Abstracts of Short Reports

Statistical properties of 1D time-dependent Hamilton systems

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I shall discuss the general theory of parametrically kicked systems, especially in nonlinear 1D Hamiltonian systems. I shall present the general Papamikos-Robnik (PR) conjecture for parametrically kicked Hamilton systems, which says that for such systems the adiabatic invariant (the action) for an initial microcanonical ensemble at the mean final energy always increases under a parametric kick. I shall also present many examples of the validity of the PR property, which is almost always satisfied, but can be broken in not sufficiently smooth potentials or in cases where we are in the energy range close to a separatrix in the phase space. The general conjecture, using analytical and numerical computations, is shown to hold true for important systems like homogeneous power law potentials, pendulum, Kepler system, Morse potential, Pöschl-Teller I and II potentials, cosh potential, quadratic-linear potential, quadratic-quartic potential, while in three cases we demonstrate the absence of the PR property: Linear oscillator enclosed in a box, sextic potential, quartic double well potential. We shall discuss the physical relevance of these results.

In the second part of the talk I shall present the results of other kinds of time-dependent systems, namely the cases of almost adiabatic (almost infinitely slow) variation, the case of unlimited linear driving of homogeneous power law potentials, where the nonlinear WKB method developed by Papamikos and Robnik (2012) can be applied, and finally also the cases of periodic driving.

References

L.D. Landau and E.M. Lifshitz 2005 *Mechanics* (Course in Theoretical Physics, Volume 1, reprinted third Edition) (Oxford: Elsevier Butterworth-Heinemann)
G. Papamikos, B.C. Sowden and M. Robnik 2012 *Nonlinear Phenomena in Complex Systems (Minsk)* **15** 227
M. Robnik and V.G. Romanovski 2000 *J. Phys. A: Math. Gen* **33** 5093
M. Robnik and V.G. Romanovski 2006 *Open Syst. & Infor. Dyn.* **13** 197-222
M. Robnik and V.G Romanovski 2008 in *"Let's Face Chaos through Nonlinear Dynamics"*, Proceedings of the 7th International summer school/conference, Maribor, Slovenia, 2008, AIP Conf. Proc. No. 1076, Eds. M.Robnik and V.G. Romanovski (Melville, N.Y.: American Institute of Physics) 65
G. Papamikos and M. Robnik 2011 *J. Phys. A: Math. Theor.* **44** 315102
G. Papamikos and M. Robnik 2012 *J. Phys. A: Math. Theor.* **45** 015206
D. Andresas, B. Batistić and M. Robnik 2014, submitted to *Phys. Rev. E*, *Preprint, arXiv:1311.1971*
D. Andresas and M. Robnik 2014, submitted to *J. Phys. A: Math. Theor.*, *Preprint, arXiv:1405.5803*

A hopping model of the energy transport in time-dependent billiards

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The energy of a particle in a time-dependent billiard can grow without limit, and under certain conditions the energy growth can be even exponentially fast in time. Time-dependent billiards are important models in many physical phenomena, for example, they serve as a model of acceleration of cosmic particles which are colliding with moving interstellar magnetic domains, as originally proposed by Enrico Fermi. Thus it is important to understand what are the general conditions that imply the exponentially fast energy growth. I shall explain the origin of the exponentially fast acceleration introducing the hopping model of the particle dynamics, in which a trajectory of the particle is represented as a path through an abstract space of invariant components of corresponding static (frozen) billiards. Such paths, which I call ζ-trajectories, are generated probabilistically in terms of time-dependent Markov transition matrices. I will show that if the number of ζ-trajectories proliferate exponentially fast in time, then the average energy of an ensemble grows exponentially in time as well. This scenario takes place if the phase space of corresponding static billiards is of the mixed type - with coexisting chaotic and regular domains. I shall also discuss cases in which the acceleration is not exponentially fast but obeys the power law, and explain the associated acceleration exponents.

References

Benjamin Batistić 2014 arXiv:1404.1747

Benjamin Batistić 2014 *PRE* **89** 022912

Exactly Solvable Counting Statistics in Open Weakly Coupled Interacting Spin Systems

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We study the full counting statistics for interacting quantum many-body spin systems weakly coupled to the environment. In the leading order in the system-bath coupling we derive exact spin current statistics for a large class of parity symmetric spin-1/2 systems driven by a pair of Markovian baths with local coupling operators. Interestingly, in this class of systems the leading order current statistics are universal and do not depend on details of the Hamiltonian. Furthermore, in the specific case of symmetrically boundary driven anisotropic Heisenberg (XXZ) spin 1/2 chain we derive explicitly the third-order non-linear corrections to the current statistics.

References

Buca B and Prosen T 2014 *Phys. Rev. Lett.* **112** 067201

Degenerate Andronov-Hopf bifurcations in systems of ODE's

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In the first part of my talk I will discuss a Hopf (or Andronov-Hopf) bifurcation for planar systems. I will describe also limit cycles on the center manifold and degenerate Hopf (Bautin) bifurcations. In the second part of the talk I will present one of the most famous problems in qualitative theory of ordinary differential equations - Hilbert's sixteenth problem on the number of limit cycles of two dimensional polynomial systems

$$\dot{x} = P_n(x, y), \quad \dot{y} = Q_n(x, y)$$

(n is the maximum degree of the polynomials on the right-hand side of the system). An essential part of the problem is the problem of estimating of the maximum number of limit cycles which can bifurcate from a singular point of center or focus type under small perturbations of coefficients of the system, the so-called cyclicity problem. The key feature of our approach is that in the case of an elementary singular point the problem of cyclicity is reduced to the algebraic problem of searching for a basis of a certain polynomial ideal. We apply this approach for solving the cyclicity problem for a subfamily of cubic systems.

References

Ferčec B and Mahdi A 2013 *Math. Comput. Simul.* **87** 55–67
Kuznetsov Yu.A 2004 *Elements of Applied Bifurcation Theory* Springer - Verlag, New York
Romanovski V G and Shafer D S 2009 *The center and cyclicity problems: A computational algebra approach* Birkhäuser, Boston

Existence, dynamics and mobility of Quantum Compactons in an extended Bose-Hubbard model

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Lattice Compactons, discrete breathers with compact support, were found for a discrete nonlinear Schrödinger (DNLS) equation extended with nearest neighbour intersite nonlinearities [1], a model originally studied with waveguide arrays in mind. These compactons were shown to exhibit very good mobility if the parameters are tuned close to the compactons stability boundary. The DNLS can also be used to model the behaviour of Bose-Einstein condensates in optical lattices, and the remarkable control over the experiments in this field of research has made it possible to study the quantum mechanics of strongly correlated atoms.

We will define the concept of a Quantum Lattice Compacton [2] and discuss the existence and dynamics, with special emphasis on mobility [3], of these in an extended Bose-Hubbard model corresponding to above-mentioned extended DNLS equation in the quantum mechanical limit. The compactons is given 'a kick' by means of a phase-gradient and it is shown that the size of this phase is crucial for the mobility of the compactons. For small phase-gradients, corresponding to a slow coherent motion in the classical model, the time-scales of the quantum tunnelings become of the same order as the time-scale of the translational motion and the classical mobility is destroyed by quantum fluctuations. For large phase-gradients, corresponding to rapid classical motion, the classical and quantum time-scales separate so that a mobile, localized coherent quantum state can be translated many sites in the lattice already for small particle numbers of the order of 10 [3].

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References

[1] M. Öster, M. Johansson, and A. Eriksson 2003 *Phys. Rev. E* **67** 056606
[2] P. Jason and M. Johansson 2012 *Phys. Rev. A* 85 011603(R)
[3] P. Jason and M. Johansson 2013 *Phys. Rev. A* 88 033605

On-off intermittency generated by infinite-modal maps

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On-off intermittency is an irregular switching phenomenon between long-term laminar behavior and instantaneous bursts. This phenomenon was discovered by Fujisaka and Yamada [1] in coupled chaotic systems, and observed in many experimental systems or mathematical models. Conventional mathematical models for on-off intermittency have been modeled using linear multiplicative noise systems. In such systems, it has been found that statistical properties such as the following are true at near the transition to transient behaviors from intermittent behaviors: (i) The stationary distribution about the distance r from the laminar state has $P(r) \sim r^{-1}$ $(r \ll 1)$ [2], (ii) The laminar duration distribution has $\rho(t) \sim t^{-3/2}$ $(t \gg 1)$ [3]. Since the above laws was observed from many experimental systems as well as mathematical models, they are regarded as the standard statistical laws for on-off intermittency.

Recently, in response to experimental examples deviating from such standard statistical laws, a probabilistic model which can change the exponents, such as -1 or $-3/2$, of the standard statistical laws by control parameters is devised [4]. However the deterministic model generating the non-standard statistical laws is not yet known. In such a situation, we found that the following one-dimensional dynamical system, which is an infinite-modal map, can generate on-off intermittency chaos:

$$x_{n+1} = x_n |x_n|^{a-1} \sin\left(b \log\left(1/|x_n|\right)\right), \quad -1 \le x_n \le 1. \tag{1}$$

where $a \in (0, 1)$ and $b > 0$ are parameters. This map originates in the dynamics for near the homoclinic orbit of the saddle-focus point in ordinary differential equations [5,6]. In this presentation, we present that the essence for the dynamics of this map is a non-linear multiplicative noise system and that it has non-standard statistical laws, according to numerical simulations.

References

[1] Fujisaka H and Yamada T 1985 *Prog. Theor. Phys.* **74** 918
[2] Yamada T and Fujisaka H 1986 *Prog. Theor. Phys.* **76** 582
[3] Heagy J F, Platt N and Hammel S M 1994 *Phys. Rev. E* **49** 1140
[4] Bertin E 2012 *Phys. Rev. E* **85** 042104
[5] Shilnikov L P 1965 *Sov. Math. Dokl.* **6** 163
[6] Arneodo A, Coullet P H, Spiegel E A and Tresser C 1985 *Physica D* **14** 327

Swarm Dynamics and Lyapunov Analysis

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The swarm dynamics is characterized by many local and global modes in the collective motions. The global response in the group dynamics is sensitively affected by the local aspects and also the local behaviors are controlled by the global information. The local-global linkage in the swarm dynamics is an essential mechanism, which leads to self-organization of the unified behavior in the swarm dynamics. In this paper, we consider a simple model to understand the linkage between local and global effects in the group dynamics by taking into account the local communicative interactions as well as the global environmental effects.

We propose the minimal model of active matters, where two parameters play the essential role in the group behavior: one is the environmental effect from the outside of the swarm, and the other is the communicative effect among individuals inside of swarm. The velocity of the i-matter is described by the following dynamics;

$$\dot{\mathbf{v}}_i(t) = \left(1 - |\mathbf{v}_i(t)|^2\right) \mathbf{v}_i(t) + \mathbf{F}^{comm}(t) + \mathbf{F}^{goal}(t) + \mathbf{F}^{env}(t) \tag{2}$$

For the sake of simplicity, the attractive effect of the goal information is shown by $\mathbf{F}^{goal}(t)$, the environmental effect from outside by $\mathbf{F}^{env}(t)$, and the communicative competence among individuals by $\mathbf{F}^{comm}(t)$. The first term is also a simplified form of the self-controlling effect to each velocity.

The Lyapunov exponents, which describe the sensitivity in the motion of each individual, is used to understand the global and local behaviors of the swarm dynamics. Increasing the size of the swarm, the exponents also increase. It will be discussed that the Lyapunov exponents (spectrum) affect the swarm dynamics of our model sensitively and the relation between the collective states and the instability. Detailed aspects of the Lyapunov exponents in the swarm dynamics are discussed in comparison with other chaotic dynamics of many-body systems.

References

LJ. Milanovic, H. A. Posch, and W. G. Hoover 1998 *Mol. Phys.* **95**(2) 281

Anomalous transport processes of inertial Brownian particles induced by white Poissonian noise

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Absolute negative mobility (ANM) is counterintuitive phenomenon: particles move in a direction opposite to a static bias force. It seems to be in contradiction to the Newton equation motion, the second law of thermodynamics and observation of motion at a macroscopic scale. However, under non-equilibrium conditions, there is no fundamental principle which excludes ANM. What are essential ingredients for the occurrence of ANM? The minimal model can be formulated in terms of one-dimensional Newton equation for a Brownian particle moving in a symmetric spatially periodic potential, driven by unbiased harmonic force and biased by a static force F [1]. The ANM response in a symmetric periodic potential is so that an average particle velocity $\langle v(F) \rangle$ obeys the relation $\langle v(F) \rangle = -\langle v(-F) \rangle$, which follows from the symmetry arguments. In particular $\langle v(0) \rangle = 0$. So, for $F = 0$ there is no directed transport in the long time regime. The non-zero static force F breaks the symmetry and therefore induces a directed motion of particles. In the lecture, we replace the static force F by a random force $\eta(t)$ of a time-independent non-zero mean value $\langle \eta(t) \rangle = \eta_0$ [2]. We assume that the particle is coupled to its environment (thermostat) of temperature T and thermal fluctuations $\xi(t)$ are included as well. As an example of the random force $\eta(t)$, we consider non-equilibrium Poissonian white shot noise, which is composed of a random sequence of δ-shaped pulses with random amplitudes. We analyse the dependence of the long-time average velocity $\langle v \rangle$ on parameters of both random forces $\eta(t)$ and $\xi(t)$. We find a rich variety of anomalous transport regimes including the absolute negative mobility around zero biasing Poissonian noise, the emergence of a negative differential mobility and the occurrence of a negative nonlinear mobility (for values of bias η_0 far from zero). As a feasible physical system, we propound a setup consisting of a single resistively and capacitively shunted Josephson junction driven by both a time periodic current and a noisy current. In this case the phase difference between the macroscopic wave functions of the Cooper electrons in both sides of the junction translates to the Brownian particle coordinate and the voltage across the junction translates to the particle velocity. For such a system, the anomalous transport characteristics can be measured, thus putting our predictions to a reality check.

References

[1] L. Machura, M. Kostur, P. Talkner, J. Luczka and P. Hänggi, Phys. Rev. Lett. 98 (2007) 040601
[2] J. Spiechowicz, J. Luczka and P. Hänggi, J. Stat. Mech. (2013) P02044