Can chaos be useful in quantum mechanics?

From quantum information to quantum chaos and back

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Summary

- Quantum information: basic facts
  - Quantum computer
  - Quantum teleportation
  - ‘No-go’ for quantum clonning
- Quantum chaos: two-slit experiment
- Parametric stability of quantum dynamical systems: The fidelity
- Theory of quantum fidelity
- Can it help in designing roboust quantum algorithms
  - Example: Improved quantum Fourier transform

Quantum information

“*Hilbert space is a big place.*” (Carlton Caves)

qubit: An abstract two-level quantum system

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

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$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

$n-$qubit register = coherent superposition of 2^n states

$$|\psi\rangle = \sum_{r=0}^{2^n-1} c_r|r\rangle.$$

Quantum computer: What is that?

A machine, which

- in a *finite* number of steps applies *selected unitary transformation* U - quantum algorithm - on an *arbitrary* $n-$qubit state

$$|\psi_{\rm f}\rangle = U|\psi_{\rm i}\rangle.$$

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- performs measurements of *arbitrary* qubit in a register.

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- Capability of a mesurement of arbitrary individual qubit state

Some promising candidate technologies

SYSTEM (QUBIT)	$\tau_Q\ (s)$	$\tau_{op}\ (s)$	τ_Q/τ_{op}
nuclear spin	$10^{-2} - 10^{8}$	$10^{-3} - 10^{-6}$	$10^{5} - 10^{14}$
electron spin	10^{-3}	10^{-7}	10^{4}
Ion trap (In^{+})	10^{-1}	10^{-14}	10^{13}
electron charge - Au	10^{-8}	10^{-14}	10^{6}
electron charge - GaAs	10^{-10}	10^{-13}	10^{3}
quantum dot	10^{-6}	10^{-9}	10^{3}
photon - optical resonator	10^{-5}	10^{-14}	10^{9}
photon - microwave resonator	10^{0}	10^{-4}	10^{4}

Universal set of quantum gates

1-qubit gates:

$$X = \begin{array}{c} \\ |0\rangle \\ |1\rangle \end{array} \begin{array}{c} \begin{array}{cc} |0\rangle & |1\rangle \end{array} \\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{array} \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad R_\varphi = \begin{pmatrix} e^{\mathrm{i}\varphi} & 0 \\ 0 & e^{-\mathrm{i}\varphi} \end{pmatrix}$$

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2-qubit gates:

$$CNOT = \begin{matrix} & |00\rangle & |01\rangle & |10\rangle & |11\rangle \\ |00\rangle & 1 & 0 & 0 & 0 \\ |01\rangle & 0 & 1 & 0 & 0 \\ |10\rangle & 0 & 0 & 0 & 1 \\ |11\rangle & 0 & 0 & 1 & 0 \end{matrix}$$

Quantum algorithms

$$U = U_T \cdots U_2 U_1$$

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Examples of *efficient* quantum algorithms:

- DFT (Deutsch 1994), $T = \mathcal{O}(n^2)$

$$U \left\{ \sum_{r=0}^{2^n-1} c_r |r\rangle \right\} = \sum_{r=0}^{2^n-1} \left\{ \frac{1}{2^{n/2}} \sum_{s=0}^{2^n-1} e^{2\pi i r s / 2^n} c_s \right\} |r\rangle$$

- Integer factorization (Schor 1994), $T = \mathcal{O}(n^2)$.
- Search in an unstructered list of 2^n elements (Grover 1996), $T = \mathcal{O}(2^{n/2})$.

EPR paradox

Entangled pair (EP) of two qubits at places A and B

$$\boxed{A \;\; \Uparrow\Downarrow} \qquad \cdots \qquad \boxed{B \;\; \Uparrow\Downarrow}$$

$$EP_{\mathrm{AB}} = \frac{1}{\sqrt{2}}(|0\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}} + |1\rangle_{\mathrm{A}}|1\rangle_{\mathrm{B}}) = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

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Nonlocality of quantum mechanics:
Measurement of a qubit at place A triggers instantaneous transition of a qubit B into an identical state (0 or 1).

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Can such EP be used as a resource to transport quantum information?

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Algorithm: **(1)** Alice applies $CNOT$ onto her qubits:

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(2) Alice then applies H on the first qubit:

$$\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle) =$$
$$|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) +$$
$$|10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)$$

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$$\begin{aligned} U\left\{|\psi\rangle_A \otimes |0\rangle_B\right\} &= |\psi\rangle_A \otimes |\psi\rangle_B \\ U\left\{|\phi\rangle_A \otimes |0\rangle_B\right\} &= |\phi\rangle_A \otimes |\phi\rangle_B \end{aligned}$$

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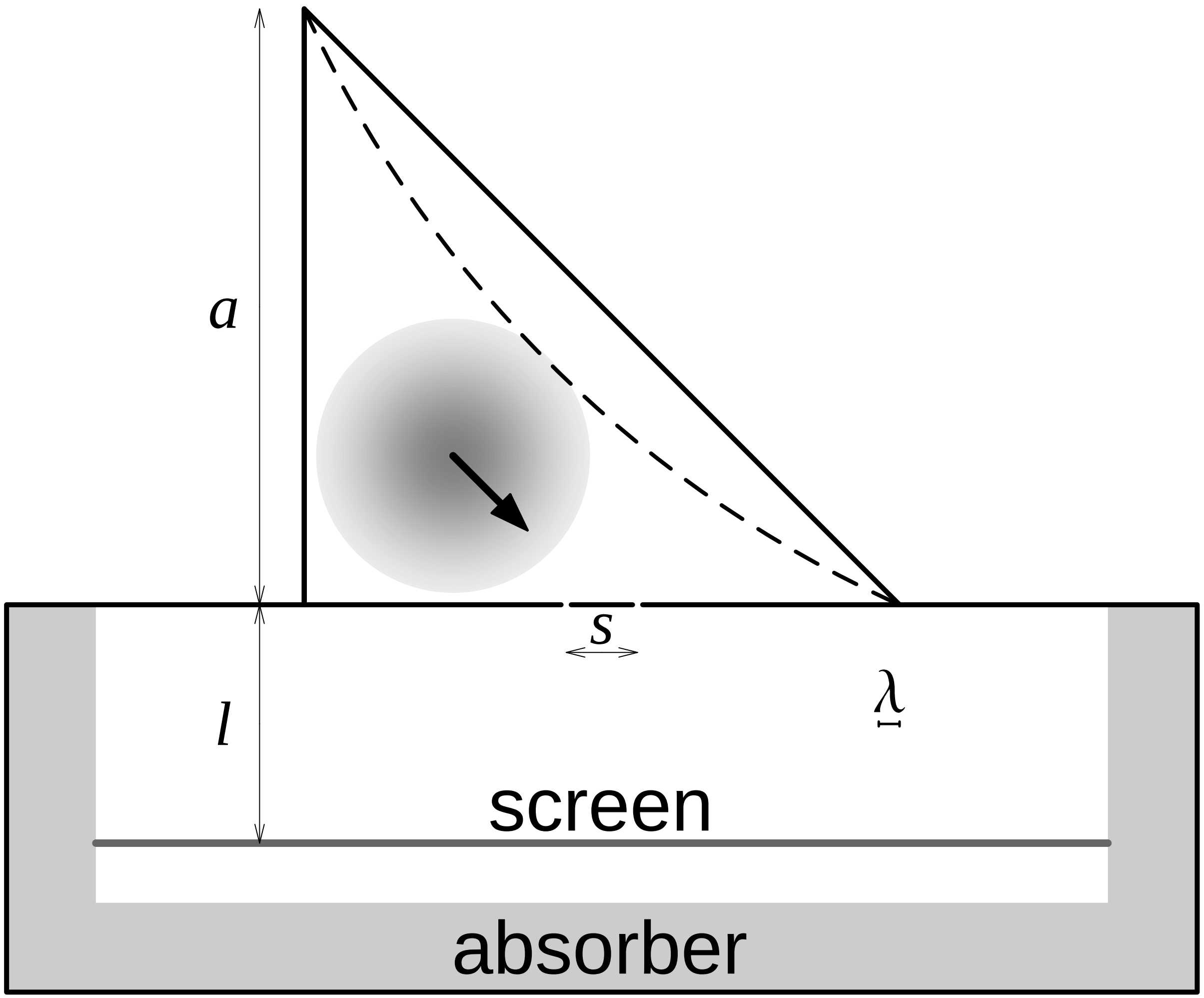
Perf. inner product $+$ *unitarity* of $U \Rightarrow \langle\psi|\phi\rangle = \langle\psi|\phi\rangle^2$

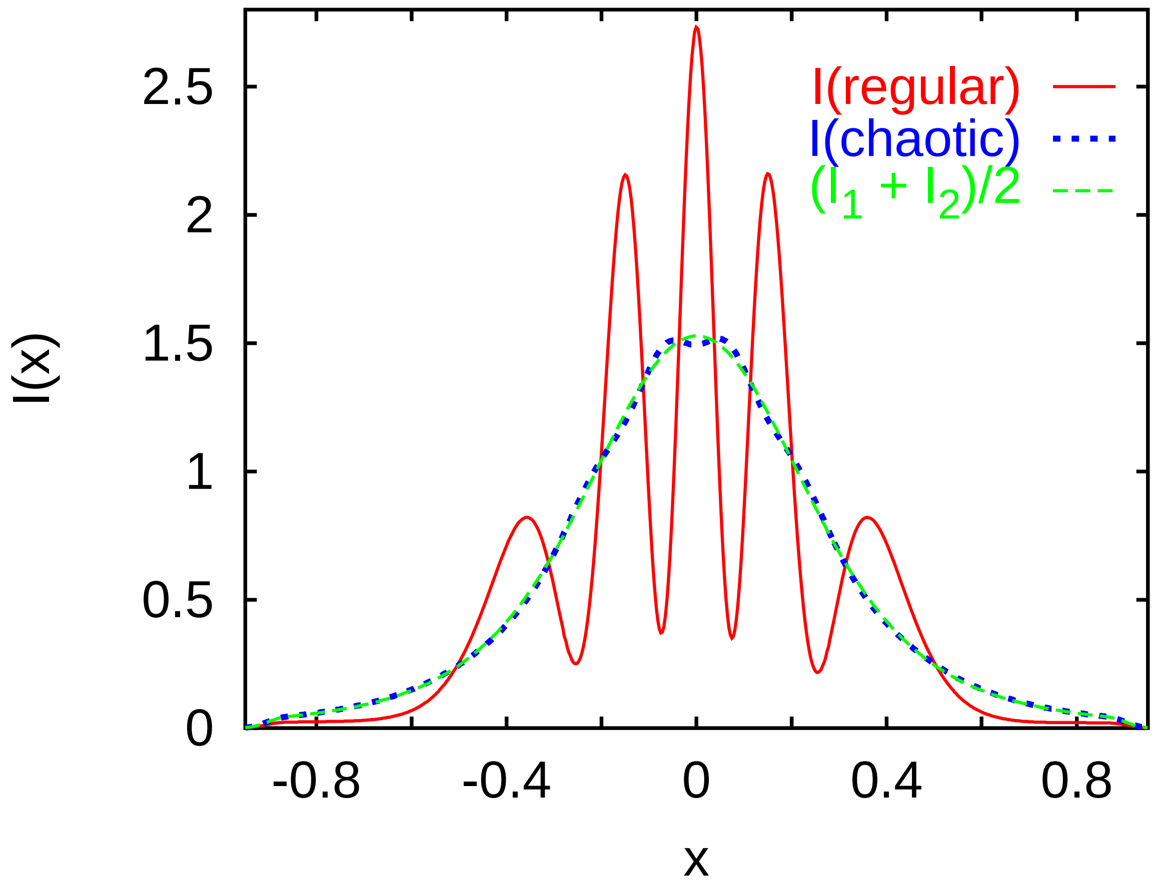
$\Rightarrow$ We can only copy mutually orthogonal states — equivalent to copying of classical informattion.

Quantum chaos: 2-slit experiment



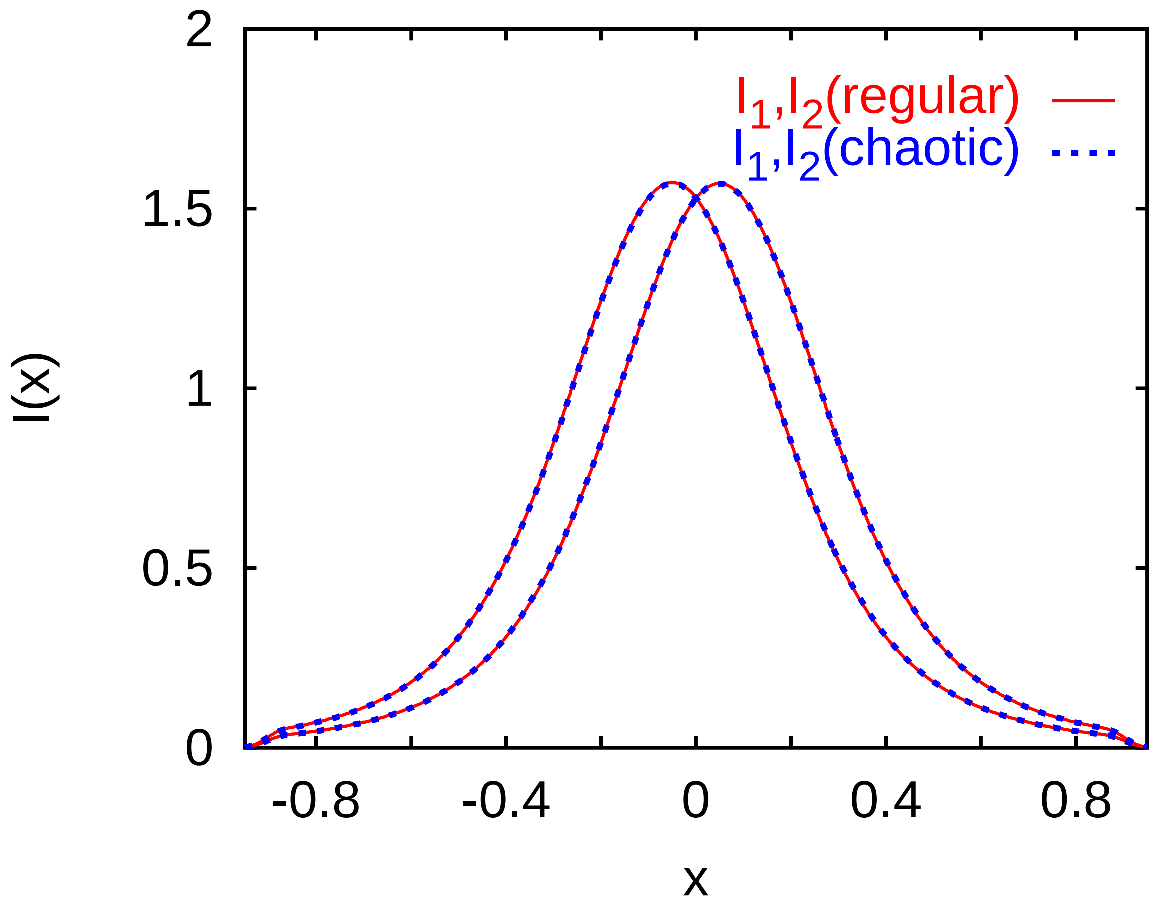
Quantum chaos: 2-slit experiment

Integrated probability current on the screen when both slits open:



Quantum chaos: 2-slit experiment

Integrated probability current on the screen when a single slit open:



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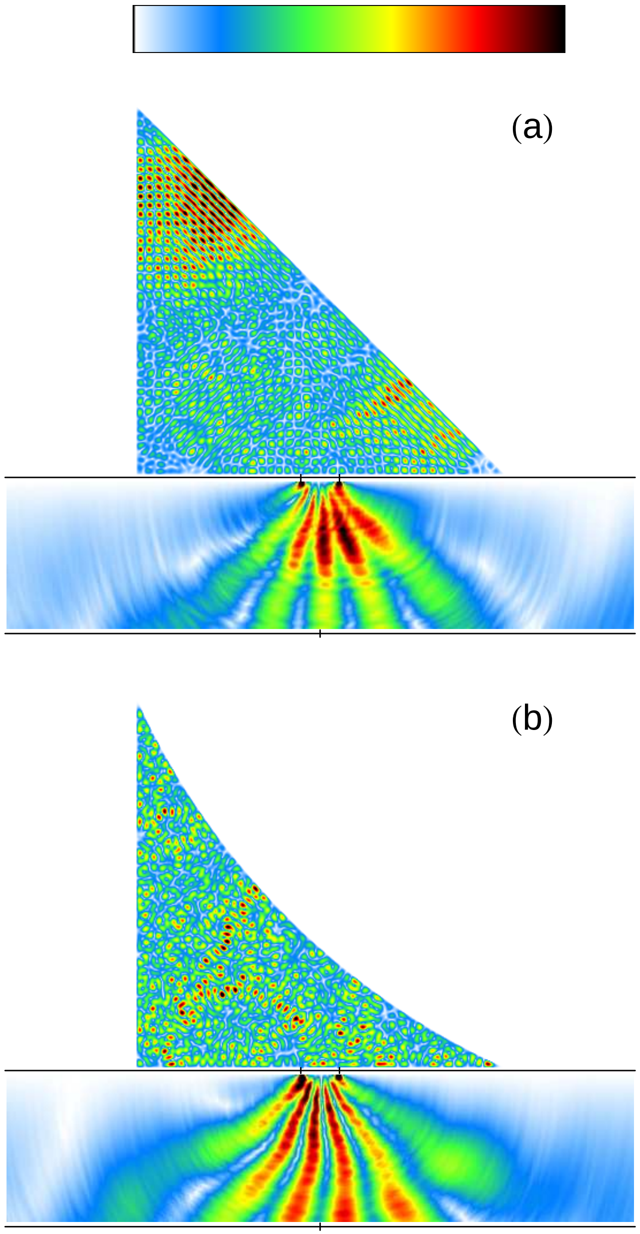

Quantum fidelity

Sensitivity of quantum dynamics/algorithm U with respect to perturbing Hamiltonian/quantum gates.

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\begin{aligned}
|\psi(t)\rangle &= U^t|\psi\rangle, \\
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U_\delta &= U \exp(-iV\delta/\hbar).
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Definition (Quantum fidelity):

$$F(t) = |\langle\psi_\delta(t)|\psi(t)\rangle|^2 = |\langle\psi|U_\delta^{-t}U^t|\psi\rangle|^2 = |\langle M_\delta(t)\rangle|^2$$

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is an expectation value of unitary echo operator

$$M_\delta(t) = U_\delta^{-t}U^t.$$

Linear response theory of quantum fidelity

Step 1 • Echo operator as an ordered product

Let $V_t := U^{-t} V U^{t}$, and $U_\delta^\dagger U = \exp(iV\delta/\hbar)$, and

$$
\begin{aligned}
M_\delta(t) &= U_\delta^{-t} U^{t} \\
&= U_\delta^{-(t-1)} U^{t-1} U^{-(t-1)} U_\delta^\dagger U U^{t-1} \\
&= M_\delta(t-1) \exp(iV_{t-1}\delta/\hbar) \\
&= M_\delta(t-2) \exp(iV_{t-2}\delta/\hbar) \exp(iV_{t-1}\delta/\hbar) \\
&\ldots \\
&= \exp(iV_0\delta/\hbar) \exp(iV_1\delta/\hbar) \cdots \exp(iV_{t-1}\delta/\hbar)
\end{aligned}
$$

Theory of quantum fideliy

Step 2 • power series expansion in δ

$$M_\delta(t) = \mathbb{1} + \sum_{m=1}^{\infty} \frac{i^m \delta^m}{m! \hbar^m} \hat{\mathcal{T}} \sum_{t_1, t_2 \ldots t_m = 0}^{t-1} V_{t_1} V_{t_2} \cdots V_{t_m}.$$

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Step 3 • Put $F(t) = |\langle\psi|M_\delta(t)|\psi\rangle|^2$ to obtain convergent $\delta-$expansion of quantum fidelity.

Linear response

To 2nd order, δ^2, quantum fidelity writes in terms of temporal auto-correlation function of the perturbation

$$
\begin{aligned}
F(t) &= 1 - \frac{\delta^2}{\hbar^2} \sum_{t',t''=0}^{t-1} C(t',t'') + \ldots \\
C(t',t'') &= \langle V_{t'} V_{t''} \rangle - \langle V_{t'} \rangle \langle V_{t''} \rangle
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A simple general rule:

The stronger decay of correlation,

the slower the decay of fidelity, and vice versa.

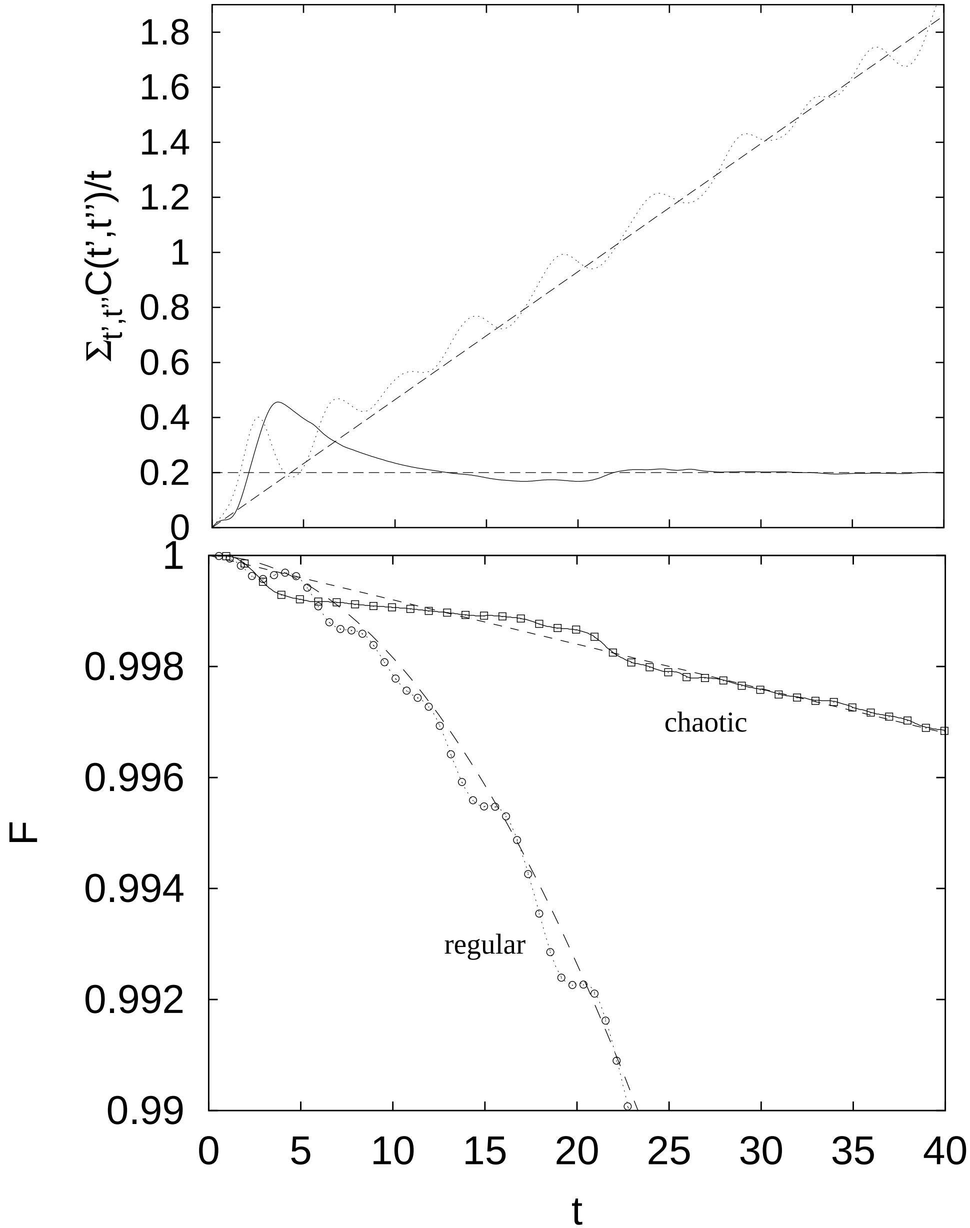
Experiment: JC model

A spin-J in one oscillator mode of EM field:

$$H = \hbar\omega a^\dagger a + \hbar\epsilon J_\mathrm{z} + \frac{\hbar}{\sqrt{2J}}\left(G(aJ_+ + a^\dagger J_-) + G'(aJ_- + a^\dagger J_+)\right)$$

Classical limit: $J \to \infty, \hbar \to 0, \hbar J = 1.$
Perturbation: ‘detuning’ $V = J_\mathrm{z}$.
Initial state: coherent state

$$|\psi\rangle = e^{\alpha a^\dagger - \alpha^* a}|0\rangle_2 \otimes (1 + |\tau|^2)^{-J} e^{\tau J_-}|0\rangle_1$$


1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
$\Sigma_{t',t''}C(t',t'')/t$
1
0.998
0.996
0.994
0.992
0.99
F
chaotic
regular
0
5
10
15
20
25
30
35
40
t

Beyond linear response: semiclassics

- Chaotic classical limit, arbitrary initial state:

$$\boxed{F_{\rm em}(t) = \exp(-t/\tau_{\rm em}), \qquad \tau_{\rm em} = \frac{\hbar^2}{\delta^2 \sigma}.}$$

$\sigma = \lim_{t\to\infty} \frac{1}{t} \sum_{t',t''=0}^{t-1} C(t',t'')$ is a transport coefficient.

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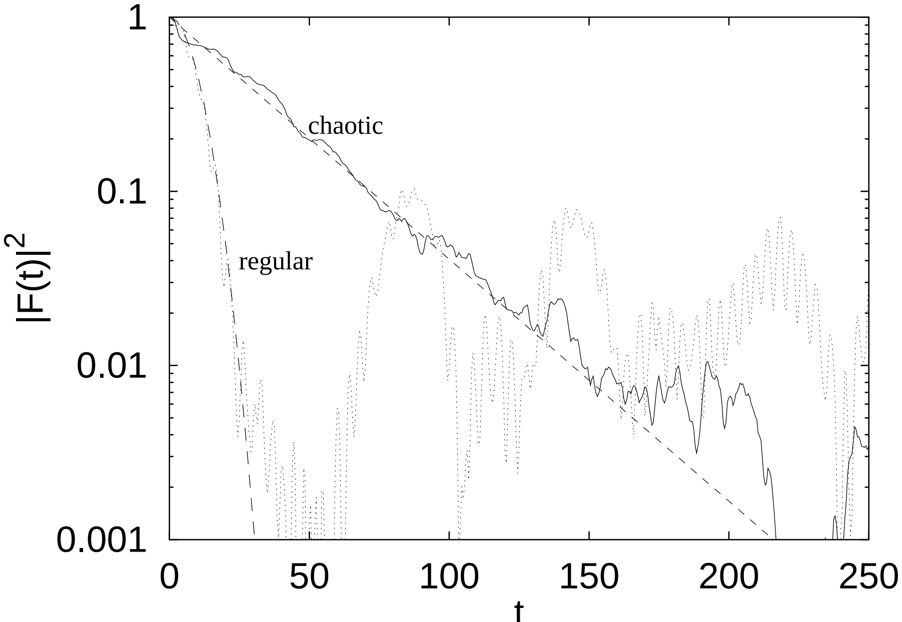
- Non-ergodic classical dynamics, coherent init.state:

$$F_{\rm ne}(t) = \exp\left(-t^2/{\tau_{\rm ne}}^2\right), \qquad \tau_{\rm ne} \sim \hbar^{1/2}\delta^{-1}.$$



Beyond linear response: semiclassics

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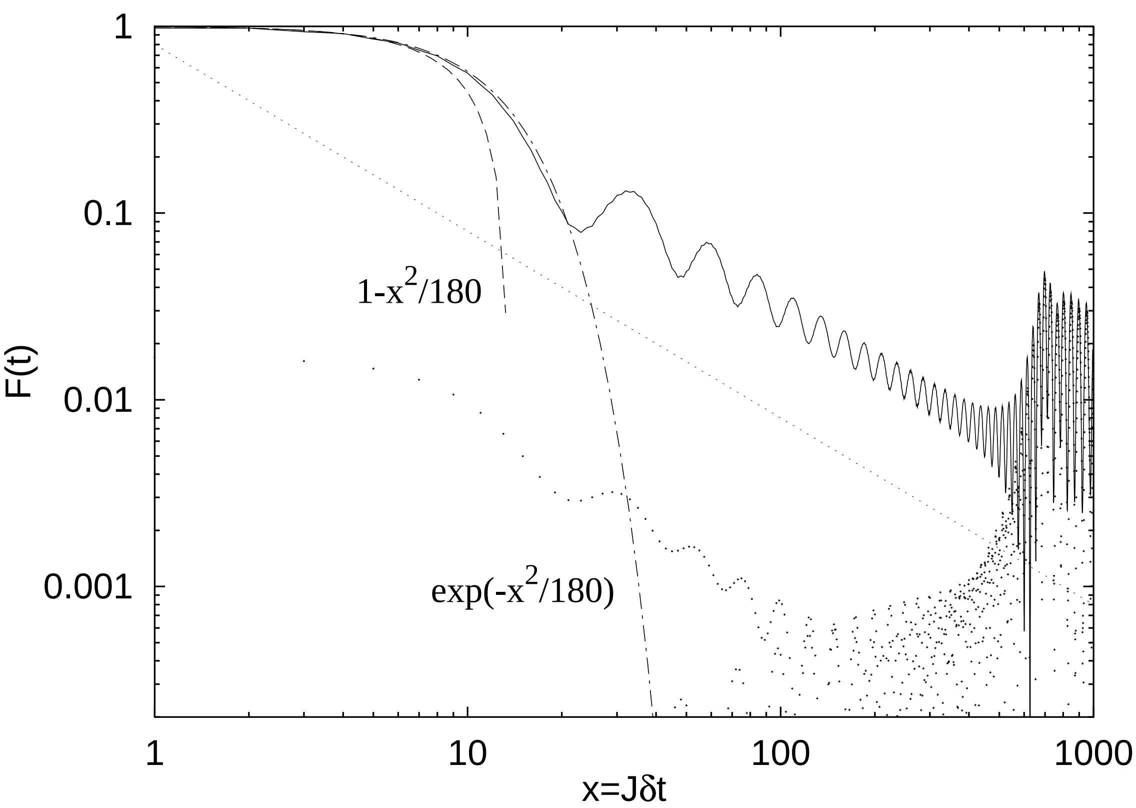

Partial conclusions

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Partial conclusions

- “Chaotic” quantm dynamics is more stable w.r.t. perturbations in the Hamiltonian than “regular”. “Chaoticity” in quantum dynamics is characterized in terms of decaying temporal correlations.
- The same conclusion *does not* hold in classical dynamics. The “paradox” is a consequence of non-commutativity of the limits $\delta \longrightarrow 0$ and $\hbar \longrightarrow 0$.
- Can this lesson be used for a design/optimization of quantum algorithms?

QA as a dynamical system

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Perturbation $V(t)$, $U_\delta(t) = U(t)\exp(-i\delta V(t))$.
Fidelity (linear reposnse):

$$F = 1 - \delta^2 \sum_{t,t'=1}^{T} C(t,t')$$

where $C(t,t') = \langle\psi|U(0,t)V(t)U(t,t')V(t')U(t',0)|\psi\rangle$ is temporal correlator of the generator of perturbation.

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The problem of optimization: Representation of unitary transformations in terms of a sequence of quantum gates $U(t)$ is *not* unique. We seek for the "most chaotic" quantum algorithm, which would minimize the correlation sum.

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Let us assume:

- Random initial state $|\psi\rangle$
- Random static perturbation $\langle V_{jk} V_{lm} \rangle = 2^{-n} \delta_{jm} \delta_{kl}$:

$$C(t,t') = \left| 2^{-n}\, \mathrm{tr}\, U(t,t') \right|^2 .$$

Quantum Fourier transformation

Write the matrix

$$U_{jk} = \frac{1}{\sqrt{N}} \exp(2\pi i jk/N),$$

$N = 2^n$, in terms of $T = n(n+1)/2$ 1-qubit and 2-qubit gates

$$\mathrm{A}_j = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}_j, \quad \mathrm{B}_{jk} = \mathrm{diag}\{1, 1, 1, e^{i\pi/2^{|k-j|}}\}_{jk}.$$

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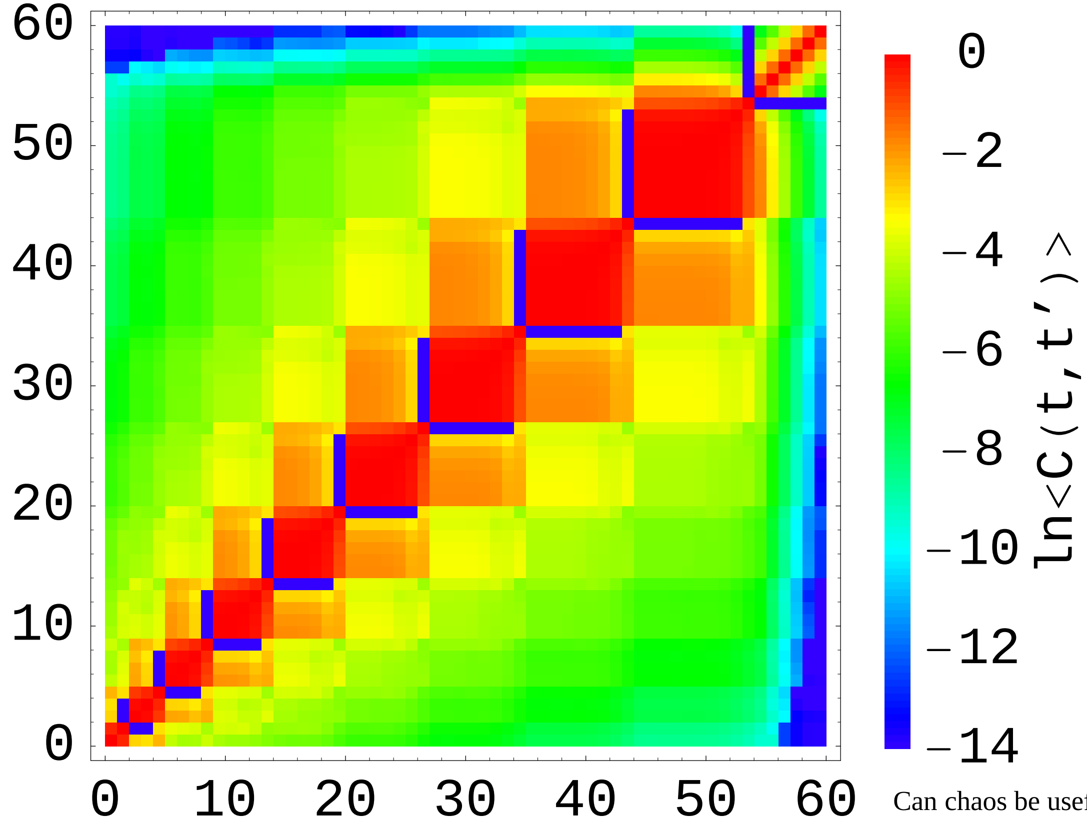
E.g., for $n = 4$:

$$U = \mathrm{T}_{03}\mathrm{T}_{12}\mathrm{A}_0\mathrm{B}_{01}\mathrm{B}_{02}\mathrm{B}_{03}\mathrm{A}_1\mathrm{B}_{12}\mathrm{B}_{13}\mathrm{A}_2\mathrm{B}_{23}\mathrm{A}_3.$$

The correlator

Blocks of B-gates result in long-tails of the correlator, and consequently, fast decay of fidelity, $\sum_{t,t'} C(t,t') \propto n^3$.

Example for $n = 10$:



Improved QFT

Replace almost diagonal B-gates in terms of a pair of new gaites

$$B_{jk} = R_{jk} G_{jk}.$$

Then, redistribute the gates which commute.

We now have $T \approx n^2$ elementary gates, e.g. for $n = 4$:

$$U = \mathrm{T}_{03}\mathrm{T}_{12}\mathrm{A}_0\mathrm{R}_{01}\mathrm{R}_{02}\mathrm{R}_{03}\mathrm{G}_{01}\mathrm{G}_{02}\mathrm{G}_{03}\mathrm{A}_1\mathrm{R}_{12}\mathrm{R}_{13}\mathrm{G}_{12}\mathrm{G}_{13}\mathrm{A}_2\mathrm{R}_{23}\mathrm{G}_{23}\mathrm{A}_3$$

The correlator

Improved QFT exhibits much faster decay of correlations, $\sum_{t,t'} C(t,t') \propto n^2$.
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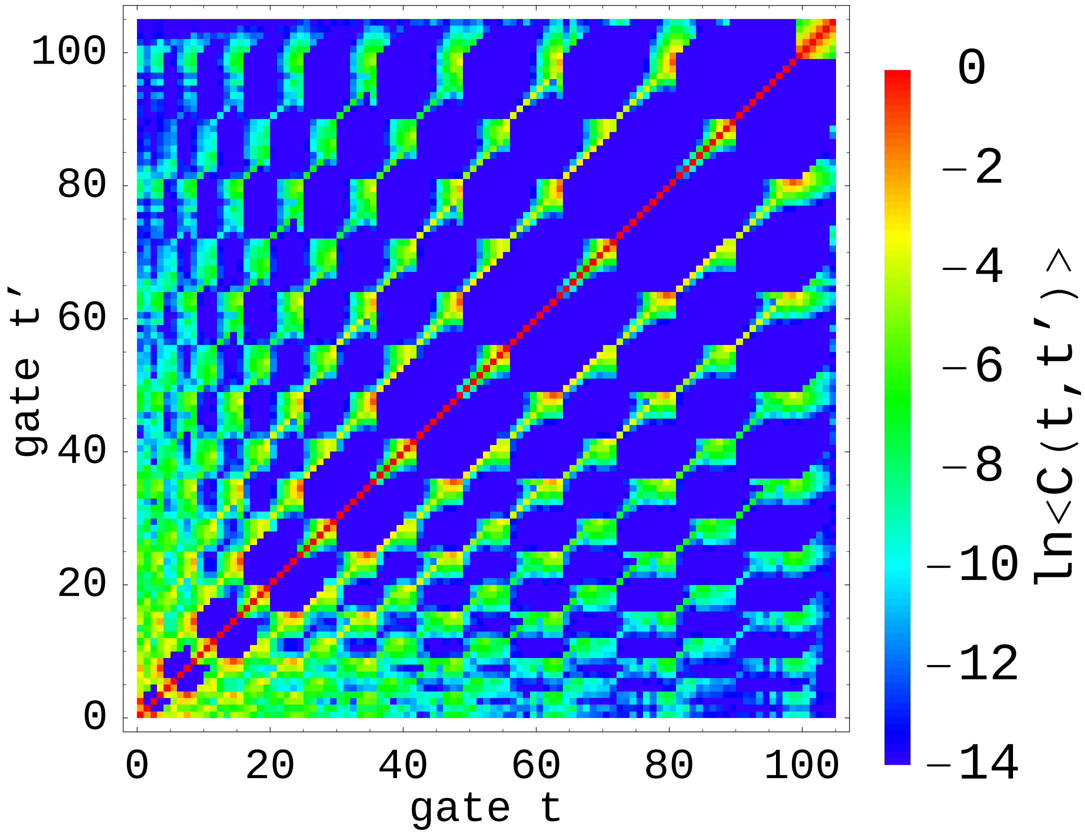
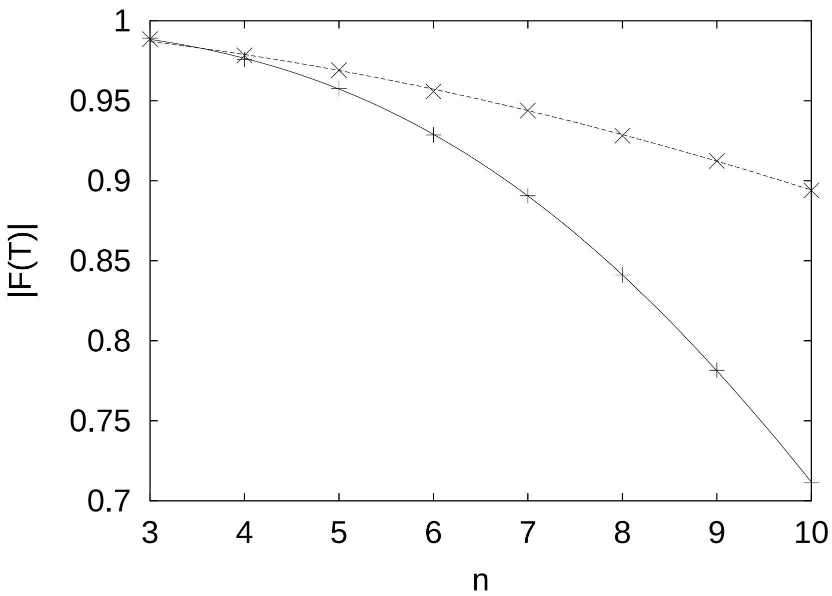
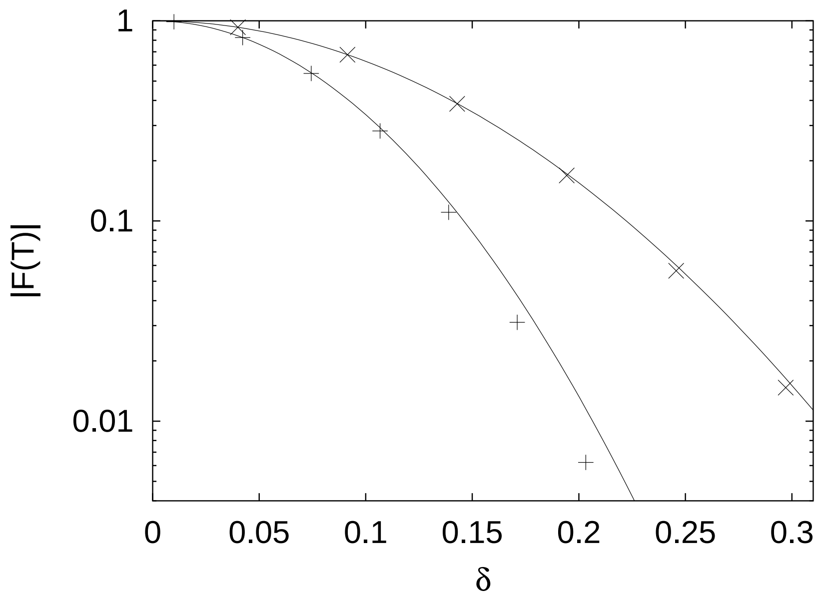


Improvement of quantum fidelity

Dependence on the number of qubits (for $\delta = 0.04$) and on the strength of perturbation (for $n = 8$):





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- We proposed, how this knowledge can be used in a design of robust quantum information processing.
- Alternative interpretation of quantum fidelity in terms of a *Loschmidt echo* helps in understanding dynamical origin of a macroscopic irreversibility.

Key references

General review on quantum information:

- M. A. Nielsen in I. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press 2000.

Key references

General review on quantum information:

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