips of growing branches of a DBM-like colony as the characteristic fingernail structure. We have also found that the concentric-ring-like colony is formed as a consequence of alternate repetition of advancing and resting of the growing interface which consists of active cells.

Based on our observations, we have tried to construct a phenomenological but unified model which produces characteristic colony patterns. It is a reaction-diffusion type model for the population density of bacterial cells and the concentration of nutrient. The essential assumption is that there exist two types of bacterial cells; *active* cells that move actively, grow and perform cell division, and *inactive* ones that do nothing at all. We have varied values of the initial nutrient concentration and the ratio of diffusion rate of bacterial cells to that of nutrients. They correspond to the specified nutrient concentration in agar plates and the substrate softness in our experiments, respectively. Our model is found to be able to globally reproduce all the colony patterns seen in the experimentally obtained morphological diagram, and is phenomenologically quite satisfactory.

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Stochastic resonance

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Stochastic resonance (SR) is commonly said to occur when a weak periodic signal in a nonlinear system is enhanced by an increase of the ambient noise intensity; a stronger definition requires that the signal/noise *ratio* (SNR) should also increase. SR has been much in the news recently, partly on account of its wide occurrence in many areas of science. After being introduced as a possible explanation of the earth's ice-age cycle (Benzi et al 1981, Nicolis 1982), SR has subsequently been observed or invoked in contexts that include lasers (e.g. McNamara et al 1988), passive optical systems, tunnel diodes, a Brownian particle in an optical trap, a magnetoelastic ribbon, crayfish and rat mechanoreceptors, a bistable SQUID (superconducting quantum interference device), arrays of SR elements, ion channels, magnetic systems, the El Nino phenomenon, social ills, various types of bistable electronic model including coexisting periodic attractors, subcritical bifurcations, systems with thresholds, transient dynamics, a quantum 2-level system, an array of coupled bistable systems, a system driven by quasimonochromatic (harmonic) noise, excitable neurons, and chemical systems. There have been several general scientific articles (e.g. Moss and Wiesenfeld 1995, Bulsara and Gammaitoni 1996) and two largely complementary topical reviews (Dykman et al 1995, Gammaitoni et al 1998).

We present a succinct introductory review of SR discussing its physical nature, the insights that can be obtained by treating it as a linear response phenomenon (Dykman et al, 1990), and the way in which electronic experiments (Luchinsky et al 1998) have tested the approximate analytic theory and extended the known range of occurrence of SR far beyond the systems characterised by static bistable potentials in which it was originally discovered.

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Large fluctuations and optimal paths

Most of the interesting and important events that occur in fluctuating nonlinear dynamical systems, e.g. the transitions between attractors giving rise to stochastic resonance or transport in Brownian ratchets, are attributable to *large rare fluctuations.* Their analysis requires both evaluation of the probability of the system occupying a state remote from the attractor and identification of the fluctuational path through which it reaches that point. Starting from Boltzmann (1904), a huge body of theory has been developed during this century; the modern understanding dates from Onsager and Machlup (1953). But it was not until recently that a method was established (Dykman et al 1992) by which *optimal paths*, i.e. the most probable fluctuational paths, can be measured experimentally.

Recent progress in the area, with particular reference to experiments on optimal paths in far-from-equilibrium systems, will be discussed. Like rays in geometrical optics or WKB trajectories in quantum mechanics, the pattern of optimal paths displays singularities. The reasons why some of these are physically observable in electronic experiments, and others are not, will be accounted for though the topological insights introduced by Dykman et al (1994). Measurements of an exit location distribution for a noisy nonconservative system (Luchinsky et al 1999), leading to the observation of *saddle-point avoidance*, will be discussed.

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Chaos in economic systems

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The basic assumption in most economic theory is that agents act in accordance with rational expectations about the future. This implies that all relevant information supplied to a market is immediately taken into account, and the noise-like behavior observed in many economic time series is considered to represent this fast and accurate adjustment process. Nonetheless, the question remains whether the concept of efficient markets is consistent with actual economic conditions. The systematic phase shifts and lags observed between various economic variables bear evidence of dynamic phenomena with relatively long adjustment times, and asymmetries in the temporal variations indicate the presence of nonlinear interactions.

In order to explore the nature of human decision making behavior in complex, nonlinear systems, we have performed a series of role playing experiments with a simulated microeconomic system. MIT students of management and experienced managers from major US companies were asked to operate a four-stage production distribution chain to ensure a stable supply of beer. The observed performance was systematically sub-optimal, leading in many cases to operational costs of more than ten times the optimal. By virtue of the built-in delays and nonlinear constraints, most players were unable to ensure a stable operation, and large-scale fluctuations with characteristic amplitudes and phase lags developed.

We have formulated a four-parameter anchoring and adjustment heuristics for the ordering decisions in the game. With econometrically estimated parameters for each individual participant, this heuristics is found to reproduce the behavior of the system quite well. At the same time it explains the sources of poor performance in the game in term of insufficient levels of inventories, too aggressive stock adjustments, or inadequate account of supply line variations.

Computer simulations with the estimated ordering heuristics produce extremely complicated modes of behavior. For many parameter combinations, including combinations within the range of realistic policies, the system shows chaotic, hyperchaotic and higher-order hyperchaotic solutions. It is also possible to observe various forms of chaos-chaos intermittency.

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Synchronization of chaotic oscillators

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The ability of interacting, nearly identical chaotic oscillators to synchronize their motions has been demonstrated for a variety of different physical, chemical, and biological systems, and applications for chaos suppression, for monitoring of dynamical systems, and for different communication purposes are actively being pursued by many investigators. An interesting question concerns the starting conditions for which synchronization can be achieved. Other questions relate to the sensitivity of the synchronized state to a small parameter mismatch between the interacting oscillators and to the behavior of the system, once synchronization breaks down.

Riddling of the basin of attraction for the synchronized chaotic state arises when particular orbits embedded in this state become unstable to asynchronous perturbations while the state remains attractive on the average. Under these conditions the neighborhood of any point from which the trajectories are attracted to the synchronized state will contain initial conditions that are repelled from it. On-off intermittency is an extreme form of intermittent bursting that can sometimes be observed on the other side of the blowout bifurcation where the synchronized state loses its average attraction.

Considering as a simple example two coupled logistic maps, the lecture presents three different scenarios for the onset of riddling and for the subsequent transformations of the basins of attraction. It is demonstrated how the concepts of absorbing and mixed absorbing areas play an essential role in explaining the different types of behavior: The transition from local to global riddling, the distinction between different types of noise response, and the occurrence of on-off intermittency. Considering hereafter a system of two coupled Rössler systems it is shown how similar phenomena arise in time continuous systems.

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Chaos in living systems

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Physiological control systems offer one of the most interesting areas of application for nonlinear dynamics. Complementary to the concept of homeostasis that has dominated physiological thinking for such a long time, unstable phenomena are increasingly being recognized as significant for the regulation and function of normal physiological systems. Many hormonal systems, for instance, operate in a pulsatile mode with the release of insulin, growth hormone and luteinizing hormone typically occurring with one-three hour intervals.

By regulating the excretion of salts and water the kidneys play an important role in regulating the arterial blood pressure. To protect their own function and secure a relatively constant supply of blood, the kidneys also dispose of mechanisms that can compensate for variations of the arterial pressure. This ability rests at least partly with controls in the individual nephron, primarily the so-called tubuloglomerular feedback (TGF). This is a negative feedback that adjusts the diameter of the incoming arteriole in response to variations in the chloride concentration of the fluid that leaves the nephron.

Experiments on rat kidneys performed at The Department of Medical Physiology, University of Copenhagen have demonstrated that the time delays associated with the TGF mechanism tend to produce oscillations in the nephron pressure and flow regulation with a typical period of 20-30 sec. For rats with elevated blood pressure, the oscillations are often chaotic. Moreover, both regular and chaotic oscillations can be observed to synchronize between neighboring nephrons.

The lecture presents a detailed physiological model of the feedback controls of the individual nephron. A bifurcation analysis of this model reveals the existence of so-called crossroad structures of overlapping saddle-node and period-doubling bifurcations that can explain the experimental observations. The model is subsequently extended to consider the coupling between two neighboring nephrons that share the same afferent arteriole. The coupled nephron model reveals a variety of different synchronization phenomena including in-phase and anti-phase synchronization of regularly oscillating nephrons and complete and partial synchronization of chaotic nephrons.

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Asymptotic expansions beyond all orders: asymptotic description of chaotic dynamics

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Heteroclinic or homoclinic structures are the well-known mechanism for a genesis of chaos in conservative dynamical systems. While numerical iterations of low-dimensional maps easily provide these complicated structures, it is very difficult to derive them analytically, as addressed by Poincaré more than a century ago.

However, the difficulty can now be overcome by using a scheme of the asymptotic expansions beyond all orders. The scheme was recently sharpened with a help of the theoretical tool of Borel summability and Stokes phenomena. The pioneering works by Hakim and Mallick(1993) and Gelfreich et al(1994) exploited the asymptotics beyond all orders to elucidate the angle of separatrices splitting of the standard map.

It is highly desirable to develop the scheme further to describe the chaotic dynamics, i.e., the stretching and folding of the unstable manifold impinging upon the hyperbolic fixed point. Tovbis et al (1998), Nakamura and Hamada(1996) and Nakamura (1997) attempted to describe analytically the chaotic dynamics for the Henon and cubic maps. However, they failed to derive the Stokes multiplier precisely. In order to overcome this problem, some new systematic approach should be invented.

In this talk, dealing with the standard and cubic maps., i.e., time-discrete dynamical systems with cosine and double-well potentials, respectively, we shall provide an asymptotic analytical description of the complicated heteroclinic and homoclinic structures on extremely fine scales and compare the issue with that of an exact numerical iteration of the maps. Technical problems around the Borel summability and Stokes phenomena are also discussed in detail. The talk is based on the latest work by Nakamura and Kushibe (1999).

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Semiclassical theory of mesoscopic 3-dimensional billiards: orbital magnetism and persistent current

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In the mesoscopic billiards, chaos is known to play an important role in the orbital diamagnetism as pointed out by Nakamura(1993). While subsequent papers by Oppen(1994) and Richter et al(1996) dealt with mesoscopic 2-d billiards from the semiclassical viewpoint, quite few works have appeared about the rich phenomena in orbital diamagnetism of 3-d billiards.

In this talk, the magnetic susceptibility of ballistic 3-d billiards is investigated on the basis of the independent-electron model, for both single-billiard and ensemble-averaged cases. Using a semiclassical method, three kind of 3-d billiards subjected to weak uniform magnetic field are analyzed: spherical shell billiards; chaotic 3-d billiards but having SO(2) rotational symmetry; completely chaotic 3-d billiards without any continuous symmetry. We particularly elucidate the role of geometric symmetry in the susceptibility. According to different symmetries, different trace formulas (e.g., the Gutzwiller trace formula extended so as to incorporate continuously-degenerate periodic orbits) are exploited.

We first calculate the magnetic susceptibility for a spherical shell at finite temperature. Periodic orbits lying on great circular planes that share the center with both of the inner and outer spherical shells give rise to the susceptibility. The obtained result is compared with the corresponding results for a SO(2) rotational-symmetric chaotic billiard and for a completely chaotic 3-d billiard. Furthermore, the asymmetry in the magnetic response is found for 3-d billiard with SO(2) symmetry.

The analysis is also made on the persistent current in the corresponding three kinds of 3-d "shell" billiards. The talk is based on the latest works by Ma and Nakamura (1999).

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Quantum mesoscopic systems: I. Random matrix theory of the conductance of a chaotic cavity

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In this first lecture, the conductance of a ballistic quantum dot (having classical chaotic dynamics and being coupled by ballistic point contacts to two electron reservoirs) will be described on the single assumption that its scattering matrix is a member of Dyson's circular ensemble. The weak-localization corrections to the average conductance and the universal amplitude of the conductance fluctuations will be calculated. This example will show how the methods of random matrix theory can be implemented for describing quantum transport when the electron-electron interaction remains negligible (non interacting quasi-particles). The results discussed in this lecture have been published in Jalabert, Pichard and Beenakker (1994) and in Baranger and Mello (1994). Reviews for the random matrix theory of quantum transport can be found in Pichard (1991), Stone, Mello, Muttalib and Pichard (1991), Beenakker (1997) and Guhr, Müller-Groeling, and Weidenmüller (1998).

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Quantum mesoscopic systems: II. Chaotic mixing of the one body states by electron-electron interactions and delocalization in one dimension.

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The effect of electron-electron interactions when the one body states are localized will be introduced in a simple limit: two interacting particles (TIP) in a disordered chain. This problem was introduced by Shepelyansky (1994) and has been studied by Pichard et al (1996-1998). The TIP localization length will be estimated and the TIP spectral statistics will be discussed. The strength of the interaction leading to a maximum mixing of the one body states and to a maximum enhancement of the TIP localization length will be derived. Then, we will consider the ground state properties of a finite density of spinless fermions with short range interactions in a disordered chain. The enhancement of the persistent current driven by an Aharonov-Bohm flux when the system is between the Anderson (no interaction) and the Mott limits (correlated array of charges for strong interaction) will be discussed following Schmitteckert et al (1998).

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Quantum mesoscopic systems: III. Two dimensional gases of charges with Coulomb repulsions: A new metal between the Fermi glass and the Wigner crystal.

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The ground state and the low energy excitations of electron (or hole) gases confined in a two dimensional random potential are the subject of this third lecture. Without interaction, one cannot have a two dimensional metal according to the scaling theory of localization. Experiments pioneered by Kravchenko et al (1994) (and nowadays confirmed by many others experimental groups) have given evidences for unexpected insulator-metal transitions. We will review the results obtained by numerical studies of small clusters: When the Coulomb energy to Fermi energy ratio r_s is small, the system is an Anderson insulator while one has a pinned Wigner crystal in the large r_s limit. For intermediary r_s, there are growing numerical (Benenti et al 1999) and experimental evidences supporting the existence of a new metal with an ordered flow of enhanced persistent currents.

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Integrability and transport in quantum many body systems

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The particle transport in macroscopic many-fermion quantum systems at finite temperatures will be discussed. It will be argued that there is a fundamental difference between the integrable and nonintegrable systems [1]. Even when the current is not a conserved quantity the integrable models show a dissipationless transport, they can in fact behave as ideal conductors in the metallic regime and as ideal insulators in the insulating regime, while nonintegrable systems are expected to behave as generic conductors or resistors [2]. In this connection the role of conservation laws will be pointed out [3] and a relation to the level statistics will be examined.

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Quantum Poincaré mapping

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The lecture will give a review of the recent theoretical work on the quantization of Poincaré mapping on the *surface of section* (SOS) of general bounded and autonomous Hamiltonian systems.

We will begin, historically, to describe the *semi-classical* construction by Bogomolny [1] of the approximate *quantum Poincaré mapping* (QPM). Then we will formulate the scattering approach to quantization, which has been introduced for billiards by Smilansky et al [2], and which can be reinterpreted as an approach to exact quantization on SOS. We will show [3] that one can generally construct the *exact* QPM $T(E) = T_\uparrow(E)T_\downarrow(E)$, as the product of two scattering operators $T_{\uparrow,\downarrow}$ of the associated scattering problems which are obtained by amputing upper/lower half of configuration space along the configurational SOS and attaching a semi-infinite flat 'wave-guide' instead. Although this form of QPM enables one to write an exact *quantization condition* (QC) as $\det(1 - T(E)) = 0$ and has the correct semi-classical limit of Bogomolny (as $\hbar \to 0$), it has a disadvantage of not being strictly unitary operator, so it does not conserve probability, and hence, strictly speaking, it cannot be interpreted as an evolution operator on SOS.

However, we propose a simple and almost unique technique of *unitarization* of QPM [4] $T(E) \longrightarrow \mathcal{T}(E)$, which yields the strictly unitary QPM $\mathcal{T}(E)$ on the Hilbert space of L^2 functions over the configurational SOS (which has one degree of freedom less than the full space problem), and which at the same time preserves (i) the (semi)classical limit, as well as (ii) the exact QC, namely, E is an eigenenergy iff $\det(1 - \mathcal{T}(E)) = 0$, or alternatively, iff one of the eigenphases $\varphi_n(E)$ of the unitary QPM $\mathcal{T}(E)$ crosses zero, $e^{-i\varphi_n(E)} = 1$.

The theoretical ideas will be worked out explicitly on an example of a *semi-separable* system (Prosen 1996), a simple two-degree-of-freedom dynamical system which is separable above and below SOS, but is discontinuous on SOS, hence in full space it is non-separable, non-integrable and possibly chaotic (depending on parameters).

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Quantum chaos, ergodicity and mixing in generic many-body systems in thermodynamic limit

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We address an old unsolved problem of (quantum) statistical mechanics, namely the justification of the ergodic hypothesis, and consequently, the 'derivation' or dynamical understanding of macroscopic statistical laws, e.g. the transport laws such as Ohm's, Fourier's or Fick's law, from the (non-integrable) microscopic equations of motion. To this end we propose to study (i) the dynamics (direct time-evolution), and (ii) the existence of dynamical conservation laws (integrals of motion), of simple quantum-many body dynamical system for which thermodynamic limit can be approached numerically. This we do [1] by means of efficient (and presumably novel) numerical methods, namely, by (i) multi-dimensional (anti)symmetrized Fast Fourier Transformation, and (ii) solving Galerkin-like variational problems in truncated operator Hilbert spaces of quantum observables of an infinite many-body system, respectively.

We define [1] a generic but simple non-integrable quantum *many-body* system of *locally* interacting particles, namely a kicked $t-V$ model of spinless fermions on 1-dim lattice (equivalent to a kicked Heisenberg XX-Z spin$-1/2$ chain). Statistical properties of dynamics (quantum ergodicity, quantum mixing [2]) and the nature of quantum transport in *thermodynamic limit* are considered as the kick parameters (which control the degree of non-integrability) are varied.

We find and demonstrate *ballistic* transport and non-ergodic, non-mixing dynamics (implying infinite conductivity at all temperatures) in the *integrable* regime of zero or very small kick parameters (as conjectured already in [3]), and more generally and important, also in *non-integrable* regime of *intermediate* values of kicked parameters, whereas only for sufficiently large kick parameters we recover quantum ergodicity and mixing implying normal (diffusive) transport. Based on numerical results we propose a (dynamical) phase transition with an order parameter D (charge stiffness, which is the time-averaged current auto-correlation function) from *non-ergodic* and *non-mixing* quantum dynamics (ordered phase, $D > 0$) to ergodic and mixing dynamics (disordered phase, $D = 0$) in the thermodynamic limit.

Similarity with order-to-chaos transition of generic non-integrable one-particle quantum systems in semi-classical limit is striking, and we mention a result [4] on close formal relation between dynamics of a particular class of non-linear quantum many body systems (in thermodynamic limit) and an associated class of *one-body* quantum systems (in classical limit).

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Chaotic dynamics and statistical mechanics in Hamiltonian systems with many degrees of freedom

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We discuss the connection between chaotic dynamics and equilibrium statistical mechanics in systems with many degrees of freedom. The talk will focus in particular on the links between microscopic chaos and phase transitions in the Hamiltonian Mean Field model (HMF), Antoni and Ruffo (1995). The latter is a simple toy model of N fully-coupled rotators which shows a second order phase transition. Recently the equilibrium properties and the dynamical behavior of HMF have been investigated numerically and analytically in several papers by Latora, Rapisarda and Ruffo (1998),(1999) . Our main result is that both the largest Lyapunov exponent and Kolmogorov-Sinai entropy show a peak at the critical energy U_c. This result is checked to be valid in the limit $N \to \infty$. Scale law behaviors for the Lyapunov exponents are derived, and an important link between chaoticity and the thermodynamical fluctuations, expressed by kinetic energy fluctuations or specific heat, is found. The canonical thermodynamical Vlasov solution has been successfully checked in numerical simulations. In general, chaotic dynamics provides the mixing property in phase space necessary for obtaining equilibration. However relaxation to equilibrium can be very slow and grows with N close to the critical point where metastable states are found. Possible connection to anomalous transport properties and Levy walks will be discussed in connection to the out-of-equilibrium regime. Our results seem to be very general and confirm a previous pioneering work by Bonasera, Latora and Rapisarda 1995. Lately similar features have been found also in other models by Caiani et al (1998), Anteneodo and Tsallis (1998), Torcini and Antoni (1999). The results discussed for this toy model can be very useful in order to understand the multifragmentation phase transition in excited nuclei, Atalmi et al (1998), and clusters , Farizon et al (1998), and could be important also in astrophysics for autogravitating systems, Yawn and Miller (1997). In general this new scenario provides an interesting bridge between Hamiltonian chaos in systems with many degrees of freedom and statistical mechanics.

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Chaos in quantum transport: From electron billiards to Coulomb blockade

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We review aspects of classical and quantum transport through electronic mesoscopic systems. In the first part of the talk we discuss properties of open conductors which represent realizations of non-interacting chaotic quantum billiards. We introduce semiclassical concepts which allow us to relate quantum coherence effects in the conductance to properties of classical irregular scattering in these billiards (Baranger et al. 1993, Stone 1995, Richter 1999). We highlight the achievements of these approaches for the understanding of related recent experiments on transport through semiconductor nanostructures. We furthermore discuss problems of the standard semiclassical transport theory to account adequately for weak localization, a quantum correction to the average conductance, in ballistic systems. We outline new theoretical advances which may cope with these problems and introduce the concept of the Ehrenfest time in the context of chaotic transport (Aleiner and Larkin 1996). These approaches enable us to extract from the *quantum* weak localization correction the mean Liapunov exponent of the *classical* counterpart of a ballistic mesoscopic system (Yevtushenko et al. 1999).

Upon reducing the coupling of the quantum dot (billiard) to the leads, interaction effects become important (Kastner 1992). In the second part of the talk we thus describe novel interaction phenomena in this *Coulomb blockade* regime; namely strong fluctuations in the conductance-peak heights and spacings. (Experimentally) observed fluctuations in the related ground state energies do not follow Wigner-Dyson (GOE- or GUE-) statistics (Sivan et al. 1996, Simmel et al. 1999) and call for refined random matrix approaches and a more careful consideration of level statistics of interacting electrons in chaotic quantum dots. We review recent theoretical approaches and discuss in particular the interplay between electron-electron interactions and the confinement geometry (integrable/chaotic) of ballistic quantum dots. We do this on the basis of both exact diagonalization for few particles and self-consistent Hartree-Fock calculations for a larger number of electrons.

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Topics in quantum chaos of generic systems I & II

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We review the main ideas and results in the stationary problems of quantum chaos in generic (mixed) systems, whose classical dynamics has regular (invariant tori) and chaotic regions coexisting in the phase space. First we discuss the universality classes of spectral fluctuations (GOE/GUE for ergodic systems, and Poissonian for integrable systems). We explain the problems in the calculation of the invariant (Liouville) measure of classically chaotic components, which has recently been studied by Robnik et al (1997) and by Prosen and Robnik (1998). Then we describe the Berry-Robnik (1984) picture, which is claimed to become exact in the strict semiclassical limit $\hbar \to 0$. However, at not sufficiently small values of $\hbar$ we see a crossover regime due to the localization properties of stationary quantum states where Brody-like behaviour with the fractional power law level repulsion is observed in the corresponding quantal energy spectra.

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Chaotic Jung scattering maps versus topological chaos of the Hamiltonian flow

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The classical scattering map proposed by Jung is the classical analog of the quantum S-matrix. It therefore contains the information we can obtain by asymptotic measurement for a classical scattering system. A detailed analysis of the relation of integrability and chaos for the Jung map and the Hamiltonian flow shows that the connection is complicated and dependent of the choice of the "unperturbed Hamiltonian" H_0. For the typical choice of H_0, namely the free particle, we can readily construct examples where the flow is integrable, but the scattering map is chaotic. This is due to the incompatibility of the integrals of motion of the Hamiltonian H with those of H_0. To transfer the integrability of H to the integrability of the Jung map it is necessary and sufficient that H_0 conserves at least asymptotically the integrals of motion of H.

This implies that it is not particularly simple to distinguish chaos or integrability from scattering functions. Some simple examples illustrate that selfsimilar and even fractal structures in the scattering functions may result from integrable Hamiltonians.

We may pass from classical to quantum mechanics if we realize that the Jung map is a canonical map of channel space (considered as a symplectic manifold or phase space) onto itself and that the S-matrix is the unitary representation of this map. This implies that the integrability properties of this map determine those of the S-matrix. The criteria of integrability carry over easily to the quantum case.

There is a further interesting aspect that results from the interpretation of the S-matrix as a representation of the Jung map. The statistical properties of the spectrum of the S-matrix not determined by integrability or chaos in the Hamiltonian flow, but rather by the corresponding properties of the Jung map. This can be understood in terms of a series of arguments centered around the concept of *structural invariance*, which establish the connection between chaos and random matrix theory; This connection is based on a probabilistic argument which allows for all the exceptions and special cases that exist in this context. It is short of being a proof because the existence of an invariant measure on the group of invertible canonical transformations has to be established.

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Unitary quantum Poincaré maps and Rydberg molecules

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Multichannel quantum defect theory (MQDT) has proven very successful for the calculation of Rydberg molecules. This theory uses results from scattering theory to obtain information about states near, but below threshold. In a particular approximation for the classical system, Lombardi showed that MQDT provides the exact quantum solution. As this approximate system may be characterized by a symplectic or canonical map, MQDT can find a much more transparent interpretation as a unitary representation of this map. A more careful analysis shows that indeed MQDT is a true quantum Poincaré map (QPM). Remember that Bogomolny's definition of a QPM is semi-classical. By consequence the map becomes unitary only in the limit $\hbar = 0$. Prosen has proposed a very elegant unitarization of the QPM at the price of obtaining an infinite dimensional matrix. In our case the QPM is constructed directly from unitary transformations between body fixed and space fixed frames as well as the phase shifts typical of MQDT, thus yielding a finite unitary matrix by construction.

Lombardi's result for the classical approximation has ingredients that are essential for MQDT to become the exact solution, but beyond that he uses the approximation that the absolute value of the electron angular momentum does not change. The latter approximation is not always good though it is very convenient because the Poincaré section is then two-dimensional, *i. e.* we can actually draw it. Yet this approximation is not essential to the reinterpretation of MQDT as a QPM

The result has several interesting applications beyond the fact that it affords a new understanding of MQDT.

First it can serve as a paradigm for a QPM and may even give a hint, when and to what extent we really have to use semi-classics to construct a QPM.

Second the interpretation as a QPM allows for a much more stable construction of eigen-values and eigen-functions of energy from the unitary matrix given by the QPM.

Third the evolution of the eigen-phases as a function of energy is essential to the possible transfer of properties of the eigen-phase spectrum to the energy spectrum. This transfer is of basic interest for the understanding of the relation of chaotic classical dynamics and random matrix theory, as the latter is easier to understand for circular ensembles.

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Rings: Generic structures of rotating systems

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From Galilieos discovery until a few years ago Saturns rings were a singular phenomenon in the sky. This has radically changed since we know that all major planets carry rings. This leads to the suspicion that such ring structures are universal phenomena. Yet it is not at all clear what universality class we look at.

In the present contribution we shall demonstrate that one of the two universal scenarios for a sudden appearance upon variation of a parameter of a localized manifold in a Hamiltonian scattering system, will always lead to ring structures if we deal with a system rotating around some point in space.

Consider free motion in a rotating frame: the particle will approach the center of rotation up to the closest approach on a spiral trajectory and then leave the central region on an outgoing spiral. We now include a repulsive potential with convex potential contour lines rotating with frequency ω about a center that lies outside the potential or at least its most repulsive part. The particle may hit this potential on the ingoing or outgoing trajectory. In the latter case the interaction may throw it onto another ingoing trajectory. This will give opportunity for a second interaction if the absolute value of the Jacobi-integral (Hamiltonian in the rotating system) is not to large. This limitation results because otherwise the particle leaves the region, where such an interaction is possible before the center of interaction has swung around a sufficient angle. Thus confined trajectories will appear as we reduce (or increase for negative values) the Jacobi-integral. There are two generic ways this can occur as a function of a parameter. One is the sudden appearence of hyperbolic horseshoes with infinite exponents. The other and the one we are interested in is the saddle-center bifurcation, which will give rise to stable periodic orbits in the rotating frame. As these orbits are made up of slightly distorted parts of a spiral they have ring shapes, and these non-circular rings will have a precession with the frequency of rotation of the system.

We shall use the model of a hard disc rotating about a center outside of the disc to illustrate our general findings. This model has the particular advantage that it does not allow for the alternate generic scenario that we mentioned. A single disc will produce a wide ring with a very involved structure, but the introduction of a second disc moving on an inner circle with an incommensurable frequency will be shown to produce narrow rings with structure.

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Complex trajectory description for quantum tunnelling I

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Tunnelling phenomenon is one of the most familiar quantum effects. The wavepacket in quantum mechanics can penetrate even into the region where the classical trajectory cannot reach. The barrier which prevents the transition to classically forbidden regimes is always formed by the energy potential in case of one degree of freedom system. However, in higher dimension, in addition to the energy, there sometimes appear dynamical barriers (KAM Tori) which are usually coexisting with chaotic components in the phase space. The classical trajectories are also confined dynamically if they are placed in such regions initially. The quantum wavepacket can also go over such dynamical barriers, and tunnelling penetration out of dynamically formed barriers is called "dynamical tunnelling" (Heller & Davis 1981, Bohigas, Tomsovic & Ullmo 1993, Creagh 1998).

The purpose of our talk is to show how the essential difference between tunnelling in one dimension and that in more than one-degree-of-freedom systems is explicitly understandable in terms of the semiclassical methodology which has been elaborated extensively in recent studies of quantum chaos. The main tool we will use is the complex semiclassical theory in which not only usual real classical orbits but also complex classical trajectories play a key role in describing the purely quantum mechanical phenomena such as tunnelling (Miller 1974).

We first introduce a simple model (kicked rotor), which is suitably designed to test the dynamical tunnelling as purely as possible and observe the time evolution of its wavefunction, especially by focusing on the tunnelling tail. The complex semiclassical analysis is then performed, aimed at the interpretation of various features discovered in the tunnelling wavefunction in chaotic systems. The procedure of the complex semiclassical analysis is almost the same as the real one, but in contrast with the real semiclassics, finding a definite rule to select *dominant complex orbits* becomes a crucial task, since in chaotic systems there appear vastly many number of candidate complex trajectories, all of which can contribute to the semiclassical propagator, but not all of which contribute equally in weights.

The main message we will present in our first lecture is that it is indeed possible to select such important complex orbits under a certain rule, and the origin of various remarkable features observed in the tunnelling tail in chaotic systems, which are completely absent in integrable systems, can be explained by the group of complex manifolds forming the chain-like structure in the initial value representation of complex trajectories (Shudo & Ikeda 1998).

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Complex trajectory description for quantum tunnelling II

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The subject of our second lecture is to present some theoretical backgrounds for the semiclassical description of tunnelling in the presence of chaos, especially to discuss the *contributing and non-contributing problems* in the complex semiclassical approach.

In applying the complex semiclassical method, we inevitably face the problem of how to select the contributing complex (tunnelling) paths from all the candidates which satisfy the saddle point condition in principle. There would essentially be two distinct origins:

The first one is concerned with how to find the most largely weighted complex paths among vastly many complex candidates which should *a priori* be included in the semiclassical superposition. The weight of an individual contribution is almost determined by the imaginary part of the action along each classical trajectory, and the paths with large imaginary actions may be negligible in the semiclassical sum. This problem is purely classical, in particular it should be solved in the context of the *complex classical dynamics* (Milnor 1990).

We will show that the most relevant complex classical trajectories in describing the tunnelling effect in chaotic systems are very closely related to the *Julia set* in the corresponding complex dynamical system. More precisely, it can be shown that the *Laputa chain*, which is the most important phenomenological object introduced to explain all the features of tunnelling in chaotic systems, is nothing but the intersection between the initial Lagrangian manifolds and the complex stable manifolds of saddles in the Julia set (Shudo & Ikeda 1999).

The second contributing and non-contributing problem arises from the *Stokes phenomenon* in multi-dimensions. The Stokes phenomenon is the discontinuous change of the asymptotic solutions, and it occurs quite generically in the saddle point method or the differential equation with large parameters. Due to the Stokes phenomenon, not all of the complex classical trajectories necessarily contribute to the final semiclassical propagator.

Owing to the recent development of so-called *exact WKB analysis* which allows us to treat asymptotic expansions on the analytical basis via Borel-Laplace transform (Voros 1983), the Stokes phenomenon now becomes well defined concept. We will give a prescription for analyzing the Stokes phenomenon in multi-dimensions, where the crossing of the Stokes curves is an essentially new event while it is absent in 2nd-order differential equations. It will be shown how the Riemann sheet structure of the Borel transform(or 'adjacency' in another context (Berry & Howls 1991)), which is a crucial information in our exact WKB understanding of the Stokes phenomenon, is determined by the *Stokes graph* (Shudo & Ikeda 1999).

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Critical bifurcation surfaces of discrete multi-dimensional dynamics

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The purpose of this lecture is to construct the three-dimensional analytical representation of the general procedure of linear local bifurcation analysis developed in Sonis, 1993, on the basis of classical Routh-Hurvitz conditions of asymptotic stability combined with the movements of fixed point in the space of orbits. The bifurcation phenomena are defined by the position of the boundaries of attraction of the fixed point. It will be proven that the domain of attraction of the fixed point of 3D discrete dynamics is bounded by three critical bifurcation surfaces: the divergence surface corresponding to the case in which one of the eigenvalues of the Jacobi matrix of the linear approximation of the dynamics equals to 1; the flip surface corresponding to the existence of the eigenvalue -1, and the flutter surface corresponding to the pair of complex conjugated eigenvalues with absolute values equal to 1. The crossing of these surfaces by the movement of the fixed point will generate the plethora of all possible bifurcation phenomena. This scheme of local bifurcation analysis will be demonstrated by numerous application to bifurcation analysis of the Henon map, the different examples of Socio-Spatial discrete Dynamics and the theoretical example of the bifurcation analysis of the T. Puu Dyopoly and Triopoly adjustment dynamics.

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Nonlinear socio-ecological dynamics and first principles of collective choice behavior of "Homo Socialis"

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Socio-ecological dynamics emerged from the field of Mathematical Social Sciences and opened up avenues for re-examination of classical problems of collective behavior in Social and Spatial sciences. The "engine" of this collective behaviour is the subjective mental evaluation of level of utilities in the future, presenting sets of composite socio-economic-temporal-locational advantages. These dynamics present new laws of collective non-local multi-population behavior which are the meso-level counterparts of the utility optimization in individual behavior. The central core of the socio-ecological choice dynamics includes the following first principle of the collective choice behavior of "Homo Socialis" based on the existence of "collective consciousness": the choice behavior of "Homo Socialis" is a collective meso-level choice behavior such that the relative changes in choice frequencies depend on the distribution of innovation alternatives between adopters of innovations. This hypothesis expands essentially the view point of the social statistical mechanics by including into the consideration the collective conscience of "human molecules", arising from the social interactions and informational mass media effects. The mathematical basis of the Socio-Ecological Dynamics includes two mutually complemented analytical approaches both based on the use of computer modeling as a theoretical and simulation tool. First approach is the "continuous approach" - the systems of ordinary and partial differential equations reflecting the continuous time Volterra ecological formalism in a form of antagonistic and/or cooperative collective hyper-games between different subsets of choice alternatives. Second approach is the "discrete approach" - systems of difference equations presenting a new branch of the non-linear discrete dynamics - the Discrete Relative m-population/n-innovations Socio-Spatial Dynamics (Dendrinos and Sonis, 1990). The generalization of the Volterra mathematical formalism leads further to the meso-level variational principle of collective choice behavior determining the balance between the resulting cumulative social spatio-temporal interactions among the population of adopters susceptible to the choice alternatives and the cumulative equalization of the power of elites supporting different choice alternatives. This balance governs the dynamic innovation choice process and constitutes the dynamic meso-level counterpart of the micro-economic individual utility maximization principle. The above described law of elite competition can be extended to each many-dimensional iteration processes. Thus is the existence of the extreme principle describing the discrete space-time transfer from the past to the future. This principle represents the collective synergetic interactions between the constituencies of the iteration process. Moreover, this principle is the discrete time analog of the law of meso-level variational principle of collective choice behavior for the continuous time Socio-Ecological Dynamics.

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Impulses and waves in biological systems: Exploring stability of membrane potential in excitable cells

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Excitable cells are either stable or oscillate. Subthreshold stimulation produces a transient departure from equilibrium while suprathreshold stimulation produces a major deparation from equilibrium that requires some time to return to the rest state. The major electrical features of an excitable cell are an insulating cell membrane and gated channels that permeate the membrane and control the flow of ions down concentration gradients. A capacitor is formed by the cell membrane surrounded by conducting fluid on either side while the gated channels form non-linear resistors. The parallel combination of capacitors and Na and K channels define a cellular reaction process which controls the electrical "state" of a cell. We explore the behavior of a simple reaction diffusion process (Fitzhugh-Nagumo Model) of the form:

$$\frac{\partial U}{\partial t} = f(U) - W + \nabla^2 U$$

where $f(U) = U(1 - U^2)$ and

$$\frac{\partial W}{\partial t} = \alpha U - W + \gamma$$

Phase plane analysis reveals the possibility of two stable equilibria and 1 unstable equilibria, depending on medium properties. The functions, f(U) and W are comparable to ionic currents that flow through channels that penetrate the membrane of excitable cells. By altering the channel conductances, the cell membrane potential will either be stable or will oscillate. Genetic diseases, that alter the properties of membrane ion channels can lead to shifts in the equilibrium of excitable cells from stable to unstable. Epilepsy and the cardiac long QT syndrome are two such examples. Experimental studies of membrane potential in cardiac cells display shifts in equilibrium when treated with drugs that block the flow of ions in membrane ion channels. This lecture will link physical and mathematical models of excitable membranes and ion channel blockade with the behavior of the *minimal* model (Fitzhugh-Nagumo) and its associated phase plane analysis of cellular stability and illustrate how genetic mutants of ion channels lead to cardiac and cerebral arrhythmias that appear quite complex.

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Impulses and waves in biological systems: Exploring front formation and propagation

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Isolated excitable cells are either stable or oscillate. Diffusive coupling between excitable cells introduces a new property: the spatial extent of the excitation must exceed a "liminal" region that is a function of media properties. In a uniform medium, fronts initiated by excitation smaller than the liminal region will collapse while fronts initiated by excitation of a region larger than the liminal region will expand. We explore the behavior of a simple reaction diffusion process of the form:

$$\frac{\partial U}{\partial t} = f(U) - W + \nabla^2 U$$

where $f(U) = U(1 - U^2)$ and

$$\frac{\partial W}{\partial t} = \alpha U - W + \gamma$$

This lecture will focus on a number of interesting questions related for formation and propagation of waves in an excitable medium, specifically wave patterns associated with cardiac arrhythmias. In homogeneous and uniform excitable media in the rest state, there are two responses to stimulation: if the stimulus is less than a critical value, a collapsing wave is initiated while if the stimulus is greater than a critical value, and continuous expanding wave is initiated. Under some conditions, it is possible to initiate a partial wave, i.e a wave that expands in some directions and collapses in other directions. Trailing a propagating wave is a region known as the vulnerable region, where the non-uniformity of the medium properties can result in incomplete front formation following suprathreshold stimulation. Depending on the time and location of stimulation after the passage of a wave, stimulation results in either: 1) a decaying front; 2) an expanding front; and 3) a wave fragment (wavelet) that expands in some directions and collapses in other directions. In two and three dimensional media,the wave fragment can evolve to a rotating spiral wave. The duration of the period of vulnerability is VP $= L/V$ where L is the spatial extent of the suprathreshold stimulus field and V is the velocity of the *conditioning* wave . We will show that drugs that reduce the excitability of isolated cells (classified as antiarrhythmic drugs) will increase the VP, leading to dramatic increases in the rate of spontaneous and potentially fatal arrhythmias. This new finding was confirmed in studies of cardiac tissue.

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Topics in nonlinear dynamics of the human cardiovascular system

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We review the main results of analysis of time series measured simultaneously at different sites of the human cardiovascular system. Because our primary interest centres on the dynamics of blood distribution, characteristics during one cycle of blood through the system, which on average takes about one minute, are considered.

First we discuss results of nonlinear time series analysis with regard to system determinism and complexity. The analysis in the time domain and in the phase space revealed highly deterministic and almost conservative nature of cardiovascular control system on time scale of minutes (Stefanovska and Krošelj, 1997, Bračič and Stefanovska, 1998a, Stefanovska and Bračič, 1999a, 1999b).

Then we present results of time-frequency analysis. The wavelet transform, a method with logarithmic frequency resolution, was used to study oscillations in cardiovascular signals (Bračič and Stefanovska, 1998b, Kvernmo, Stefanovska, Bračič, Kirkeboen and Kvernebo, 1998, Stefanovska, Bračič and Kvernmo, in print). The heart rate variability (HRV), derived from the signal resulting from electrical activity of the heart (ECG), bears the variations of instantaneous heart rate and RR intervals in the ECG. Several central and peripheral oscillators contribute to the variations between subsequent heart beats. Analysis of signals measured from respiration, cardiac function and blood flow, all revealed the existence of five almost periodic frequency components, illustrating that all the cardiac periodicities are propagated as traveling waves.

The physical description of the cardiovascular control system depends essentially on the current understanding of physiological origin of observed oscillations. The evidence that endothelial activity is manifested as oscillations in blood flow with a characteristic frequency of around 0.01Hz was recently demonstrated (Stefanovska, Bračič and Kvernmo, in print). The mode of changes in the characteristics of oscillations with aging, physical activity, diabetes, after myocardial infarction, or general anaesthesia also provides insight into the physiological and physical characteristics of the system. In addition, the strength of phase and frequency couplings among the oscillations, modified in different states of the cardiovascular system, is discussed.

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Cardiorespiratory interactions

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Oscillators that have a standard waveform and amplitude to which they return after small perturbation are known as limit-cycle oscillators. They incorporate a dissipative mechanism to dump oscillations that grow too large and source of energy to pump up those that become too small. Coupled, they continuously perturb each other and everyone of them permanently tends toward a limit cycle with time-varying amplitude and characteristic frequency.

Signals derived from the human cardiovascular system have been found to contain several periodic components (Akselrod *et al*, 1981), thereby raising the question of whether the blood distribution system and its regulatory mechanisms can be described as a set of coupled oscillators (Stefanovska and Bračič, 1999). On minute time scale the peripheral blood flow was shown to contain five characteristic frequencies (Bračič and Stefanovska, 1998). The physiological origin of two of them is easy to evidence by simultaneous measurements of the heart and respiratory activity. The heart rate at rest ranges from 0.6 Hz in sportsmen to 1.6 Hz in subjects with impaired cardiovascular systems and the respiratory frequency from 0.15 Hz to 0.35 Hz. As early as 1733, Hales observed that changes in heart rate were related in a regular manner to the respiratory pattern, and in 1847 Ludwig documented that the heart rate increased on inspiration and decreased on expiration. The respiratory modulation of the heart rate, known as "respiratory sinus arrhythmia", raises questions not answered so far: how strongly those two systems are coupled, how strongly they modulate each others amplitudes and characteristic frequencies, does phase synchronisation exist at rest in healthy and diseased states? Raschke (1987) analysed effects of mutual interactions and demonstrated that during sleep inspiration starts preferably at a certain phase of the cardiac cycle. Glass and Mackey (1988) concluded that they are weakly coupled and generally not phase locked. In contrast, Rosenblum *et al* (1998) revealed hidden phase-locked regimes between the cardiac and the respiratory rhythms.

We discuss mutual frequency and amplitude modulation of the heart and respiratory cycles. The couplings are not symmetrical, the respiratory system is imposing stronger influence to the cardiac system than vice versa. The other three oscillatory processes involved in the regulation of blood distribution system also influence the heart and respiratory activity. Consequently, a stable phase locking among those two oscillators can not be obtained in healthy cardiovascular system. In addition, we present states of coupling between the two oscillatory systems modified by aging, diabetes, myocardial infarction, or general anaesthesia.

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Nonlinear dynamics, bifurcation, and symmetry

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In many applications of nonlinear dynamics, the model system possesses a degree of symmetry. Symmetry affects the 'generic' behaviour of the system, and its presence should be taken into account when analysing the predictions of the model. The course will provide a three-lecture introduction to some basic theory, including fundamental existence theorems for the existence of symmetry-breaking These include criteria for the existence of symmetry-breaking branches of steady and periodic states, and some basic concepts related to 'symmetric chaos'. We emphasize the role of symmetry as a general framework for such problems.

1 Summary of Course Structure

1. *Steady State Bifurcation*

   Invariant functions and equivariant mappings. Group actions, isotropy subgroups, fixed-point spaces. The isotropy lattice. Genericity of absolutely irreducible representations. The Equivariant Branching Lemma.

   Examples: $\mathbf{O}(2)$-symmetric systems, $\mathbf{D}_n$-symmetric systems $\mathbf{S}_N$-symmetric systems.

   Applications: Speciation, convection, Taylor-Couette flow.

2. *Hopf Bifurcation to Periodic States*

   Loop space, circle group action by phase shift, spatio-temporal symmetries. Liapunov-Schmidt reduction. Genericity of Γ-simple representations. The Equivariant Hopf Theorem.

   Examples: $\mathbf{O}(2)$-symmetric systems, $\mathbf{D}_3$-symmetric systems.

   Applications: Oscillating hosepipe, animal locomotion, convection.

3. *Symmetric Chaos*

   Pointwise and setwise symmetry of an attractor. Symmetry on average. Detectives. Collisions of attractors. Transverse Liapunov exponents.

   Examples: The cubic logistic map, $\mathbf{D}_3$-symmetric maps.

   Applications: Oscillator arrays, the Faraday Experiment.

2 Steady State Bifurcation

Steady state bifurcation occurs when an equilibrium of a dynamical system becomes unstable as a parameter λ is varied. In suitable circumstances, the generic result is the creation of a branch of 'new' equilibria. We study such behaviour when the dynamical system has symmetry.

Specifically, by ‘ has symmetry’ we mean the following. Let Γ be a Lie group of linear transformations of $\mathbf{R}^n$. We say that f is Γ-*equivariant* if

$$f(\gamma x, \lambda) = \gamma f(x, \lambda) \tag{5}$$

for all $\gamma \in \Gamma$. Consider a Γ-equivariant ODE

$$\frac{dx}{dt} + f(x, \lambda) = 0 \tag{6}$$

where $x \in \mathbf{R}^n$, $\lambda \in \mathbf{R}$. For simplicity, assume that $f(0, \lambda) \equiv 0$, so there exists a ‘trivial branch’ of solutions $x = 0$. The *linearization* of f is

$$L_\lambda = D_x f|_{0,\lambda} .$$

Local bifurcation at $\lambda = 0$ occurs when the trivial branch undergoes a change of linear stability, so that L_0 has eigenvalues on the imaginary axis (often called *critical eigenvalues*).

There are two cases:

- Steady-state bifurcation: L_0 has a zero eigenvalue.
- Hopf bifurcation: L_0 has a complex conjugate pair of eigenvalues $\pm i\omega$.

In the first lecture we look at steady-state bifurcation. The central result here is the Equivariant Branching Lemma, which asserts the existence of branches of equilibria with certain kinds of symmetry.

The basic examples are $\mathbf{O}(2)$-symmetric systems, $\mathbf{D}_n$-symmetric systems, and $\mathbf{S}_N$-symmetric systems. Here $\mathbf{O}(2)$ is the orthogonal group in the plane, $\mathbf{D}_n$ is the dihedral group, and $\mathbf{S}_N$ is the symmetric group of degree N.

Applications include speciation in evolutionary systems (modelled as a dynamic on phenotypic space), convection, and Taylor-Couette flow of a fluid between rotating cylinders.

3 Hopf Bifurcation

Hopf bifurcation leads to a branch of time-periodic solutions. It is analogous to steady-state bifurcation, but now there is an extra symmetry given by an action of the circle group. Intuitively, this represents shifting the phase of a solution. The problem becomes $\Gamma \times SS$-equivariant.

The analogue of the Equivariant Branching Lemma is the Equivariant Hopf Theorem. It is proved by posing the problem on loop space — a Banach space of periodic functions — upon which there is a circle group action by phase shift, corresponding to spatio-temporal symmetries.

Applications include the oscillating hosepipe of circular cross-section, with both standing and rotating waves; animal locomotion, where the patterns with which animals move their legs when they walk, trot, run, and so on can be classified and organised; and convection.

4 Symmetric Chaos

Symmetric systems can have chaotic attractors, and here the symmetry of the attractor represents the symmetry 'on average' over long periods of time of the corresponding states. It is important to distinguish between pointwise and setwise symmetry of an attractor.

The symmetry of an attractor can be found experimentally by using the theory of 'detectives'.

Important phenomena include collisions of attractors, in which conjugate versions of a chaotic attractor can suddenly merge, with a change of symmetry.

Examples include the cubic logistic map

$$f(x, \lambda) = \lambda x(1 - x^2)$$

and $\mathbf{D}_3$-symmetric maps.

Applications include chaotic dynamics of oscillator arrays, synchronisation of coupled chaotic oscillators, and the Faraday Experiment, in which a dish of fluid is vibrated in the vertical direction.

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Microwave studies of chaotic billiards and disordered systems

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Chaotic billiards are ideal model systems to study the quantum mechanical properties of classically chaotic systems. Billiards with hard walls are conceptionally particularly simple. The classical trajectories can be calculated elementarily, and the Schrödinger equation reduces to the free wave equation with Dirichlet boundaries at the walls. Nevertheless the calculations soon become elaborately, in particular, if spectra or wave functions are to be determined in dependence of one external parameter, usually the billiard length or the position of an inserted scatterer. Here analogous experiments are an interesting alternative. They make use of the equivalence of the stationary Schrödinger equation, and the time-independent wave equation, the Helmholtz equation. In two-dimensional microwave billiards this equivalence is complete, including the boundary conditions.

After a short introduction into the problematic of the quantum mechanics of classically chaotic systems, shortly termed 'quantum chaos', different types of billiard experiments are introduced. The main part of the talk is devoted to microwave billiards. Results on global and local billiard spectral level dynamics are presented as well as measurements in three-dimensional chaotic microwave resonators. Here the equivalence between Schrödinger equation and electromagnetic wave equation does no longer hold. The talk ends with a discussion of transmission and localization in disordered systems.

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Microwave billiards as scattering systems

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It is impossible to study a system without disturbing it by the measuring process. To determine, e. g., the spectrum of a microwave billiard, we have to drill a hole into its wall, introduce a wire, and irradiate a microwave field. Due to the presence of the antenna a rectangular microwave cavity, e. g., is no longer integrable, but has become pseudointegrable.

The measurement thus unavoidably yields an unwanted combination of the system's own properties and those of the measuring apparatus. The mathematical tool to treat the coupling between the system and the environment is provided by scattering theory, which has originally been developed in nuclear physics. Later this theory has been successfully applied to mesoscopic systems and microwave billiards as well.

In this talk scattering theory will be applied to billiard systems, resulting in the billiard equivalent of the Breit-Wigner function, well-known from nuclear physics for many years. The consequences of the coupling for the spectral statistics will be discussed, as well as for resonance depths and widths. In the last part results on transmission through billiards with open channels are presented. Here we have a close correspondence to mesoscopic systems, which can be linked to scattering theory via the Landauer formula expressing the conduction through mesoscopic devices in terms of transmission probabilities.

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Phase resetting in medicine and biology – Stochastic approach and applications

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Synchronization processes are of great importance in several branches of science, e.g., in biology, medicine and physics. We present a new theoretical approach to phase resetting and stimulation-induced synchronization and desynchronization in a population of interacting oscillators in the presence of noise (Tass 1999).

The theoretical investigations of spontaneously emerging dynamics of populations of interacting oscillators have revealed numerous significant results (Kuramoto 1984). From the standpoint of neuroscience, however, there is still an enormous need for theoretical studies addressing stimulation-induced transient synchronization and desynchronization processes. The impact of stimuli on synchronized neuronal oscillators is of great importance for the study of brain functioning (Steriade, Jones, Llinás 1990, Singer, Gray 1995, Hari, Salmelin 1997) and of therapeutic stimulation techniques in neurology and neurosurgery (Benabid et al. 1991, Blond et al. 1992).

In Winfree's (1980) pioneering topological approach to phase resetting the influence of noise and synchronizing couplings was neglected. For this reason with methods from synergetics (Haken 1983) and statistical mechanics we, first, study patterns of synchrony emerging in the presence of noise (Kuramoto 1984, Tass 1999). Second, the impact of periodic and, in particular, single pulsatile stimuli on different synchronized states is investigated in detail (Tass 1999).

We encounter a variety of characteristic stimulation-induced dynamical features, for instance, different sorts of transient desynchronization and resynchronization processes. Moreover, data analysis tools are presented which enable us to apply the theory to design and evaluate stimulation experiments. Finally, we propose improvements to stimulation techniques as used by neurologists and neurosurgeons in the context of Parkinson's disease (Tass 1999).

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Detection of cerebral synchronization processes

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Synchronization of neuronal oscillatory activity is important under both physiological and pathological conditions. For instance, according to animal experiments synchronization appears to be a basic mechanism for coordinating different neuronal populations during complex tasks like visual pattern recognition (Singer, Gray 1995) and motor control (Roelfsema et al. 1997). By means of magnetoencephalography (MEG) and electroencephalography (EEG) the cerebral activity can noninvasively be measured in humans (Hämäläinen et al. 1993). To study synchronization processes of this kind in humans two data analysis tools (Tass et al. 1998, Tass 1999) were developed which are applied to MEG data or EEG data or directly to the cerebral current density obtained by suitable inverse methods (cf. Hämäläinen et al. 1993).

The first method makes it possible to detect $n : m$ phase synchronization in noisy non-stationary data (Tass et al. 1998). It is designed for detecting self-synchronization in terms of an adjustment of the rhythms of ongoing oscillations. To this end a sliding window analysis is performed, where one needs a window length corresponding to at least eight to ten periods of the oscillation. This method was developed based on the concept of phase synchronization of chaotic oscillators (Rosenblum, Pikovsky, Kurths 1996, Pikovsky, Rosenblum, Kurths 1996).

The second method aims at analyzing short-term synchronization processes which are induced by a stimulus (Tass 1999). Accordingly, it is designed to detect so-called stimulus-locked $n : m$ transients which are stimulus locked epochs displaying a stereotyped time course of the $n : m$ phase difference (with and without conduction delay) (Tass 1999). Its time resolution reaches down to the millisecond range, and, hence, this method can cope with short synchronous epochs which are important in the context of cerebral information processing. This data analysis tool originates from the theoretical approach to phase resetting and stimulation-induced synchronization and desynchronization in clusters of interacting oscillators which was presented in the first lecture.

The two methods provide us with complementary information concerning spontaneously emerging synchronization as well as stimulus-induced synchronous epochs. Analyzing the two types of synchronization in MEG and EEG data opens up new and promising possibilities for the study of brain functioning. Theory and physiological applications are presented in detail.

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