Theoretical and computational methods in dynamical systems and fractal geometry

Teoretične in računske metode v dinamičnih sistemih in fraktalni geometriji

Hotel PIRAMIDA, Maribor, Slovenia
7 April 2015 - 11 April 2015

Book of Abstracts

Knjiga povzetkov

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FAKULTETA ZA NARAVOSLOVJE IN MATEMATIKO •
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Invited Speakers

Povabljeni govorci

Prof. Dr. Colin Christopher

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Prof. Dr. Ewa Stróżyna

Prof. Dr. Joan Torregrosa

Dr. Domagoj Vlah

Prof. Dr. Donming Wang

Prof. Dr. Henryk Żołądek

Prof. Dr. Darko Žubrinić

Prof. Dr. Vesna Županović

SCHEDULE

URNIK

Tuesday, 7 April 2015	
17:30-19:00	**5 minutes introductory talks, general discussion**
19:00	**Dinner**

Wednesday, 8 April 2015	
09:00-09:10	**Opening**
Chairman	*Jaume Llibre*
09:10-10:00	**Marko Robnik**
10:00-10:50	**Radu Constantinescu**
10:50-11:20	*Tea & Coffee*
11:20-12:10	**Josef Diblík**
12:10-13:00	**Valery Gromak**
13:00-15:00	*Lunch*
Chairman	*Radu Constantinescu*
15:00-15:50	**Armengol Gasull**
15:50-16.40	**Siniša Slijepčević**
16:40-17:00	*Tea & Coffee*
17:00-17.50	**Christoph Lhotka**
17:50-18.40	**Natalia Maslova**
18:45	*Dinner*

Thursday, 9 April 2015	
Chairman	*Armengol Gasull*
09:00-09:50	**Jaume Llibre**
09:50-10:40	**Henryk Żołądek**
10:40-11:10	*Tea & Coffee*
11:10-12:00	**Colin Christopher**
12:00-12:50	**Donming Wang**
12:50-15:00	*Lunch*
Chairman	*Valery Gromak*
15:00-15:50	**Vladimir Gerdt**
15:50-16:40	**Joan Torregrosa**
16:40-17:00	*Tea & Coffee*
17:00-17:50	**Darko Žubrinić**
17:50-18:40	**Dana Constantinescu**
19:00	*Conference Dinner*

Friday, 10 April 2015	
Chairman	*Henryk Żołądek*
09:00-09:50	**Vesna Županović**
09:50-10:40	**Bojan Kuzma**
10:40-11:10	*Tea & Coffee*
11:10-12:00	**Agnieszka Malinowska**
12:00-12:50	**Ewa Stróżyna**
12:50-15:00	*Lunch*
Chairman	*Vladimir Gerdt*
15:00-15:50	**Aliaksandr Hryn**
15:50-16:40	**Maja Resman**
16:40-17:00	*Tea & Cofee*
17:00-17:50	**Brigita Ferčec**
17:50-18:40	**Valery Romanovski**
18:40	*Dinner*

Saturday, 11 April 2015	
Chairman	*Valery Romanovski*
09:00-09:30	**Maša Dukarić**
09:30-10:20	**Domagoj Vlah**
10:20-12:00	**Closing Discussion**

IMPORTANT LINKS

Everything about Faculty of Natural Sciences and Mathematics:

http://fnm.uni-mb.si/

Everything about University of Maribor:

http://www.um.si/

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Vse o Fakulteti za naravoslovje in matematiko:

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ABSTRACTS

POVZETKI

Integrability of Lotka Volterra equations

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The Lotka Volterra equations in two dimensions form one of the simplest of families of non-linear systems. However, their behaviour is still quite rich. We survey some results old and new on the integrability of these systems - in particular the existence of algebraic curves and the use of monodromy arguments. We also consider some comparable results for three dimensional Lotka Volterra systems.

Fractional dynamics. Applications to the study of some transport phenomena

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We present basic elements of fractional calculus and the way to use it in dynamical modeling. We derive the fractional transport equation and we analyze some examples coming from physics (heat transport in fusion plasma experiments [1], [2], [3]) and economics (dynamics of prices in markets with jumps, growth and inequality processes, volatility of financial markets [4], [5], [6]). We present a numerical method for solving 2D transport equation and we apply it for the study of specific transport equations which describe phenomena that occur in tokamaks [7].

Keywords: Fractional dynamical systems, fractional transport equation, tokamak

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1. A. V. Chechkin, V. Yu Gonchar, M. Szydlowski, *Fractional kinetics for relaxation and superdiffusion in a magnetic field*. Physics of Plasmas **9** (1) (2002). 78–88.

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Control and optimization techniques for ”jerk” type circuits

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The paper investigates a specific type of nonlinear dynamical systems represented by electronic circuits with nonlinear elements, known as Chua circuits [1]. A diode with a nonlinear intensity-voltage characteristic or other electronic elements are used as nonlinear elements, and, because of this nonlinearity, the circuit generates interesting stochastic signals. Such circuits become chaotic oscillators and they have important applications in communication technologies, biology, neurosciences, and in other fields. Despite the simplicity of the circuit, the system of nonlinear differential equations arising when the electric laws are written down is very rich in the dynamical states, with interesting transitions from chaos to regular dynamics. The most general form of the differential system which corresponds to chaotic circuits in the same class with Chua is:

$$\begin{aligned}\dot{x} &= a(y - f(x))\\ \dot{y} &= bx + cy - g(x,z)\\ \dot{z} &= mz + h(x,y)\end{aligned}$$

In fact, we will study not directly the system from before, but the only one equivalent differential equation of third order which can be obtained from the system. The equation belongs to the jerk type equations and seems to be very interesting in respect with the dynamics generated. We will consider the case when:

$$f(x) = thx; \;\; g(x,z) = 0; \;\; h(x,y) = 0.$$

The interest will be given to the problem of controlling the chaotic behavior, in the sense of synchronization of the irregular and complex dynamics of the circuit with that of a coupled system which present periodic orbits or steady states. The main results which will be reported will concern the optimization of the dynamics using a quadratic control term. Other interesting results concern the possibility of attaching a Lagrangian function and transforming the equation in a variational one.

Keywords: chaos control, synchronization, Chua circuit.

References

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2. Mohammad Ali Khan, *J. Information and Computing Science Vol. 7*, No. **4** (2012), 272–283.

Increasing divergent solutions to certain systems of difference equations with delays

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We consider a homogeneous system of difference equations with deviating arguments in the form

$$\Delta y(n) = \sum_{k=1}^{q} \beta^k(n)[y(n-p_k) - y(n-r_k)]$$

where $n \geq n_0$, $n_0 \in \mathbb{Z}$, p_k, r_k are integers, $r_k > p_k \geq 0$, q is a positive integer, $y = (y_1, \ldots, y_s)^T$, $y\colon \{n_0 - r, n_0 - r + 1, \ldots\} \to \mathbb{R}^s$ is an unknown discrete vector function, $s \geq 1$ is an integer, $r = \max\{r_1, \ldots, r_q\}$, $\Delta y(n) = y(n+1) - y(n)$, and $\beta^k(n) = (\beta^k_{ij}(n))^s_{i,j=1}$ are real matrices such that $\beta^k_{ij}\colon \{n_0, n_0+1, \ldots\} \to [0, \infty)$, and $\sum_{k=1}^{q} \sum_{j=1}^{s} \beta^k_{ij}(n) > 0$ for each admissible i and all $n \geq n_0$. Discussed is the behavior of monotone solutions of this system for $n \to \infty$. The existence of solutions in an exponential form is proved and estimates of solutions are given. Sufficient conditions for the existence of unbounded monotone solutions are determined. The scalar case is discussed as well.

Local integrability and linearizability of a 3-dimensional quadratic system

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We study integrability and linearizability of three dimensional system of the form

$$\begin{aligned}\dot{x} &= \quad x + a_{12}xy + a_{13}xz + a_{23}yz\\ \dot{y} &= \quad -y + b_{12}xy + b_{13}xz + b_{23}yz\\ \dot{z} &= \quad -z + c_{12}xy + c_{13}xz + c_{23}yz.\end{aligned}$$

Necessary and sufficient conditions for existence of two functionally independent analytic integrals of this system were obtained. For the proof of integrability and linearizability the method of Darboux and the normal form theory were used. Some Darboux factors used for linearizability are obtained from first integrals of systems. The problem of existence of only one analytic first integral was investigated as well.

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3. Z.Hu, M. Han, V.G. Romanovski, *Local integrability of a family of three-dimensional quadratic systems*, Physica D: Nonlinear Phenomena **265** (2013), 78-86.

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Integrability of complex planar systems with homogeneous nonlinearities

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The problem of integrability of systems of differential equations is one of central problems in the theory of ODE's. Although integrability is a rare phenomena and a generic system is not integrable, integrable systems are important in studying various mathematical models, since often perturbations of integrable systems exhibit rich picture of bifurcations.
In this talk we discuss conditions for the existence of a local analytic first integral for a family of quintic systems having homogeneous nonlinearities studied in [1], i.e.

$$\begin{aligned} \dot{x} &= x - a_{40}x^5 - a_{31}x^4y - a_{22}x^3y^2 - a_{13}x^2y^3 - a_{04}xy^4, \\ \dot{y} &= -y + b_{5,-1}x^5 + b_{40}x^4y + b_{31}x^3y^2 + b_{22}x^2y^3 + b_{13}xy^4 + b_{04}y^5, \end{aligned} \tag{1}$$

where x, y, a_{jk}, b_{kj} are complex variables.

One of important mechanisms for integrability is the so-called time-reversibility (or just reversibility). We will describe an approach to find reversible systems within polynomial families of Lotka-Volterra systems with homogeneous nonlinearities.

References

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Limit cycles for 3-monomial differential equations

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We study planar polynomial differential equations that in complex coordinates write as $z' = Az + Bz^k\bar{z}^l + Cz^m\bar{z}^n$. We prove that for each natural number p there are differential equations of this type having at least p limit cycles. Moreover, for the particular case $z' = Az + B\bar{z} + Cz^m\bar{z}^n$, which has homogeneous nonlinearities, we show examples with several limit cycles and give a condition that ensures uniqueness and hyperbolicity of the limit cycle. The talk is based on a joint work with Chengzhi Li and Joan Torregrosa.

Hidden Lagrangian Constraints and Differential Thomas Decomposition

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Models with singular Lagrangians play a fundamental role in quantum mechanics, quantum field theory and elementary particle physics. Singularity of such models is caused by local symmetries of their Lagrangians. Gauge symmetry is the most important type of local symmetries and it is imperative for all physical theories of fundamental interactions. The local symmetry transformations of a dynamical (resp. field-theoretical) differential equation relate its solutions satisfying the same initial (Cauchy) data. For dynamical systems with only one independent variable the initial data include (generalized) coordinates and velocities whereas for field-theoretical models they include the field variables, their spatial and the first-order temporal derivatives ('velocities'). The presence of local symmetries in a singular model implies that its general solution satisfying the initial data depends on arbitrary functions.

A distinctive feature of singular Lagrangian models is that their dynamics is governed by the Euler-Lagrange equations which have differential consequences in the form of (hidden) constraints for the initial data. This is in contrast to regular constrained dynamics whose constraints are external with respect to the Euler-Lagrange equations.

Given a model Lagrangian, it is very important to verify whether it is singular, and if so to compute the hidden constraints that follow from the Euler-Lagrange equations. Knowledge of constraints is necessary for the local symmetry analysis, for well-posedness of initial value problems and for quantization of the model.

In the present talk we consider Lagrangian models whose Lagrangians (mechanics) and Lagrangian densities (field theory) are differential polynomials. Under this condition we show that the differential Thomas decomposition, being a characteristic one for the radical differential ideal generated by the polynomials in Euler-Lagrange equations, provides an algorithmic tool for verification of singularity and for computation of hidden Lagrangian constraints. Unlike the traditional linear algebra based methodology used in theoretical and mathematical physics for computation of linearly independent hidden Lagrangian constraints, our approach takes into account rank dependence of the Hessian matrix on the dynamical (field) variables and outputs the complete set of algebraically independent constraints.

Some approaches to construction of hierarchies of Painleve type equations

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The six Painlev'e equations were first discovered in a classification problem of nonlinear ordinary differential equations. Although Painlev'e equations were first discovered from strictly mathematical considerations, now they have arisen in a variety of important physical applications. They possess hierarchies of rational solutions and one-parameter families of solutions expressible in terms of the classical special functions, for special values of the parameters. Further the Painlev'e equations admit symmetries under affine Weyl groups which are related to the associated Backlund transformations. In the general case the Painlev'e transcendent may be thought of a nonlinear analogues of the classical special functions. We discuss different methods for obtaining of hierarchies of differential equations that are generalizations of the Painlev'e equations, such as Painlev'e method of small parameter, methods of nonlinear chains and symmetry reduction of some soliton equations, methods of isomonodromic deformation of linear systems. In particular, we consider Schlesinger and Garnier equations which are generalization of the Painlev'e equations and some their solutions.

On the estimation of limit cycles number for some planar autonomous systems

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The talk is devoted to the investigation of limit cycles for planar autonomous systems depending on the real parameter $a \in J \subseteq R$

$$\frac{dx}{dt} \equiv P(x,y) = y, \quad \frac{dy}{dt} \equiv Q(x,y,a) = \sum_{j=0}^{l} h_j(x,a)y^j, \tag{2}$$

$l \geq 1$, in some region $\Omega = \{(x,y) : x \in I \subseteq R, y \in R\}$, under the assumption that the functions $h_j : I \times J \to R$ are continuous in the first variable and continuously differentiable in the second variable.

Our purposes are to derive precise global upper bounds for the number of limit cycles of (1) and to localize their position as well as to construct systems (1) with prescribed number of limit cycles. It means that mentioned estimations hold in the whole region Ω for all $a \in J$.

The main tool for our investigations is Dulac-Cherkas function $\Psi(x,y,a)$ satisfying the inequality

$$\Phi \equiv k\Psi div f + \frac{\partial \Psi}{\partial x}P + \frac{\partial \Psi}{\partial y}Q > 0 (< 0),\ \forall (x,y) \in \Omega,\ f = (P,Q) \tag{3}$$

where $0 \neq k \in R$.

The talk present algorithms for the construction of Dulac-Cherkas functions in the form $\Psi(x,y,a) = \sum_{i=0}^{n} \Psi_i(x,a)y^i$, $n \geq 1$, under the assumption that the functions $\Psi_i : I \times J \to R$ are continuously differentiable in both variables. These algorithms use analytical and numerical approaches. Their applications are demonstrated for some classes of system (1) in the cases $l = 3$ and $l = 5$ such as generalized Kukles systems [1] and pendulum systems.

References

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Applications of commuting graphs

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A commuting graph of an algebra A is a simple graph whose vertex set consists of all noncentral elements from A and where two disjoint vertices are connected if the corresponding elements in A commute. We will discuss some problems related to commuting graphs and review its role in a recent classification of surjective maps which preserve commutativity on n-by-n complex matrices.

The use of dissipative normal forms and averaging methods in celestial dynamics

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Weakly dissipative, nearly integrable dynamical systems are at the core of celestial dynamics. In this talk we outline two stability theorems in these systems based on normal form theory. The talk includes real world applications to orbital and rotational dynamics: motion of dust and rotation of celestial bodies close to resonance and subject to non-gravitational forces.

On the equilibrium points of an analytic differentiable system in the plane. The center–focus problem and the divergence

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We shall recall briefly how can be the local phase portraits of the equilibrium points of an analytic differential system in the plane, and we shall put our attention in the center-focus problem, i.e. how to distinguish a center from a focus. This is a difficult problem which is not completely solved. We shall provide some new results using the divergence of the differential system.

Krause's model of opinion dynamics on time scales

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We analyse bounded confidence models on time scales. In such models each agent takes into account only the assessments of the agents whose opinions are not too far away from his own opinion. We prove a convergence into clusters of agents, with all agents in the same cluster having the same opinion. The necessary condition for reaching a consensus is given. Simulations are performed to validate the theoretical results.

Finite simple groups that are not spectrum critical

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The spectrum of a finite group G is the set $\omega(G)$ of all element orders of G. A finite group G is $\omega(G)$-critical (or spectrum critical) if for any subgroups K and L of G such that K is a normal subgroup of L, the equality $\omega(L/K) = \omega(G)$ implies $L = G$ and $K = 1$. In [1] the definition of $\omega(G)$-critical group was introduced and the following question was formulated: If G is a finite simple group not isomorphic to $P\Omega_8^+(2)$ or $P\Omega_8^+(3)$ then G is $\omega(G)$-critical, isn't it? We have obtained the negative answer to this question. Moreover, we have proved the following theorem.
Theorem. Let G be a finite simple group and K and L be subgroups of G such that K is a normal subgroup of L. Then $\omega(L/K) = \omega(G)$ if and only if $K = 1$ and one of the following conditions holds:

(1) G is $PSp_4(q)$ and L is $PSL_2(q^2) < t >$ where t is a field automorphism of order 2 of $SL_2(q^2)$;

(2) G is $PSp_8(q)$ and L is $SO_8^-(q)$ where q is even;

(3) G is $P\Omega_8^+(2)$ and L is $P_7(2)$;

(4) G is $P\Omega_8^+(3)$ and L is $P_7(3)$.

References

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Classifications of parabolic germs and epsilon-neighborhoods of orbits

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We consider analytic germs of parabolic diffeomorphisms $f : (C, 0) \to (C, 0)$. The question is if we could recognize a germ using the functions of the (directed) areas of the epsilon-neighborhoods of its orbits. We show that the formal class can be read from only finitely many terms in the asymptotic expansion of the (directed) area function in epsilon. We further discuss analytic properties of this function. We concentrate on the coefficient of the quadratic term in the expansion, as a function of the initial point. It satisfies a cohomological equation similar to the trivialisation equation.

Statistical properties of one-dimensional time-dependent Hamiltonian oscillators

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Recently the interest in time-dependent dynamical systems has increased a lot. In this talk I shall present most recent results on time-dependent one-dimensional Hamiltonian oscillators. The time-dependence describes the interaction of an oscillator with its neigbourhood. While the Liouville theorem still applies (the phase space volume is preserved), the energy of the system changes with time. We are interested in the statistical properties of the energy of an initial microcanonical ensemble with sharply defiend initial energy, but uniform distribution of the initial conditions with respect to the canonical angle. We are in particular interested in the change of the action at the average energy, which is also adiabatic invariant, and is conserved in the ideal adibatic limit, but otherwise changes with time. It will be shown that in the linear oscillator the value of the adiabatic invariant always increases, implying the increase of the Gibbs entropy in the mean (at the average energy). The energy is universally described by the arcsine distribution, independent of the driving law. In nonlinear oscillators things are different. For slow but not yet ideal adiabatic drivings the adiabatic invariant at the mean energy can decrease, just due to the nonlinearity and nonisochronicity, but nevertheless increases at faster drivings, including the limiting fastest possible driving, namely parametric kick (jump of the parameter). This is so-called PR property, following Papamikos and Robnik *J. Phys. A: Math. Theor.* **44** *(2011) 315102*, proven rigorously to be satisfied in a number of model potentials, such as homogeneous power law potential, and many others, giving evidence that the PR property is always sastified in a parametric kick, except if we are too close to a separatrix or if the potential is not smooth enough. The local analysis is possible and the PR property is formulated in terms of a geometrical criterion for the underlying potential. We also study the periodic kicking and the strong (nonadiabatic) linear driving of the quartic oscillator. In the latter case we employ the nonlinear WKB method following Papamikos and Robnik *J. Phys. A: Math. Theor.* **45** *(2012) 015206* and calculate the mean energy and the variance of the energy distribution, and also the adiabatic invariant which is asymptotically constant, but slightly higher than its initial value. The key references for the most recent work are Andresas et al (2014), given below.

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Integrability of polynomial systems of ODEs

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The problem of finding systems with one or few independent first integrals inside families of polynomial systems of ODEs depending on parameters is considered. Computational approaches for computing necessary conditions of integrability and invariant surfaces are proposed. Interconnection of time-reversibility and integrability is discussed and algorithms for finding time-reversible systems inside of parametric polynomial families are described.

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Description of two-dimensional attractors of some dissipative infinite-dimensional dynamical systems

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We consider a general class of infinite-dimensional dynamical systems, including dissipative dynamics of Frenkel-Kontorova models (one-dimensional coupled infinite chains in a periodic potential), as well as scalar reaction diffusion equations on infinite domains. We prove that the attractor of these systems is at most 2 dimensional, by introducing a new, topological Lyapunov function on the phase space. In the examples we numerically show that the fractal dimension of the attractor in many cases seems to be between 1 and 2.

We use the description of the attractor to give rigorous characterization of dynamical (Aubry) phase transition for the dynamics, depending on e.g. forcing parameter of the system. We distinguish two phases, following the solid state physics terminology: the pinned and depinned phase, and show that the attractor in the depinned phase consists of a single limit cycle.

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Normal forms for germs of vector fields with quadratic leading part. The polynomial first integral case

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The investigated problem is to find a formal classification of the vector fields of the form $\dot{x} = ax^2 + bxy + cy^2 + \ldots, \dot{y} = dx^2 + exy + fy^2 + \ldots$ using formal changes of coordinates, but not using the change of time. We consider the first case - with the polynomial first integral. In the proofs we avoid complicated calculations. The method we use is effective and it is based on the method presented in our previous work with H. Żołądek, where the case of Bogdanov-Takens singularity was studied. We consider homological operators, analogues of ad_V, acting on transversal and tangential parts of a vector field. The kernels and cokernels of those operators is used in the several cases which appear here. We provide the final list of non-orbital normal forms in the considered case.

Center, weak-focus and cyclicity problems for planar systems

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The center-focus problem consists in distinguishing whether a monodromic singular point is a center or a focus. For singular points with imaginary eigenvalues, usually called *nondegenerate singular points*, this problem was already solved by Poincaré and Lyapunov, see [3]. The solution consists in computing several quantities called commonly the *Poincaré–Lyapunov constants*, and study whether they are zero or not.

Despite the existence of many methods, the solution of the center-focus problem for simple families, like for instance the complete cubic systems or the quartic systems with homogeneous nonlinearities, has resisted all the attempts. For this reason, we propose to push on this question in another direction. We study this problem for a natural family of differential systems with few free parameters but arbitrary degree. We consider planar systems with a linear center at the origin that in complex coordinates the nonlinearity terms are formed by the sum of few monomials. For some families in this class, we study the center problem, the maximum order of a weak-focus and the cyclicity problem. Several centers inside this family are done. The list includes a new class of Darboux centers that are also persistent centers. We study if the given list is exhaustive or not.

For small degrees we provide explicit systems with weak foci or high-order centers that, after perturbation, give new lower bounds for the number of limit cycles surrounding a single critical point. These lower bounds are higher than the corresponding Hilbert number known until now for these degrees.

The talk will be a review of the results [1,2].

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Fractal analysis of oscillatory solutions of a class of ordinary differential equations including the Bessel equation

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In this talk we investigate oscillatority of functions using the fractal dimension. We apply this approach to some common objects of interest in that subject. These objects that we investigate, are chirp-like functions, Bessel functions, Fresnel oscillatory integrals and some generalizations.

We first, from the point of view of fractal geometry, study oscillatority of a class of real C^1 functions $x = x(t)$ near $t = \infty$. A fractal oscillatority of solutions of second-order differential equations near infinity is measured by *oscillatory* and *phase dimensions*, defined as box dimensions of the graph of $X(\tau) = x(\frac{1}{\tau})$ near $\tau = 0$ and trajectory $(x, \dot{x})$ in $\mathbb{R}^2$, respectively, assuming that $(x, \dot{x})$ is a spiral converging to the origin. The box dimension of a plane curve measures the accumulation of a curve near a point, which is in particular interesting for non-rectifiable curves. The phase dimension has been calculated for a class of this oscillatory functions using formulas for box dimension of a class of nonrectifiable spirals. Also, the case of rectifiable spirals have been studied. A specific type of spirals that we called *wavy spirals*, converging to the origin, but with an increasing radius function in some parts, emerged in our study of phase portraits.

We further study the phase dimension of a class of second-order nonautonomous differential equations with oscillatory solutions including the Bessel equation. We prove that the phase dimension of Bessel functions is equal to $\frac{4}{3}$, and that the corresponding trajectory is a wavy spiral, exhibiting an interesting behavior. The phase dimension of that specific generalization of the Bessel equation has been also computed.

Then we study some other class of second-order nonautonomous differential equations, and the corresponding planar and spatial systems, again from the point of view of fractal geometry. Using the phase dimension of a solution of the second-order equation we compute the box dimension of a spiral trajectory of the corresponding spatial system, lying in Lipschitzian or Hölderian surfaces. This phase dimension of the second-order equation is connected to the asymptotics of the associated Poincaré map.

Finally, we obtain a new asymptotic expansion of generalized Fresnel integrals $x(t) = \int_0^t \cos q(s)\, ds$ for large t, where $q(s) \sim s^p$ when $s \to \infty$, and $p > 1$. The terms of the expansion are defined via a simple iterative algorithm. Using this we show that the box dimension of the related q-clothoid, also called the generalized Euler or Cornu spiral, is equal to $d = 2p/(2p-1)$. This generalized Euler spiral is defined by generalized Fresnel integrals, as component functions, where $x(t)$ is as before and $y(t) = \int_0^t \sin q(s)\, ds$. Furthermore, this curve is Minkowski measurable, and we compute its d-dimensional Minkowski content.

Algebraic Computation and Qualitative Analysis of Dynamical Systems

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In this talk, we provide a brief review of algebraic methods based on resultants, triangular sets, Groebner bases, quantifier elimination, and real solution classification and discuss their applications to the analysis of stability and bifurcations of dynamical systems. Examples of biological dynamical systems are given to illustrate the advantages of the presented symbolic computational approach.

The case CD45 revisited

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In my paper "Eleven small limit cycles in a cubic vector field" (Nonlinearity 8) the existence of eleven small amplitude limit cycles in a perturbation of some special cubic plane vector field with center was proved. I will present a new and corrected proof of that result.

Lapidus zeta functions and their applications

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The theory of 'zeta functions of fractal strings' has been initiated by the first author in the early 1990s, and developed jointly with his collaborators during almost two decades of intensive research in numerous articles and several monographs. In 2009, the same author introduced a new class of zeta functions, called 'distance zeta functions', which since then, has enabled us to extend the existing theory of zeta functions of fractal strings and sprays to arbitrary bounded (fractal) sets in Euclidean spaces of any dimension. A natural and closely related tool for the study of distance zeta functions is the class of 'tube zeta functions', defined using the tube function of a fractal set. These three classes of zeta functions, under the name of 'fractal zeta functions', exhibit deep connections with Minkowski contents and upper box dimensions, as well as, more generally, with the complex dimensions of fractal sets. Further extensions include zeta functions of relative fractal drums, the box dimension of which can assume negative values, including minus infinity.

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Fractal analysis of bifurcations of dynamical systems

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In this talk I would like to give an overview of results concerning fractal analysis of dynamical systems, obtained by scientific group at University of Zagreb and our collaborators. Bifurcations of limit cycles are related to the 16th Hilbert problem. It asks for an upper bound or the number of limit cycles, of polynomial vector fields in the plane, as a function of the degree of the vector field. The problem is still open. It is of special interest to determine how many limit cycles can bifurcate from a given limit periodic set in a generic unfolding. This number is called the cyclicity of the limit periodic set. The cyclicity is classically obtained by studying the multiplicity of fixed points of Poincaré map. We establish a relation between cyclicity of a limit periodic set of a planar system and fractal properties of the Poincaré map of a trajectory of the system. A natural idea is that higher density of orbits reveals higher cyclicity. The study of density of orbits is where fractal analysis is applied. Classical fractal analysis associates box dimension and Minkowski content to bounded sets. They measure the density of accumulation of a set, see [10].

In the paper [11], the cyclicity of weak foci and limit cycles is directly related to the box dimension of any trajectory. It was discovered that the box dimension of a spiral trajectory of weak focus signals a moment of Hopf and Hopf-Takens bifurcation. The result was obtained using Takens normal form. In [12], box dimension of spiral trajectories of weak focus was related to the box dimension of its Poincaré maps. Results were based on [2] and [3]. This article also shows that generic bifurcations of 1-dimensional discrete systems are characterised by the box dimension of orbits. Fractal analysis of Hopf bifurcation for discrete dynamical systems, called Neimark-Sacker bifurcation, has been completed in [4].

In the above continuous cases, the Poincaré map was differentiable, which was crucial for relating the box dimension and the cyclicity of a limit periodic set. The problem in hyperbolic polycycle case is that the Poincaré map in not differentiable, but the family of maps in generic bifurcations has an asymptotic development in a so-called Chebyshev scale. We introduced in [5] the appropriate generalizations of box dimension, depending on a particular scale for a given problem.

The cyclicity was concluded using the generalized box dimension in the case of saddle loops.

The box dimension has been read from the leading term of asymptotic expansion of area of ε-neighborhoods of orbits. If we go further into the asymptotic expansion we can make formal classification of parabolic diffeomorphisms using fractal data given in the expansion, see [7], and also [8].

Analogously it is possible to study singularities of maps, see [1]. We study geometrical representation of oscillatory integrals with analytic phase function and smooth amplitude with compact support. Geometrical and fractal properties of the curves defined by oscillatory integral depend on type of critical point of the phase. Methods in [9] include Newton diagrams and resolution of singularities.

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