Disorder-Tailored Delocalization

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We derive disorder fields tailored by the details of a choice of a delocalized wave function. We first investigate the unidirectional Hatano-Nelson chain and its localization properties under M-base diagonal disorder with variable weights. The spectrum forms loops in the complex plane and the loop parameter is a good quantum number similar to a momentum. All eigenstates are subexponentially 'localized', i.e. the logarithm of the absolute value of the wave function performs a random walk in space, and are characterized by a corresponding length scale ξ_{sel} as shown in 1998 by Silvestrov for the general Hatano-Nelson chain. For $M = 2$ real-valued binary disorder with equal weights the model was solved in [1], yielding Cassini oval spectral loops and a diverging subexponential localization length for two eigenstates and for disorder weaker than a critical value set by the hopping strength. When the disorder field for any M and arbitrary weights is confined to circles in the complex plane with radius equal to the hopping strength, the circle center will belong to the spectrum and to one of the spectral loops, and host a plane-wave-like eigenstate with diverging ξ_{sel}. We generalize to tailoring on-site disorder for a given eigenstate at a given energy, for any lattice dimension, and any hopping field (both ordered and disordered), for Hermitian and non-Hermitian systems. We exemplify by constructing a one-dimensional chain with Anderson-localized eigenstates hosting a completely delocalized one with random phases. The localization length diverges as $1/|E|^{2/3}$ upon approaching the delocalized state. Our method can be used for the systematic construction of disorder fields which host predefined eigenstates with arbitrary properties [2].

References

[1] Supriyo Ghosh and Sergej Flach, *Spectral topology and delocalization in disordered Hatano-Nelson chains*, Zeitschrift für Naturforschung A **81** 421 (2026)

[2] Yeongjun Kim, Supriyo Ghosh and Sergej Flach, *Disorder-tailored delocalization*, arXiv:2609.15271 (2026)