Abstracts of Invited Lectures

Regular and Chaotic Motion in the Linear and Non-Linear Schrödinger Equation

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In this lecture we compare and contrast the regular and chaotic dynamics as described by linear and nonlinear Schroedinger equations. The linear Schrödinger equation (LSE) predicts a strictly regular dynamics, yet classical particle chaos emerges from its short-wavelength (or semiclassical) limit. The non-linear Schrödinger equation (NLSE) which in the limit of weak interparticle interaction reduces to the LSE, on the other hand, features deterministic wave chaos. We have recently shown that wavepackets separate exponentially in Hilbert space allowing for the determination of a Lyapunv exponent in direct analogy to phase space trajectories of classical particles[1]. The Gross-Pitaevskii equation (GPE) describing Bose-Einstein condensate of ultracold quantum gases on the mean-field level is a prominent example of such a NLSE. The existence of wave chaos raises fundamental questions as to the stability of Bose-Einstein condensates and the validity of a mean-field description for the time evolution. After all, the GPE is only an approximation to the exact many-body Schroedinger equation which, in turn, is a LSE and thus strictly regular.

References

[1] Brezinova I et al. 2011 Phys. Rev. A **83** 043611

Dynamical chaos, Entanglement generation and Complexity of quantum motion

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Given two quantum systems how can we decide which one is more complex than the other? In classical mechanics the notion of complexity is rooted in the local exponential instability which leads to dynamical chaos. Such property is absent in quantum mechanics. Here we discuss the deep implications that chaos and entanglement have in characterizing quantum many-body dynamical complexity. Finally a classical approach to entanglement is discussed.

References

Balachandran V, Benenti G, Casati G and Gong J 2010 *Phys. Rev. E* **82** 046216.
Casati G, Resslen J and Guarneri I 2011 *Classical Dynamical Theory of Quantum entanglement* preprint

Modern String Theory and Particle Physics

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and

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We review developments leading to the unification of string theories (M-theory), with an emphasis on particle physics implications. We introduce extended objects - Dirichlet branes - and highlight an important geometric role that these objects play in deriving particle physics from string theory. Constructions of string solutions with Dirichlet branes, that have features of the Standard Model with three families of quarks and leptons, are reviewed. We also highlight recent developments, where new stringy non-perturbative effects due to Euclidean D-brane instantons were introduced, and focus on their implications for neutrino masses and other couplings of the Standard Model.

References

Cvetič M and Halverson J 2011 “TASI Lectures: Particle Physics from Perturbative and Non-perturbative Effects in D-braneworlds,” arXiv:1101.2907 [hep-th]

Blumenhagen R , Cvetič M, Kachru S and Weigand T 2009 “D-Brane Instantons in Type II Orientifolds,” Ann. Rev. Nucl. Part. Sci. **59**, 269 [arXiv:0902.3251 [hep-th]]

Cvetič M, Garcia-Etxebarria I and Halverson J 2011 “On the computation of non-perturbative effective potentials in the string theory landscape: IIB/F-theory perspective,” arXiv:1009.5386 [hep-th]

Blumenhagen R, Cvetič M and Weigand T 2007 “Spacetime instanton corrections in 4D string vacua: The Seesaw mechanism for D-Brane models,” Nucl. Phys. B **771**, 113 [arXiv:hep-th/0609191]

Turbulence, in six easy (but unfinished) pieces

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Lecture 1: Dynamics

We start with a recapitulation of basic notions of dynamics; flows, maps, local linear stability, heteroclinic connections, qualitative dynamics of stretching and mixing and symbolic dynamics.

References
Chapters "Overture" to "Cycle stability."
(all chapters refer to P. Cvitanović *et al.*, *Classical and Quantum Chaos*, `ChaosBook.org`)

Lecture 2: Periodic orbit theory

A motion on a strange attractor can be approximated by shadowing the orbit by a sequence of nearby periodic orbits of finite length. This notion is here made precise by approximating orbits by primitive cycles, and evaluating associated curvatures. A curvature measures the deviation of a longer cycle from its approximation by shorter cycles; the smoothness of the dynamical system implies exponential (or faster) fall-off for (almost) all curvatures. The technical prerequisite for implementing this shadowing is a good understanding of the symbolic dynamics of the classical dynamical system. The resulting cycle expansions offer an efficient method for evaluating classical and quantum periodic orbit sums; accurate estimates can be obtained by using as input the lengths and eigenvalues of a few prime cycles.

References
Chapters "Trace formulas" to "Cycle expansions."

Lecture 3: The best of all partitions

All physical systems are affected by some noise that limits the resolution that can be attained in partitioning their state space. For chaotic, locally hyperbolic flows, this resolution depends on the interplay of the local stretching/contraction and the smearing due to noise. Our goal is to determine the 'finest attainable' partition for a given hyperbolic dynamical system and a given weak additive white noise. That is achieved by computing the local eigenfunctions of the Fokker-Planck evolution operator in linearized neighborhoods of the periodic orbits of the corresponding deterministic system, and using overlaps of their widths as the criterion for an optimal partition. The Fokker-Planck evolution is then represented by a finite transition graph, whose spectral determinant yields time averages of dynamical observables.

References
D. Lippolis and P. Cvitanović, "How well can one resolve the state space of a chaotic map?", *Phys. Rev. Lett.* **104**, 014101 (2010); arXiv.org:0902.4269

Lecture 4: Symmetries and dynamics

Dynamical systems often come equipped with symmetries, such as the reflection symmetries of various potentials. Symmetries simplify the dynamics in a rather beautiful way:

If dynamics is invariant under a set of discrete symmetries G, the state space $\mathcal{M}$ is *tiled* by a set of symmetry-related tiles, and the dynamics can be reduced to dynamics within one such tile, the *fundamental domain* $\mathcal{M}/G$. If the symmetry is continuous, the dynamics is reduced to a lower-dimensional desymmetrized system $\mathcal{M}/G$, with "ignorable" coordinates eliminated (but not forgotten). We reduce a continuous symmetry by slicing the state space in such a way that an entire class of symmetry-equivalent points is represented by a single point.

In either case, families of symmetry-related full state space cycles are replaced by fewer and often much shorter "relative" cycles. In presence of a symmetry the notion of a prime periodic orbit has to be reexamined: it is replaced by the notion of a relative periodic orbit, the shortest segment of the full state space cycle which tiles the cycle under the action of the group. Furthermore, the group operations that relate distinct tiles do double duty as letters of an alphabet which assigns symbolic itineraries to trajectories.

References

Chapters "World in a mirror" and "Relativity for cyclists."
S. Froehlich and P. Cvitanović, "Reduction of continuous symmetries of chaotic flows by the method of slices," *Comm. Nonlinear Sci. and Numerical Simulation*, (2011) submitted; arXiv.org:1101.3037
R. Gilmore and C. Letellier, *The Symmetry of Chaos* (Oxford U. Press, Oxford 2007)

Lecture 5: Hopf's dynamical vision of turbulence

As a turbulent flow evolves, every so often we catch a glimpse of a familiar pattern. For any finite spatial resolution, the system follows approximately for a finite time a pattern belonging to a finite alphabet of admissible patterns. In "Hopf's vision of turbulence," the long term turbulent dynamics is a walk through the space of such unstable patterns.

References

E. Hopf, "A mathematical example displaying features of turbulence," *Commun. Appl. Math.* **1**, 303 (1948)
B. Hof, C.W.H. van Doorne, J. Westerveel, F.T.M. Nieuwstad, H. Faisst, B. Eckhardt, H. Wedin, R.R. Kerswell and F. Waleffe, "Experimental demonstration of travelling waves in pipe flow," *Science* **305**, 1594 (2004)

Geometry of turbulence

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In the world of moderate Reynolds number, everyday turbulence of fluids flowing across planes and down pipes a velvet revolution is taking place. Experiments are almost as detailed as the numerical simulations, DNS is yielding exact numerical solutions that one dared not dream about a decade ago, and dynamical systems visualization of turbulent fluid's state space geometry is unexpectedly elegant.

We shall take you on a tour of this newly breached, hitherto inaccessible territory. Mastery of fluid mechanics is no prerequisite, and perhaps a hindrance: the lecture is aimed at anyone who had ever wondered why - if no cloud is ever seen twice - we know a cloud when we see one? And how do we turn that into mathematics?

References

J.F. Gibson *et al.*, “Movies of plane Couette,” `ChaosBook.org/tutorials`
J.F. Gibson, J. Halcrow and P. Cvitanović, “Visualizing the geometry of state space in plane Couette flow,” *J. Fluid Mech.* **611**, 107 (2008); `arXiv:0705.3957`
P. Cvitanović, E. Siminos and R. L. Davidchack, “On state space geometry of the Kuramoto-Sivashinsky flow in a periodic domain,” *SIAM J. Appl. Dyn. Syst.* **9**, 1 (2010); `arXiv:0709.2944`

Dynamics of heterogeneous populations of coupled oscillators

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Large populations of coupled nonlinear oscillators have been playing crucial roles in a variety of disciplines of science and technology. Their interesting behaviors, e.g. synchronization, clustering, and spatiotemporal chaos, have been extensively studied experimentally as well as theoretically. Real populations of coupled oscillators are more or less heterogeneous in the sense that each constituent oscillator has its inherent dynamics. Of many different classes of such heterogeneous systems, particularly important is the one such that a population consists of oscillators with distributed values of a bifurcation parameter. Over the years we have been studying the behavior of one typical example of this class of heterogeneous populations, in which the system is composed of two groups of oscillators: one is the collection of spontaneous oscillators beyond a supercritical Hopf bifurcation and the other is formed by damped oscillators below it. Such a study is of significance in the context of checking the robustness of the behavior of the system against defects, for example, and has lead to discoveries of not a few novel phenomena, which include aging transitions, clustering, disorder-induced phase coherence, and so on.

The purpose of this talk is to give a brief review of these theoretical studies and present some results of our ongoing studies on a more generalized class of heterogeneous populations.

References

Daido H and Nakanishi K 2004 *Phys. Rev. Lett.* **93** art. no. 104101
Daido H and Nakanishi K 2006 *Phys. Rev. Lett.* **96** art. no. 054101
Daido H and Nakanishi K 2007 *Phys. Rev. E* **75** art. no. 0562067; **76** art. no. 049901(E)
Daido H 2008 *Europhys. Lett.* **84** art. no. 10002
Daido H, Kawata N, Sano Y and Yamaguchi S 2008 *AIP Proceedings* No. 1076 33
Daido H 2009 *Europhys. Lett.* **87** art. no. 40001
Daido H 2011 *Phys. Rev. E* **83** art. no. 026209

Water delivery in the Early Solar System

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Essential for the development of life on our Earth is water. We investigate what are the reasons that water in such quantities is on the surface of our planet. The questions we need to answer in this connection are

- When the terrestrial planets formed how much was their content of water?
- Why don't we find water in the same quantities on the other terrestrial planets?
- What happended to the water when a mars-sized object hit the Earth and the Moon formed?
- What happened during the Late Heavy Bombardement (LHB)
- Where from came water after the LHB?
- What is the role of the comets from the Oort Cloud?

In this lecture we will discuss all these points more or less in detail. Although we have many interesting ideas about the water delivery no real satisfying answers can be given.

The weak password problem: chaos, criticality, and encrypted p-CAPTCHAs

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Vulnerabilities related to weak passwords are a pressing global economic and security issue. We report a novel, simple, and effective approach to address the weak password problem [1]. Building upon chaotic dynamics, criticality at phase transitions, CAPTCHA recognition, and computational round-off errors we design an algorithm that strengthens security of passwords. The core idea of our method is to split a long and secure password into two components. The first component is memorized by the user. The second component is transformed into a CAPTCHA image and then protected using evolution of a two-dimensional dynamical system close to a phase transition, in such a way that standard brute-force attacks become ineffective. We expect our approach to have wide applications for authentication and encryption technologies, as worldwide reactions suggest [2].

References

[1] T.V. Laptyeva, S. Flach, K. Kladko, arXiv:1103.6219 (2011).
[2] http://www.pks.mpg.de/ flach/html/password.html

The Nature of Temporal Fluctuations in Musical Rhythms

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When musical rhythms are performed by humans, the underlying beats are not entirely accurate in time, but deviate to a certain extent from those given by an ideal beat pattern, the deviations being a fundamental characteristic of music played by humans. Professional audio software applications therefore include a so-called 'humanizing' feature, which allows to add ('white-noise') temporal fluctuations to a given audio sequence. The nature of these fluctuations in human music, however, has never been scrutinized as yet.

We have examined the correlation properties of deviations from the exact beats for various combinations of hand, feet, and vocal performances, by both amateur and professional musicians [1]. In all cases, the interbeat intervals exhibit long-range correlations indicating that scaling laws are a generic feature of musical rhythms performed by humans. We also ask what is the role of these correlations in musical perception. Listeners showed a high preference for music with long-range correlated temporal deviations over uncorrelated humanized music. Based on these findings we have obtained patents for a novel concept of humanizing musical sequences.

References

[1] H. Hennig, R. Fleischmann, A. Fredebohm, Y. Hagmayer, A. Witt, J. Nagler, F. J. Theis, and T. Geisel, to be published.

Order statistics and the Lyapunov spectra of some classes of high-dimensional billiard systems

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Consider a system made out of a possibly large number of identical copies of a two-dimensional dispersive billiard table and let us further assume a form of infrequent pairwise energy-preserving interaction among them. The interaction we will consider will typically be of collisional type and may therefore induce the exchange of a substantial amount of energy among the colliding pair.

The question we address is the following:

> What is the spectrum of Lyapunov exponents of such a system?

It turns out this question is closely related to a famous problem in probability theory, first addressed by Laplace in his attempt to construct an error function towards the end of the 18th century:

> What is the distribution of the ordered lengths of a fixed number of random divisions of the unit interval?

Very strong thermal convection
or
On the sensitive dependence on the experimental conditions and theoretical interpretation

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There is currently exciting progress in experimental and theoretical study of strongly driven, high Rayleigh number turbulent heat convection [1,2,3]. An unexpected and quite surprising multitude of different states has been found in high precision experiments by Ahlers, Funfschilling and Bodenschatz. These experiments even added to the longstanding mystery of differently scaling convective heat transport in two seemingly identical Rayleigh-Bénard experiments with cryogenic helium [4,5].

The emphasis of this talk is put on the ideas to physically understand and explain these multiple state observations, cf. [6], why that is possible and which the properties of the various states might be. The main idea is the varying role and dominance of the turbulent bulk flow and the boundary layer structures, being either laminar with different dominance of transport via thermal plumes or via thermal fluctuations or also turbulent, for either turbulent bulk or log-profile boundary layers of the velocity field, see [6].

References

[1] Ahlers G, Grossmann S and Lohse D 2009 *Rev. Mod. Phys.* **81** 503

[2] Ahlers G, Funfschilling D and Bodenschatz E 2009 *New J. Phys.* **11** 123001

[3] Ahlers G January 2010 *lecture at the Euromech Colloquium in Les Houches, see www.hirac4.cnrs.fr/HIRAC4-Talks-files/Ahlers*

[4] Chavanne X, Chilla F, Castaing B, Hebral B, Chabaud B and J. Chaussy 1997 *Phys. Rev. Lett.* **79** 3648

[5] Niemela J, Skrbek L, Sreenivasan K and Donnelly R 2000 *Nature* **404** 837

[6] Grossmann S and Lohse D 2011 *Physics of Fluids* **23** 045108

Introduction to Econophysics

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At first sight, it seems a bit far–fetched that physicists work on economics problems. A closer look, however, reveals that the connection between physics and economics is rather natural — and not even new! Many physicists are surprised to hear that the mathematician Bachelier developed a theory of stochastic processes very similar to the theory of Brownian motion which Einstein put forward in 1905. Bachelier did it in the context of financial instruments, and he was even a bit earlier than Einstein. Moreover, not all physicists know that financial time series were a major motivation for Mandelbrot when he started his work on fractals. Mathematical modeling in physics and economics, in particular finance, is similar. The famous Black–Scholes theory for stock options, for example, is a beautiful application of stochastic processes, leading to a partial differential equation for the option price which is formally just a heat equation.

In the last 15 or 20 years, the physicists' interest in economical issues grew ever faster, and the term "econophysics" was coined. Econophysics developed into a recognized subject. The crucial reason for this was the dramatic improvement of the data situation, a wealth of data became available and (electronically) accessible. This is an indispensable prerequisite for theoretical physicists whose key competence is mathematical modeling based on empirical information. Moreover, complex systems moved into the focus of physics research. The economy certainly qualifies as a complex system and poses serious challenges for basic research. Simultaneously, economics started to develop into a more quantitative science. Although some economists are still sceptical and doubt that physics approaches are useful in their field, there is a growing number of economists who appreciate the long–standing competence of physicists in model building based on empirical data. From a more practical viewpoint, the need to quantitatively improve economical risk management is a driving force in econophysics. To manage risk, statistical features have to be understood and modeled. Not surprisingly, many econophysicists come from statistical physics.

This course gives an introduction to econophysics. The presentation starts from scratch, no background in economics is needed. The course consists of five lectures:

1. Basic Concepts
   We begin with explaining markets, particularly financial markets, efficiency, arbitrage and risk. Price and return distributions are shown. Simple stochastic processes are constructed and their limitations are discussed

2. Detailed Look at Stock Markets and Trading
   The limited descriptive power of standard stochastic processes is seen when carefully analyzing empirical stock market data. Concepts such as order book, market and limit orders as well as liquidity are explained. Various correlations in the time series of a given stock are studied. A much deeper understanding of stock market trading is achieved.

3. Financial Correlations and Portfolio Optimization
   In addition to the above mentioned correlations, there are also (cross) correlations between different stocks, because the companies depend on each other. Important information about markets can be obtained from them. Furthermore, they have a considerable impact on investments, more precisely on how to choose a portfolio comprising shares of different stocks. Methods to optimize such portfolios are presented. The rôle of a special kind of "noise" is discussed.

4. Quantitative Identification of Market States
   Qualitatively, it is plausible that markets can function in different states which emerge and stabilize after dramatic events. The (cross) correlations are used to quantitatively identify and extract such different market states. A particular focus is given on the still ongoing financial crisis.

5. Credit Risk
   A major reason for the present problems in the world economy was a credit crisis, that is, the failure of many individuals and companies to make promised payments. Models for credit risk are presented and evaluated in detail. It is shown that the benefit of "diversification" is vastly overestimated.

Econophysics already comprises a broad spectrum of activities. As time is limited, some of those will not be touched in these lectures. Nevertheless, the material presented in the course provides an overview of major directions in econophysics research. The field develops quickly, implying that not all of the topics in the course can be found in text books appropriate for a physics audience. Some good text books written by physicists are listed below, further literature will be given in the course.

References

Mantegna R.N and Stanley H.E. 2000 *An Introduction to Econophysics*, Cambridge University Press, Cambridge
Bouchaud J.P.and Potters M. 2003 *Theory of Financial Risk and Derivative Pricing*, Cambridge University Press, Cambridge
Voit J. 2001 *The Statistical Mechanics of Financial Markets*, Springer, Heidelberg

Some studies on limit cycle bifurcations for non-smooth systems

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There have been many studies on the behavior of non-smooth dynamical systems in recent years, and obtained some meaningful results. For instances, see the works [Filippov, 1988] and [Kunze, 2000] on the basic methods of qualitative theory, and [Bernardo et al., 2008] on the bifurcations in piecewise systems. Recently, Hopf bifurcations in general or some kind of non-smooth planar systems were studied, see for example [Coll et al., 2001; Du et al., 2008; Gasull and Torregrosa, 2003; Han and Zhang, 2010; Liu and Han, 2009; Yang et al., 2011; Zou et al., 2006]. In this lecture we consider a planar non-smooth system of the form

$$\dot{x} = f(x,y),\ \dot{y} = g(x,y),$$

where

$$f(x,y) = \begin{cases} f^+(x,y), \ x > 0, \\ f^-(x,y), \ x \le 0, \end{cases} \quad g(x,y) = \begin{cases} g^+(x,y), \ x > 0, \\ g^-(x,y), \ x \le 0, \end{cases}$$

and $f^\pm$ and $g^\pm$ are C^∞ functions for all $(x,y) \in R^2$. We focus on the studies of limit cycle bifurcations for the above system, including Hopf bifurcation near a focus or center, Poincaré bifurcation from a periodic annular and generalized homoclinic bifurcation.

References

M. di Bernardo, C. J. Budd, A. R. Champneys, P. Kowalczyk 2008 *Piecewise-smooth Dynamical Systems, Theory and Applications*, Springer-Verlag, Berlin
B. Coll , A. Gasull, R. Prohens 2001 *J. Math, Anal.Appl.* **253** 671
A. F. Filippov 1998 *Differential Equation with Discontinuous Righthand Sides*, Kluwer Academic Pub., Netherlands
M. Han, W. Zhang 2010 *J. Diff. Equat.* **248** 2399
M. Kunze 2000 *Non-smooth dynamical systems*, Springer-Verlag, Berlin
X. Liu, M. Han 2009 *Int. J. Bifur. and Chaos* **19** 2401
A. Gasull, J. Torregrosa 2003 *Int. J. Bifurcation and Chaos* **13** 1755
M. Han, J. Yang and W. Huang 2011 *J. Diff. Equat.* **250** 1026
X. Liu, M. Han 2010 *Int. J. Bifur. and Chaos* **20** 1379

Fast photonic non-deterministic random bit generation using on-chip chaos lasers

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Optical random bit generation using chaos in semiconductor lasers subject to delayed optical feedback has recently been developed as a method for fast generation of non-deterministic random bit sequences, which are crucially important for secure communication and computation systems. However, the chaotic laser generators which have been realized so far use discrete optical components with fiber-optic or spatial optic techniques to obtain delayed optical feedback. Miniaturized chaotic lasers have been developed by monolithic integration of a distributed feedback laser, a passive waveguide, and a semiconductor optical amplifier, on a single chip. However, it has not yet been known whether such on-chip chaotic lasers can be applied for fast, random bit generation. We present the first monolithically integrated optical random bit generator which operates at rates up to 2.08 Gbps. Our demonstration strongly suggests potential for widespread use of fast optical random bit generation with chaotic lasers.

References

Uchida A, Amano K, Inoue M, Hirano K, Naito S, Someya H, Oowada I, Kurashige T, Shiki M, Yoshimori S, Yoshimura K, and Davis P 2008 *Nature Photonics* **2** 728

Regular-to-Chaotic Tunneling and Spectral Statistics

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The first part of the talk will review recent work on the quest for a quantitative understanding of tunneling in systems with a mixed phase space. The focus will be on dynamical tunneling from a regular to a chaotic region. Theoretical approaches for predicting the direct regular-to-chaotic tunneling rate and the combination with resonance-assisted tunneling will be presented, with applications to microwave billiards and optical microcavities. In the second part the influence of regular-to-chaotic tunneling on spectral statistics is studied. In the regime of small spacings it will be shown analytically and numerically that the nearest neighbor level-spacing distribution follows a power law with a fractional exponent.

References

Bäcker A, Ketzmerick R, Löck S and Schilling L. 2008 *Phys.Rev.Lett.* **100** 104101

Bäcker A, Ketzmerick R, Löck S, Robnik M, Vidmar G, Höhmann R, Kuhl U and Stöckmann H.-J. 2008 *Phys.Rev.Lett.* **100** 174103

Bäcker A, Ketzmerick R, Löck S, Wiersig J and Hentschel M 2009 *Phys.Rev.A* **79** 063804

Bäcker A, Ketzmerick R and Löck S 2010 *Phys.Rev.E* **82** 056208

Löck S, Bäcker A, Ketzmerick R and Schlagheck P 2010 *Phys.Rev.Lett.* **104** 114101

Bäcker A, Ketzmerick R, Löck S and Mertig N 2011 *Phys.Rev.Lett.* **106** 024101

The visibility and correlation networks of calcium dynamics in pancreatic islets

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Network theory has been successfully used in exploring the structure of many complex systems in the last decade [1]. It seems that a particular organization of biological networks is common to biological systems at all scales.

Here, we shall present the construction of complex networks of pancreatic islet, a compact microorgan in which the release of insulin is under physiological conditions robustly controlled by an efficient cell-to-cell network communication.

The networks of insulin releasing beta-cells are formed based on measured time series data of calcium dynamics and positional information obtained by image analysis of confocal multiphoton functional imaging of intact islets in pancreatic slice tissues. Using the visibility algorithm [2] we will first represent single time series as networks which will then be coupled based on correlations of the calcium dynamics in the islet. We will analyze the properties of obtained islet networks and compare them with the network model of spatially embedded heterogeneous cells [3] to seek the relationship between the structure and the function of the tissue [4].

References

[1] A.-L. Barabasi, Science, **325**, 412 (2009)
[2] L. Lacasa, B. Luque, F. Ballesteros, J. Luque, and J. C. Nuno, PNAS **105**, 4972 (2008).
[3] K. Yakubo and D. Korošak, Phys. Rev. E, in press (2011).
[4] C. Zhou, L. Zemanova, G. Zamora, C. C. Hilgetag, J. Kurths, Phys. Rev. Lett. **97**, 238103 (2006).

Solvable Models in 1D Quantum Mechanics & Stochastic Dynamics

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In the first part of the lecture I shall briefly outline main approaches to solvability in quantum mechanics. Exactly solvable problems are those admitting construction of all eigenstates of significant spectrum plus continuous spectrum (if it exists) in an analytical form. A slightly weaker form of solvability is called quasi exact solvability (QES) and is referred to problems when only a few number of eigenstates can be found analytically whereas to find other states one has to use some numerical algorithms.

Then I shall concentrate myself on 1D problems and analytical approaches to their investigations. I shall start from the construction of quasi-exactly solvable (QES) problems with one additional condition on polynomial coefficients of governing ODE imposed. Then I shall consider some new results on interesting properties of some QES problems where more than one condition are imposed. In particular the possibility of the situation when one governing equation corresponds to several quantum QES problems will be discussed.

In the second part I shall briefly present the main concepts of the SUSY quantum mechanics and the interrelation between quantum mechanics and stochastic dynamics. Few examples will be demonstrated to show quite unexpected features of the constructed stochastic dynamics solutions.

References

G. Krylov and M. Robnik 2001 *J. Phys.* **A34** 5403-5415

G.G. Krylov 2008 *Acta Physica Debrecina* **42** 151

G.G. Krylov 2008. *NPCS* **11** 336

G. Krylov 2010 *NPCS* **13** 100

Optimal Topologies for Best Dynamical Responses in Complex Biological Networks

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In the last three decades biological rhythms were extensively studied. New theories and experiments in the field of non-linear dynamics brought new insights into understanding of heart and EEG rhythms, our daily cycle of walking and sleeping, various metabolic processes, and many other rhythms observed at macroscopic level (Glass 2001). To understand the origins of those rhythms cellular oscillators were studied. The basic idea was to investigate cellular oscillators first, and then coupling them to see their collective effects in tissues. However, understanding the behavior of particular oscillators in a network does not necessarily mean that one could understand complex dynamics of the networks. It has been realized that functioning of tissues and organs does not only depend on intrinsic rhythms of individual cells, but it also relies on collective activity of cell populations. Rhythms essential for life are thus a result of interactions of these cells with each other in terms of intercellular communication. The many efforts devoted to understand collective phenomena in biological systems take now advantage of the recent theory of complex networks. Complex topologies such as small-world or scale-free networks have been identified in a plethora of real-life systems (Albert and Barabási 2002). The question arises how to recognize optimal network topology which under given circumstances enables best network dynamical responses. To this purpose, we employed a mathematical model of cellular networks in which we can smoothly change the topology from a scale-free network with dominating long-range connections to a homogeneous network with dominating short-range connections, where actually only adjacent cells are connected. Since biological cells exhibit diverse temporal patterns we take into account different types of inherent dynamics of individual units constituting the network, like bistable and excitable dynamics, bursting oscillations, and also non-oscillatory cellular transients. We found that irrespectively of the type of inherent dynamics, an optimal network topology exists for which the collective behavior of coupled cells provides best results. Taking into account particular examples, and studying stochastic and coherence resonance effects the best responses were obtained for networks in the mid range between scale-free and regular topology, when a suitable number of hubs and a proper ratio between long- and short-range connections exist in the networks (Gosak et al. 2010a, 2011). Our method was also applied to real experimental data obtained in airway smooth muscle cells. Two types of cells, ones characterised by normal physiological responses and the others with pathological hypoxic responses, were exposed to high concentrations of KCl and the responses at tissue level were measured. The experimental measurements cannot be explained by differences in cellular dynamics, or in another words, knowing the individual cellular dynamics is not enough to explain the responses at tissue level. On the other hand, however, we were able to explain the experimental findings by modelling the corresponding topologies responsible for particular physiological and pathophysiological responses. We found that changes in tissue topology were mainly responsible, more than changes in particular cellular dynamics, for developing of pathological behaviour in the tissue (Gosak et al. 2010b).

References

Albert R and Barabási A-L 2002 *Rev. Mod. Phys.* **74** 47
Glass L 2001 *Nature* **410** 277
Gosak M, Korošak D and Marhl M 2010a *Phys. Rev. E* **81** 054104
Gosak M, Marhl M, Guibert C, Billaud M and Roux E 2010b *Proc. ACM Inter. Conf. Bioinf. Comput. Biol.* 475
Gosak M, Korošak D and Marhl M 2011 *New J. Phys.* **13** 013012

Statistical Features of Complex Systems —Toward establishing sociological physics—

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Complex systems have recently attracted much attention, regardless of natural sciences or sociological sciences. Members constituting a complex system evolve through nonlinear interactions among each other. This means that in a complex system the multiplicative experience or, so to speak, history that any member has had produces its present characteristics. We can then anticipate the following. If attention is paid to any statistical property in any complex system, the lognormal distribution is the most natural and appropriate for the standard or gnormalh statistics to look over the whole system. In fact, the lognormality emerges rather conspicuously when we examine, as familiar and typical examples of statistical aspects in complex systems, nursing-care period for the aged, populations of prefectures and municipalities, and our body height and weight. Many other examples are found in nature and society. Based on these observations, we would like to discuss the possibility of sociological physics.

References

Kobayashi N, Kuninaka H, Wakita J and Matsushita M Review article to be published soon in J. Phys. Soc. Jpn., and references cited therein.

Rogue waves in superfluid ^{4}He

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We discuss recent experiments on nonlinear wave interactions in superfluid ^{4}He leading to the observation [1] of rogue waves. The equivalent phenomenon on the ocean [2] involves waves that are rare, and much higher (and steeper) than all the other waves around them. For obvious reasons, they are a menace to shipping. There have been several suggestions about possible mechanisms for the creation of rogue waves. These include the combined effects of wind and currents, and the focusing effects associated with the profile of the ocean floor and nearby shorelines. Where rogue waves appear in deep water far from any shore, which they sometimes do, it seems likely that they evolve through nonlinear interactions within the "noisy background" of smaller wind-blown waves [3]. Rogue waves have been sought experimentally and/or studied in e.g. large wave tanks, optical systems, microwave systems, and superfluid ^{4}He.

We will review briefly the necessary background in turbulence and superfluidity, discuss why superfluid ^{4}He is an ideal medium for modelling nonlinear wave interactions and wave turbulence in the laboratory [4], present our observations of rogue waves [1], and consider their implications.

References

[1] Efimov V B, Ganshin A N, Kolmakov G V, McClintock P V E and Mezhov-Deglin V P 2010 *Eur. Phys. J. Special Topics* **185** 181
[2] Kharif C, Pelinovsky E and Slunyaev E 2009 *Rogue Waves in the Ocean*, Springer-Verlag, Berlin
[3] Dyachenko A I and Zakharov V E 2005 *JETP Lett.* **81** 255
[4] Ganshin A N, Efimov V B, Kolmakov G V, Mezhov-Deglin L P and McClintock P V E 2008 *Phys. Rev. Lett.* **101** 065303

Synchronization Structure in Kidney Autoregulation

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As part of an effort to understand the relation between hypertension and kidney function we have long been engaged in the study of nephron autoregulation, i.e. the mechanisms by which the individual functional unit of the kidney regulates the incoming blood flow [1]. This regulation involves two different mechanisms: A myogenic mechanism that reacts directly to changes in the arterial pressure, and a so-called tubuloglomerular feedback (TGF) that responds to changes in the salt concentration of the fluid that leaves the nephron. Due to a delay in the feedback of about 15 sec, the TGF mechanism tends to produce large amplitude oscillations in the nephron pressures and flows with periods in the 30-40 sec range. The myogenic mechanism depends on a propensity of the smooth muscle cells in the arterial wall to contract in response to an increasing blood pressure. This mechanism involves a positive feedback and gives rise to oscillations with periods in the 6-8 sec range.

The regulatory mechanisms both work through the same smooth muscle cells. This allows the oscillatory modes to interact and to synchronize with typical locking ratios of 1:4, 1:5 or 1:6. Moreover, episodes of period-2 dynamics are observed for about 50% of the experimental time traces. The nephrons are typically arranged in pairs that share part of a common arteriole. Hence, one can also observe synchronization between neighboring nephrons both of the TGF-mediated and the myogenic oscillations.

The focus of the present study is to examine the mechanisms by which a pair of neighboring nephrons moves in and out of synchrony. It is well-known that a period-doubling cascade that unfolds along the edge of a synchronization tongue displays a special scaling behavior, referred to as cyclic or C-type criticality. Our analyses show that a different structure arises in the coupled nephron system. In particular we find that the transition from synchronized periodic dynamics to quasiperiodic dynamics involves a torus bifurcation rather than the usual saddle-node bifurcation. Moreover, each period-doubling of the stable and unstable resonance modes not only leads to a new pair of saddle-node bifurcation curves, but also to a torus bifurcation curve that extends along one of the edges of the resonance zone.

References

[1] Laugesen J, Mosekilde E and Holstein-Rathlou N-H 2011 *Interface Focus* **1** 132.

The many-body problem far from equilibrium: Where statistical mechanics meets quantum information theory.

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In this series of lectures we shall outline the basic concepts of quantum information theory which have been either originally introduced recently, re-used extensively, or sometimes only rephrased in more general and cleaner contexts, in order to advance our understanding of equilibrium and non-equilibrium quantum statistical mechanics of low dimensional interacting many-body systems; such as *entanglement*, *renormalization*, *criticality*, *area laws*, *Markovian master equations* etc.

Our emphasis in particular will be on studies of locally interacting quantum chains (spin chains, fermionic or bosonic chains) far from equilibrium, either in time-dependent or in (non-equilibrium) steady state context. As such systems have recently become amenable to accurate and well controlled experimental treatments, in particular in the field of cold gases and optical lattices [1], whereas some related phenomena have long been debated in the community of solid state physics [2,3], we are thus witnessing exciting theoretical challenges. For example, one of the key fundamental questions in non-equilibrium statistical physics is to derive precise microscopic conditions under which non-equilibrium quantum transport is diffusive or ballistic.

The lectures will cover several fundamental theoretical problems and present the main ideas of the best current theoretical and numerical methods to tackle them. The main prototype toy models used in our numerous examples will be the (anisotropic) Heisenberg chain of spins 1/2 and the Ising chain of spins 1/2 in a tilted magnetic field.

Five topical lectures will cover the following material:

- **Lecture 1** *Basic concepts of quantum information theory in many-body systems.* [4,5] Entanglement and correlations. Area laws. Valence bond ground states and matrix product ansatz. Time-evolution of a many-body system. Trotter formula. Examples.
- **Lectue 2** *Renormalization.* [6] Renormalization group and efficient description of many-body quantum states, density matrix renormalization group. Variational and time-dependent approach. Quenched dynamics. The many-body time evolution problem: How far can we go?
- **Lecture 3** *Open quantum systems.* [7,8] Markovian master equations, Lindblad versus Redfield model. Non-equilibrium steady states. Quasi-free dynamics and its analytical description. Canonical quantization in the Liouville-Fock space. Non-equilibrium quantum phase transitions. [9,10,11]
- **Lecture 4** *Quantum transport problem.* Non-equilibrium transport from quantum master equation [12] versus linear response theory and Kubo formalism [3]. Examples of diffusive and ballistic quantum transport. The problem of spin diffusion. The relevance of integrability and conservation laws. Mazur inequality. The problem of microscopic derivation of the quantum Fourier law of heat conduction.
- **Lecture 5** *Quantum chaos in many body-systems.* The problem of quantum Loschmidt echoes and fidelity decay. [14] The relevance of quantum chaos for the transport problem and non-equilibrium phenomena. [13]

References

[1] Bloch I, Dalibard J and Zwerger W 2008 Rev. Mod. Phys. **80** 885
[2] Giamarchi T 2003 *Quantum Physics in One Dimension* Clarendon Press, Oxford
[3] Zotos X and Prelovšek P 2003 *Transport in one dimensional quantum systems* in “Interacting Electrons in Low Dimensions”, book series ”Physics and Chemistry of Materials with Low-Dimensional Structures”, Kluwer Academic Publishers
[4] Eisert J, Cramer M, Plenio M B 2010 Rev. Mod. Phys. **82** 277
[5] Amico L, Fazio R, Osterloh A and Vedral V 2008, Rev. Mod. Phys. **80** 517

[6] Schollwöck U 2011 Ann. Phys. (NY) **326** 96
[7] Breuer H-P and Petruccione F 2002 *The theory of open quantum systems*, Oxford University Press, New York
[8] Alicki R and Lendi K 2007 *Quantum dynamical semigroups and applications*, Springer, Heidelberg
[9] Prosen T 2008 New J. Phys. **10** 043026; Prosen T 2010 J. Stat. Mech. P07020
[10] Prosen T and Žunkovič B 2010 New. J. Phys. **12** 025016
[11] Prosen T and Pižorn I 2008 Phys. Rev. Lett. **101** 105701
[12] Prosen T and Žnidarič M 2009 J. Stat. Mech. P02035
[13] Prosen T 2007 J. Phys A: Math. Theor. **40** 7881
[14] Gorin T, Prosen T, Seligman T H, and Žnidarič M 2006 Phys. Rep. **435** 33

Chaotic examples in low-dimensional topology

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During the last three decades, Slovenian research group *Topology and Geometry* has been intensively working, in collaboration with several foreign research groups from United States, Japan and Russian Federation, in various areas of geometric topology. Among the more recently studied unsolved problems are also very interesting connections between chaos theory and geometric topology. In this lecture we shall illustrate, by means of examples, how certain methods of modern geometric topology can be used to construct very interesting and diverse examples in chaos:

(i) Using a classical example from 1930's, discovered by the celebrated British topologist J. H. C. Whitehead, of an open contractible topological 3-manifold which fails to be homeomorphic to the Euclidean 3-space R^3, we shall see how the corresponding Whitehead continuum "at infinity" is not a chaotic local attractor for one common method of embedding of this continuum;

(ii) We shall show how to modify the embedding of the Whitehead continuum from (i) so that the new embedding of this continuum becomes a chaotic local attractor;

(iii) We shall review the key techniques from modern geometric topology which are needed for verification of the intriguing properties of the special embedding of the Whitehead continuum in (ii); and also list some open problems and conjectures in this interesting and quickly developing research area.

References

Barge M, Martin J 1990 *Proc. Amer. Math. Soc.* **110** 523
Garity D J, Jubran I S, Schori R M 1997 *Houston J. Math.* **23** 33
Garity D J, Repovš 2008 *Amer. Inst. Phys. Conf. Proc.* **1076** 63
Ghrist R. W, van den Berg J B, van der Vorst R C 2003 *Invent. Math.* **152** 369
Gilmore R, Lefranc M 2002 *The Topology of Chaos* Hoboken:Wiley
Günther B 1994 *Proc. Amer. Math. Soc.* **120** 653
Kennedy J, Yorke J A, 2001 *Trans. Amer. Math. Soc.* **353** 2513
van den Berg J B, Vandervorst R C, Wójcik W 2007 *Topology Appl.* **154** 2580

Graphene Billiards

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As distinct from the (smooth) confinement potential of semiconductor-based lateral quantum dots, boundaries in graphene nanostructures, arising from abrupt lattice termination, are (locally) composed of zigzag- or armchair-type atomic arrangements, which for instance characterize nanoribbons and usually vary along the edges of a graphene quantum dot. Depending on the type of edge, namely zigzag or armchair, a ballistic graphene cavity can be regarded as representing a single Dirac billiard for massless fermions or two coupled copies of it, respectively. Hence the edges affect crucially the spectral and transport properties of graphene billiards. In particular, some edges cause effective time reversal symmetry breaking even if the system is literally time reversal invariant [1].

After a brief introduction into the relevant physics of graphene we develop a theoretical approach that is capable of handling such edge effects in graphene quantum dots. We will proceed in two steps. First, we derive an exact expression for the Green function of a mesoscopic graphene flake, where each term in the related expansion corresponds to the specific number of times the quasiparticle hits the edge. Second, we employ a semiclassical approximation for the Green function in the ballistic regime to derive Gutzwiller- and Berry-Tabor-type trace formulae for the energy spectra of chaotic and integrable, closed graphene systems [2]. Furthermore, we will consider the spectral statistics of chaotic graphene billiards, as well as the conductance of open quantum dots. In particular, we focus on graphene specific features in phase-coherence effects such as weak localization and universal conductance fluctuations.

References

[1] Wurm J, Rycerz A, Adagideli I, Wimmer M, Richter K and Baranger H U,
Symmetry Classes in Graphene Quantum Dots: Universal Spectral Statistics, Weak Localization, and Conductance Fluctuations, Phys. Rev. Lett. **102**, 056806 (2009)

[2] Wurm J, Richter K, Adagideli I,
Edge effects in graphene nanostructures: I. From the multiple reflection expansion to the trace formula for the density of states, arXive condmat/1104.4292

Quantum Chaos and Random Matrix Theories

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I shall present the fundamentals of quantum chaos, especially the statistical properties of energy spectra of the stationary Schrödinger equation. For Hamilton systems having the classical limit, we find in the semiclassical limit, that the spectral fluctuations obey the Poissonian statistics, whilst for classically ergodic and chaotic systems the statistics of the eigenvalues of the Gaussian random matrices applies. In both cases we have universality, as no free parameter appears. If the system is of the mixed type (generic system), having divided classical phase space (regular motion on invariant tori and chaotic motion on complementary initial conditions) we find in the strict semiclassical limit the statistical independence of regular and chaotic energy levels (Berry and Robnik 1984). This BR-picture rests upon the principle of uniform semiclassical condensation of Wigner functions (WF) of eigenstates, where we see in the semiclassical limit that WF fill uniformly the classically accessible phase space (regular WF on invariant tori, chaotic ones on the chaotic components), and the eigenvalues do not interact. At lower energies or larger values of the effective Planck constant we observe new effects, namely in the first place localization of chaotic eigenstates (nonuniform spreading of WF) and interaction of eigenvalues (due to the tunneling, i.e. overlap of semiclassical WF in classically forbidden regions). I shall present most recent models of random matrices which excellently describe these effects. Important examples are e.g. various 2D billiard systems and the hydrogen atom in strong magnetic field.

References

Batistić B and Robnik M 2010 *Journal of Physics A: Math. Theor.* **43** 215101

Stöckmann H.-J. 1999 *Quantum Chaos: An Introduction*, Cambridge Univ. Press

Robnik M 1998 *Nonlinear Phenomena in Complex Systems (Minsk)* **1** 1

Berry M V and Robnik M 1984 *J. Phys. A: Math. Gen.* **17** 2413

Gomez J M G, Relano A, Retamosa J, Faleiro E, Salasnich L, Vraničar M and Robnik M 2005 *Phys. Rev. Lett.* **94** 084101

Robnik M 2006 *International Journal of Bifurcation and Chaos* **16** No.6 1849

Grossmann S and Robnik M 2007 *Z. Naturforsch.* **62a** 471

Vidmar G, Stöckmann H.-J., Robnik M, Kuhl U, Höhmann R and Grossmann S 2007 *J.Phys.A: Math.Theor.* **40** 13883

Bäcker A, Ketzmerick R, Löck S, Robnik M, Vidmar G, Höhmann R, Kuhl U and Stöckmann H.-J. 2008 *Phys.Rev.Lett.* **100** 174103

Integrability of 3-dim Polynomial Systems of ODEs

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Integrability of systems of differential equations is one of central problems in the theory of ODEs. Although integrability is a rare phenomenon and a generic system is not integrable, integrable systems are important in studying various mathematical models, since often perturbations of integrable systems exhibit rich picture of bifurcations.

In our talk we first give some basics of the method of normal forms for autonomous systems

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{X}(\mathbf{x}),$$

where $\mathbf{x} = (x_1, \ldots, x_n)^T$, $X(\mathbf{x}) = (X_1(\mathbf{x}), \ldots X_n(\mathbf{x}))^T$ is an analytic vector-function whose series expansion starts from at least quadratic terms.

Then we present an efficient computational approach to find systems with first integrals within some families of polynomial systems of ordinary differential equations in the case when the matrix A of the linear approximation has one zero eigenvalue or two pure imaginary eigenvalues while the other eigenvalues have negative real parts. We apply it to find first integrals or conditions for their existence for three dimensional systems involving cubic polynomials. The procedure requires finding solutions of systems of polynomials, so we also discuss an approach to the decomposition of affine varieties using modular arithmetics.

References

Basov V V and Romanovski V G 2010 *J. Phys. A: Math. Theor.* **43** 315205-1
Bibikov Y N 1979 *Local Theory of Nonlinear Analytic Ordinary Differential Equations.* Lecture Notes in Mathematics, Vol. 702. Springer-Verlag, New York
Romanovski V G and Shafer D S 2009 *The Center and Cyclicity Problems: A Computational Algebra Approach*, Birkhäuser, Boston

Quantum Difference-Differential Equations

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Differential equations which contain the parameter of a scaling process are usually referred to by the name Quantum Difference-Differential Equations. Some of their applications to discrete models of the Schrödinger equation are presented and some of their rich, filigrane und sometimes unexpected analytic structures are revealed.

References

Ruffing A 2006 Habilitationsschrift TU München, Fakultät für Mathematik: *Contributions to Discrete Schrödinger Theory*
Meiler M and Ruffing A 2008 *Constructing Similarity Solutions to Discrete Basic Diffusion Equations*, Advances in Dynamical Systems and Applications, **3**, Nr. 1, 41–51
Ruffing A and Simon M 2004 *Difference Equations in Context of a q-Fourier Transform*, New Progress in Difference Equations, CRC press, 523–530
Ruffing A and Simon M 2008 *Analytic Aspects of q-Delayed Exponentials: Minimal Growth, Negative Zeros and Basis Ghost States*, Journal of Difference Equations and Applications, **14**, Nr. 4, 347–366
Ruffing A and Suhrer A 2010 *New Potentials in Discrete Schrödinger Theory*, Preprint 2010
Birk L, Roßkopf S and Ruffing A 2010 *Difference-Differential Operators and Gene-ralized Hermite Polynomials, Preprint 2010*

Surface Nanobubbles

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Surface nanobubbles are nanoscopic gas bubbles that form at the solid/liquid interface. They are surprisingly stable to bulk dissolution, surviving at least 11 orders of magnitude longer than the classical expectation. Here we describe a route to nucleation before providing a model for their remarkable stability. The key to the stability is that the gas in a nanobubble is of Knudsen type. This, combined with the broken symmetry created by the hard substrate and 'leaky' liquid/gas interface, leads to the generation of a bulk liquid flow which effectively forces the diffusive gas to remain local. Hence, the gas does indeed diffuse out of the nanobubble, but is trapped in this circulatory flow where it is transported back to the three-phase line for re-entry into the bubble.

References

Seddon J.R.T., Bliznyuk O., Kooij E.S., Poelsema B., Zandvliet H.J.W., and Lohse D. 2010 *Langmuir* **26** 9640

Seddon J.R.T. and Lohse D. 2011 *J. Phys. Cond. Mat.* **23** 133001

Seddon J.R.T., Kooij E.S., Poelsema B., Zandvliet H.J.W., and Lohse D. 2011 *Phys. Rev. Lett.* **106** 056101

Role of natural boundaries of KAM curves in quantum tunneling problems

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Natural boundaries are borders of analyticity of functions. In the textbook of complex analysis, one can find simple concrete examples which indeed have natural boundaries, but it is in general difficult to show that an arbitrarily given function has natural boundaries. In the early 80s, natural boundaries has been discussed in conjunction with arguments on the breakup of the KAM curves in the area-preserving map. Natural boundaries have been examined there as a mathematical tool to specify the condition for the last KAM curve.

Here we discuss possible roles of natural boundaries in quantum tunneling problem. Since tunneling is purely quantum mechanical, it may be reasonable that complex classical dynamics comes into play in its description, and the recent progress of complex semiclassical technique makes it possible. The issue of natural boundaries is often mentioned from the beginning of the study of multidimensional tunneling, but still its real role is not clear enough.

Our approach to investigate this problem is (0) to link, in analogy with one-dimensional complex dynamics, natural boundaries to the Julia set which are the most relevant objects in the complex classical description of quantum tunneling, (1) the precise identification of natural boundaries using Páde approximations to observe its manifestation in quantum dynamics (2) “complexifying” a piecewise affine map in a proper manner to develop some analytical arguments on the role of natural boundaries.

References

Costin O and Kruskal K 2005 *Comm.Pure Appl.Math.* **58** 723
Costin O and Huang M 2009 *Adv.Math.* **222** 1370
Creagh S C 1998 in *Tunneling in complex systems* ed. by S. Tomsovic (World Scientific, Singapore) p. 35
Greene J M and Percival I C 1981 *Physica D* **3** 530
Percival I C 1982 *Physica D* **6** 67
Shudo A, Ishii Y and Ikeda K S 2009 *J. Phys.A: Math.Theor.* **42** 265101
Shudo A, Ishii Y and Ikeda K S 2009 *J. Phys.A: Math.Theor.* **42** 265102

Efficient methods of chaos detection

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Determining the chaotic or regular nature of orbits of conservative dynamical systems is a fundamental problem of nonlinear dynamics, having applications to various scientific fields. The most commonly employed method for distinguishing between regular and chaotic behavior is the evaluation of the maximum Lyapunov exponent (MLE) σ_1, because if $\sigma_1 > 0$ the orbit is chaotic. The main problem of using σ_1 as a chaos indicator is that its numerical evaluation may take a long –and not known a priori– amount of time to provide a reliable estimation of the MLE's actual value. Over the years, several methods which try to avoid this problem have been introduced. In this talk we will present two such techniques, namely the smaller (SALI) and the generalized (GALI) alignment indices, which are based on the evolution of small deviations from a given orbit.

First we will recall the definition of the SALI, emphasizing its effectiveness in distinguishing between regular and chaotic motion. For the computation of the SALI one has to follow the evolution of an orbit and of two initially different unit deviation vectors from it (in contrast to the computation of the MLE where only one deviation vector is needed). The SALI is defined as the smaller norm of the vectors produced by the addition and the difference of the two evolved unit deviation vectors. The index exhibits completely different behaviors for chaotic and regular orbits, which help us distinguish between the two cases. In particular, it tends exponentially to zero for chaotic orbits, while it remains different from zero for regular ones. To illustrate the SALI's advantages we will apply it to a model of a simplified accelerator ring having sextupole nonlinearities, in order to estimate rapidly and accurately the dynamic aperture (i.e. the stability domain around the nominal circular orbit) of the system.

Then, we will focus on the GALI method. The generalized alignment index of order $k > 1$ (GALI$_k$), is defined as the volume of a generalized parallelepiped, whose edges are k initially linearly independent unit deviation vectors from the studied orbit. An efficient numerical scheme for the computation of the GALI, which is based on the singular value decomposition (SVD) algorithm, will be presented. We will also show analytically and verify numerically on particular examples of Hamiltonian systems and symplectic maps that, for chaotic orbits GALIs tend exponentially to zero with exponents that involve the values of several Lyapunov exponents, while in the case of regular orbits GALIs fluctuate around non-zero values or go to zero following power laws that depend on the dimension of the torus and the number of used deviation vectors. Then, exploiting their advantages we will demonstrate how one can use the GALIs for identifying quasiperiodic motion on low-dimensional tori.

Finally, we will discuss the numerical integration of the variational equations, which govern the evolution of deviation vectors in Hamiltonian systems. These equations have to be integrated simultaneously with the Hamilton's equations of motion which define the evolution of orbits. We will show how one can integrate this extended set of differential equations by the so-called '*tangent map (TM) method*', a scheme based on symplectic integration techniques. According to this method, a symplectic integrator is used to approximate the solution of the Hamilton's equations of motion by the repeated action of a symplectic map S, while the corresponding tangent map TS, is used for the integration of the variational equations. We will present a simple and systematic technique to construct TS. Then we will show that the TM method is superior to other commonly used numerical schemes for integrating the variational equations of low- and high-dimensional Hamiltonian systems, both with respect to its accuracy and its speed.

References

Skokos Ch 2001 *J. Phys. A* **34** 10029
Skokos Ch, Antonopoulos Ch, Bountis T C and Vrahatis M N 2003 *Prog. Theor. Phys. Supp.* **150** 439
Skokos Ch, Antonopoulos Ch, Bountis T C and Vrahatis M N 2003 *J. Phys. A* **37** 6269
Skokos Ch, Bountis T C and Antonopoulos Ch 2007 *Physica D* **231** 30
Skokos Ch, Bountis T C and Antonopoulos Ch 2008 *Eur. Phys. J. Sp. Top.* **165** 5
Manos T, Skokos Ch and Antonopoulos Ch 2011 *e-print arXiv:1103.0700*
Bountis T and Skokos Ch 2006 *Nucl. Instr. Meth. Phys. Res. - Sect. A* **561** 173
Boreux J, Carletti T, Skokos Ch and Vittot M 2010 *e-print arXiv:1007.1565*
Boreux J, Carletti T, Skokos Ch, Papaphilippou Y and Vittot M 2011 *e-print arXiv:1103.5631*

Skokos Ch and Gerlach E 2010 *Phys. Rev. E* **82** 036704
Gerlach E and Skokos Ch 2010 *e-print arXiv:1008.1890*
Gerlach E, Eggl S and Skokos Ch 2011 *e-print arXiv:1104.3127*

From neural activity to movement

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The human brain has about 85 billion neurons, of which about 15 billion are part of the cortex. These cortical neurons fire action potentials (spikes) at a rate of about 10 Hz. Thus, neurons in our cortex emit about 150 billion spikes per second. This activity underlies our sensory perceptions, our thoughts, our decisions, our actions. One of the central problems of systems neuroscience is that of *decoding* these vast spatial and temporal patterns of neural activity so as to interpret them and assign meaning to them. In the last ten years, much progress has been made in decoding activity of neurons in the motor cortex, an output area of the brain that controls movement through its projection to muscles via the spinal cord. In this talk I will report on reproducible experiments carried out by several groups since 2000. These experiments are based on the implantation of multielectrode arrays that record neural activity in awake behaving monkeys. The arrays allow us to monitor the activity of about one hundred neurons in motor cortex during the execution of sequences of reaches to provided targets. I will describe our theoretical efforts to construct models that capture the underlying relationship between neural activity and movement, and thus predict the direction and extent of a reach before the movement is executed. From a theoretical point of view, these studies have allowed us to make substantial progress in our understanding of the neural code. From a practical point of view, our increasing ability to extract information from neural signals has allowed us to translate neural activity into commands to control computer cursors and robotic manipulators. The potential of this approach to restore motor behavior in severely handicapped patients motivates pioneering interdisciplinary research in Brain Machine Interfaces (BMIs), a new area at the frontier of systems neuroscience.

References

Wessberg J, Stambaugh C R, Kralik J D, Beck P D, Laubach M, Chapin J K, Kim J, Biggs S J, Srinivasan M A and Nicolelis M A L 2000 *Nature* **408** 361

Serruya M D, Hatsopoulos N G, Paninski L, Fellows M R and Donoghue J P 2002 *Nature* **416** 141

Taylor D M, Helms Tillery S I and Schwartz A B 2002 *Science* **296** 1829

Lebedev M A and Nicolelis M A L 2006 *Trends Neurosci.* **9** 536

Santhanam G, Ryu S I, Yu B M, Afshar A and Shenoy K V 2006 *Nature* **442** 195

Fagg A H, Hatsopoulos N G, London B M, Reimer J, Solla S A, Wang D and Miller L E *IEEE Eng. Med. Biol. Soc.* **2009** 3376

Biological oscillators

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In this series of lectures we shall reconsider the long-standing question “What is life?” probably best known as formulated by Erwin Schrödinger in his book of that title.

Schrödinger’s paradox is that, in a world governed by the Second Law of Thermodynamics, all closed systems are expected to approach a state of maximum disorder – but life approaches and maintains a highly ordered state, which seems to violate the Second Law. The paradox is resolved by noting that life is not a closed system. Corresponding to the increase of order inside an organism is the decrease in order outside this organism. Overall, the Second Law is obeyed, and life maintains a highly ordered state, which it sustains by continuously increasing the disorder in the Universe.

Living systems will be considered as being thermodynamically open. A continuous exchange of energy and matter is needed to maintain a living system, and this gives rise to rate processes. The rate processes result in oscillations and the time-scales on which the processes can occur are what determine the frequencies of the oscillations. Those processes which occur on a certain time scale contribute to the specific function and structure of a system that on the macroscopic scale can be considered as an entity.

In this way living matter can be considered as ensembles of oscillators that interact internally within the ensemble, and the ensembles also interact one with another. The magnitude of their activity is proportional to the level of synchronization they achieve.

Therefore, we will briefly discuss the non-equilibrium thermodynamics approach introduced by Hermann Haken and the phase dynamics approach introduced by Yoshiki Kuramoto. We will review current developments in the field – both theoretical studies of coupled ensembles of oscillators as well as methods of analysis to study properties from measured data.

First we will illustrate the concept, discussing a living cell and its function and considering the cell as an ensemble of oscillators; then we will move on to cardiovascular dynamics and then to brain dynamics. We will summarize the lecture series by arguing that a new theory of nonautonomous dynamical systems is needed in order to advance our understanding and to provide a mathematical description of living systems. We will present the state of the art for this new and fast-developing field.

The following lectures will be included –

1. Phase dynamics theory and its application to inverse problems
2. A living cell and its dynamics
3. Cardiovascular dynamics
4. Brain dynamics
5. Recent developments in nonautonomous systems.

References

Schrödinger E 1944 *What is Life?*, Macmillan
Haken H 1975 *Rev Mod Phys*, **47** 67
Kuramoto Y, 1984 *Chemical Oscillations, Waves, and Turbulence*, Springer, Berlin
Stefanovska A, Haken H, McClintock PVE, Hožič M, Bajrović F, Ribarič S 2000 *Phys Rev Lett*, **22** 4831
Bahraminasab A, Ghasemi F, Stefanovska A, McClintock PVE, Friedrich R 2009 *New J Phys*, **11** 103051
Shiogay Y, Stefanovska A, McClintock PVE 2010 *Phys Rep*, **488** 51
Jamšek J, Paluš M, Stefanovska A 2010 *Phys Rev E*, **81** 036207
Deco G, Jirsa VK, McIntosh AR 2011 Nature Rew Neuroscience, **12** 43

Marvel SA, Kleinberg J, Kleinberg RD, Strogatz SH 2011 *PNAS*, **108** 1771
Sheppard LW, Stefanovska A, McClintock PVE 2011 *Phys Rev E*, **83** 016206
Garcia-Ojalvo J 2011 *Contemp Phys*, in press
Rasmussen M, Kloeden P 2011 *Nonautonomous Dynamical Systems*, AMS Mathematical Surveys and Monographs, In press

Quantum chaos and random matrix theory

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Random matrix theory is the standard tool to describe the universal properties of the spectra of classically chaotic systems. An overview on the existing experiments is presented, in particular from nuclear and atomic physics, mesoscopic physics, and a variety of billiard systems. On the theoretical side the basic concepts are introduced, including spectral density, level spacing distribution, two-point correlation function, spectral form factor, and related quantities. Spectral level dynamics is another important aspect, i.e. the development of the eigenvalues under the influence of an external perturbation. The course ends with an introduction into supersymmetry theory, the method of choice to perform averages over ensembles of random matrices.

References

Mehta M. L. 1991 *Random Matrices, 2nd Ed.*, Academic Press, San Diego
Stöckmann H.-J. 1999 *Quantum Chaos: An Introduction*, Cambridge University Press, Cambridge
Haake F 2010 *Quantum Signatures of Chaos, 3nd Ed.*, Springer, Heidelberg

Long-lasting neuronal desynchronization caused by coordinated reset stimulation

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A number of brain diseases, e.g. movement disorders such as Parkinsons disease, are characterized by abnormal neuronal synchronization. Within the last years permanent high-frequency (HF) deep brain stimulation became the standard therapy for medically refractory movement disorders. To overcome limitations of standard HF deep brain stimulation, we use a model based approach. To this end, we make mathematical models of affected neuronal target populations and use methods from statistical physics and nonlinear dynamics to develop mild and efficient control techniques. Along the lines of a top-down approach we test our control techniques in oscillator networks as well as neural networks. In particular, we specifically utilize dynamical self-organization principles and plasticity rules. In this way, we have developed coordinated reset (CR) stimulation, an effectively desynchronizing brain stimulation technique. The goal of CR stimulation is not only to counteract pathological synchronization on a fast time scale, but also to unlearn pathological synchrony by therapeutically reshaping neural networks.

The CR theory, results from animal experiments as well as clinical applications will be presented: Animal and human data will be shown on electrical CR stimulation for the treatment of Parkinsons disease via chronically implanted depth electrodes. Furthermore, acoustic CR stimulation for the treatment of subjective tinnitus will be explained. Subjective tinnitus is an acoustic phantom phenomenon characterized by abnormal synchronization in the central auditory system. In a multicenter proof of concept study it has been shown that acoustic CR stimulation significantly and effectively counteracts tinnitus symptoms as well as the underlying pathological neuronal synchronization processes.

References

Tass PA 2003 *Biol. Cybern.*, **94** 81-88
Tass PA, Majtanik M 2006 *Biol. Cybern.*, **94** 58-66
Tass PA, Silchenko A, Hauptmann C, Barnikol UB, Speckmann E-J 2009 *Phys. Rev. E* **80** 011902
Hauptmann C, Tass PA 2010 *J. Neural Eng.* **7** 056008

Time series analysis using wavelet for molecular dynamics simulation of proteins

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A new method to extract nonstationary features of coarse grained motions is presented for time series data of molecular dynamics simulation of proteins. We use the wavelet transformation together with the singular value decomposition (SVD). The wavelet analysis enables us to characterize time varying features of the dynamics and SVD enables us to reduce the degrees of freedom of the data. We apply our method to time series data obtained by molecular dynamics simulation for Adenylate Kinase from *Escherichia coli* (AKE), and *Thermomyces lanuginosa* lipase (TLL).

For the case of AKE, we show that the first singular vector alone can describe main features of the collective motions. Moreover, time dependence of the first singular vector reveals transient features of slow collective motions both in space and frequency. Introducing quantities which characterize similarity of such transient features, we have identified several types of collective motions. As for the space, the most typical types exhibit collective movement of the domains, and the boundaries of these collective oscillations coincide with the hinges identified previously. However, more complicated features of slow motions are also revealed, which indicates that some parts of the domains exhibit separate slow oscillations. As for the frequency, we have noticed that peaks of the spectra vary as time evolves. Such time-dependence of the spectra implies importance of nonlinear effects which result in energy transfer among collective motions.

For the case of TTL, by introducing indexes to characterize collective motion of the protein, we have obtained the following two results. First, time evolution of the collective motion involves not only the dynamics within a single potential well but takes place wandering around multiple conformations. Second, correlation of the collective motion between secondary structures shows that collective motion exists involving multiple secondary structures. We discuss future prospects of our study involving "disordered proteins".

References

Sakurai N, Toda M, Fuchigami S, and Kidera A, to be submitted
Kamada M, Toda M, Sekijima M, Takada M and Joe K 2011 *Chem. Phys. Lett.* **502** 241

Soft-Mode Turbulence and Symmetry

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Abstract

The soft-mode turbulence (SMT) is an unusual type of spatiotemporal chaos at onset analogous to the second order phase transitions in equilibrium systems. It was discovered in 1995-96 [1-4] and still remains an appealing issue. SMT arises as a result of a single supercritical bifurcation from a quiescent state. On one hand it exhibits typical features of developed turbulence, such as the Kolmogorov cascades (both normal and inverted), interplay of different spatiotemporal scales, decay of correlations, etc. On the other hand it is characterized by critical slowing down and divergence of the correlation length at the onset, typical to the second order phase transitions [5]. In the present contribution deep connection between SMT and the problem symmetry (including effects of weakly broken symmetry) is revealed. It is shown that the origin of SMT is in coupling of short-wavelength modes, related to a Turing type instability of spatially uniform states of the system, with slow long-wavelength modes detaching from a neutrally stable Goldstone mode, related to the problem symmetry. The symmetry violation may result in suppression of SMT, so that instead of SMT spatially periodic patterns arise. Symmetry-related constrains imposed on the the structure of the corresponding stability problems are discussed too. SMT may exhibit unusual scaling properties, which also are inspected. The developed theory is compared with experiment. Finally some open questions are indicated.

References

[1] H. Richter, A. Buka, and I. Rehberg 1995 *Phys. Rev. E* **51**, 5886.
[2] S. Kai, K. Hayashi, and Y. Hidaka 1996 *J. Phys. Chem.* **100**, 19007.
[3] M. I. Tribelsky and K. Tsuboi 1996 *Phys. Rev. Lett.* **76**, 1631.
[4] A. G. Rossberg, A. Hertrich, L. Kramer, and W. Pesch 1996 *Phys. Rev. Lett.* **76**, 4729.
[5] M. I. Tribel'skii 1997 *Phys. Usp.* **40**, 159.

Measure stretching exponents and cosmic ray arrival directions

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In a chaotic system, the long term stretching of the local neighbourhood of a trajectory is characterized by the Lyapunov exponent. In an ergodic system, the Lyapunov exponent is independent of the initial point, meaning that long time local stretching rates are equal everywhere. Nevertheless, when observing the stretching of some global object in phase space when evolved under the phase space flow, the long term increase of its measure is not characterized by the Lyapunov exponents but by a set of related stretching exponents that are different and typically larger than the corresponding Lyapunov exponents. I will demonstrate the origin of the discrepancy between the local and global stretching exponents and how it relates to variances in short time stretching rates.

I will then focus on the case of deterministic chaotic random maps. These can be considered as a model for cosmic rays traveling through random magnetic fields. I will demonstrate that the global length stretching exponent is the relevant quantity characterising the number of possible different directions for cosmic rays to reach a point in space (i.e. Earth) when originating from a single source.

References

Mehlig B and Wilkinson M 2004 *Phys. Rev. Lett.* **92** 250602

Veble G and Prosen T 2004 *Phys. Rev. Lett* **92** 034101

Vorobiov S, Hussain M and Veberič D 2008 *Studies of the UHECR propagation in the Galactic Magnetic Field* `arXiv:0901.1579v1`, presented at the 21st ECRS in Kosice, Slovakia, 9-12 September 2008

Fourier's Law for quasi-one-dimensional chaotic quantum systems

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We derive Fourier's law for a completely coherent quasi-one-dimensional chaotic quantum system coupled locally to two heat baths at different temperatures. We solve the master equation to first order in the temperature difference. We show that the heat conductance can be expressed as a thermodynamic equilibrium coefficient taken at some intermediate temperature. We use that expression to show that for temperatures large compared to the mean level spacing of the system, the heat conductance is inversely proportional to the level density and, thus, inversely proportional to the length of the system.

Test-tube model for rainfall

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The lack of quantitative predictions about the life cycle of clouds is one of the major obstacles to properly account for the earth albedo in climate models. A central problem is the 'condensation-coalescence bottleneck' of the growth of intermediate size droplets (see, for example, Shaw, 2003, Falkovich, Fouxon and Stepanov, 2002). Recent work addresses this problem by measurement campaigns in clouds, large-scale laboratory models, and high performance computing. Here we suggest a complementary approach by using the universality of hydrodynamic equations to map the problem to a test-tube setting. We model droplet growth by a period of Ostwald ripening, which is treated using the approach of Lifshitz and Slezov (1961), followed by a finite-time runaway growth of droplet sizes due to larger droplets sweeping up smaller ones. The theory predicts that the period Δt to arrive at precipitation is related to the temperature sweep rate ξ by $\Delta t \sim \xi^{-3/7}$, in good agreement with the experiment. The theory also resolves an apparent bottleneck in the kinetics of rain droplet growth for warm clouds in a convectively stable atmosphere.

References

R. A. Shaw, Particle-turbulence interactions in atmospheric clouds, *Ann. Rev. Fluid Mech.*, **35**, 183-227, (2003).

G. Falkovich, A. Fouxon and M. G. Stepanov, Acceleration of rain initiation by cloud turbulence, *Nature*, **419**, 151-4, (2002).

E. M. Lifshitz and V. V. Slyozov, The kinetics of precipitation from supersaturated solid solutions, *J. Phys. Chem. Solids*, **19**, 35, (1961).

A nonlinear dynamics approach to Bose-Einstein condensates

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It is well known that at sufficiently low temperatures the Gross-Pitaevskii equation (GPE), a nonlinear Schrödinger equation for the macroscopic wave function, provides an accurate description of the dynamics of dilute trapped Bose-Einstein condensates for both the ground state and the excitation spectrum. Because of its nonlinearity, the GPE can in general only be solved numerically, e.g. by imaginary time evolution. We pursue a different approach: We assume the wave function as a superposition of Gaussian wave packets, with time-dependent complex width parameters, insert it into the mean-field energy functional corresponding to the GPE, and apply the time-dependent variational principle. In this way the GPE is transformed into a system of coupled equations of motion for the complex width parameters, which can be analyzed with the methods of nonlinear dynamics, and which exhibits all the richness of phenomena typical of nonlinear systems, such as a transition from order to chaos or the appearance of different bifurcation scenarios. Even for condensates with long-range interactions (monopolar and dipolar), in addition to the short-range contact interaction, the method leads to highly accurate results for energies and wave functions, and thus turns out to be a full-fledged alternative to numerical quantum calculations. Moreover, the method is capable of giving access to regions of the space of solutions of the GPE that are difficult or impossible to investigate by conventional numerical calculations. A stability analysis of the fixed point solutions of the equations of motion even yields the low-lying quantum mechanical Bogoliubov excitation spectrum of the condensates.

References

Fabčič T, Main J and Wunner G 2009 *Phys. Rev. A* **79** 043416
Köberle P, Cartarius H, Fabčič T, Main J and Wunner G 2009 *New Journal of Physics* **11** 023017
Köberle P and Wunner G 2009 *Phys. Rev. A* **80** 063601
Rau S, Main J, Köberle P and Wunner G 2010 *Phys. Rev. A* **81** 031605(R)
Rau S, Main J and Wunner G 2010 *Phys. Rev. A* **82** 023610
Rau S, Main J, Cartarius H, Köberle P and Wunner G 2010 *Phys. Rev. A* **82** 023611
Junginger A, Main J and Wunner G 2010 *Phys. Rev. A* **82** 023602
Eichler R, Main J and Wunner G 2011 *Phys. Rev. A* in press

An Algorithm for Melnikov Functions and Applications

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In this work we study a dynamical system with a complicated nonlinearity, which describes oscillation of a turbine rotor, and give an algorithm to compute Melnikov functions for analysis of its chaotic behavior. We first derive the rotor model whose nonlinear term brings difficulties to investigating the distribution and qualitative properties of its equilibria. This nonlinear model provides a typical example of a system for which the homoclinic and heteroclinic orbits cannot be analytically determined. In order to apply Melnikov's method to make clear the underlying conditions for chaotic motion, we present a generic algorithm that provides a systematic procedure to compute Melnikov functions numerically. Substantial analysis is done so that the numerical approximation precision at each phase of the computation can be guaranteed. Using the algorithm developed in this paper, it is straightforward to obtain a sufficient condition for chaotic motion under damping and periodic external excitation, whenever the rotor parameters are given. This is a joint work with Jianxin Xu and Rui Yan.

References

Ashwin P and Mei Z 1998 *SIAM J. Numer. Anal.* **35** 2055
Childs D 1993 *Turbomachinery Rotordynamics*, John Wiley & Sons, New York
Davis P J and Rabinowitz P 1975 *Methods of Numerical Interaction*, Academic Press, New York
Espelid T O and Overholt K J 1994 *Numer. Algorithms* **8** 83
Goodwin M J 1989 *Dynamics of Rotor-Bearing Systems*, Unwin Hyman, London
Govaerts W, Kuznetsov Yu A and Sijnave B, *SIAM J. Numer. Anal.* **38** 329
Guckenheimer J and Holmes P 1983 *Nonlinear Oscillations: Dynamical Systems and Bifurcations of Vector Fields*, Springer-Verlag, New York
Guckenheimer J, Myers M and Sturmfels B 1997 *SIAM J. Numer.Anal.* **34** 1
Guckenheimer J and Meloon B, *SIAM J. Sci. Comput.* **22** 951
Kim C.-S. and Lee C.-W. 1993 *Vibration of Rotating Systems*, DE-Vol. 60, ed. Wang K W and Segalman D, ASME, New York, 325
Kuang J, Tan S and Leung A Y T 2002 *Int. J. Control* **75** 328
Liu L, Moore G and Russell R D 1997 *SIAM J. Sci. Comput.* **18** *69*
Tong X, Tabarrok B and Rimrott F P J 1995 Int. J. Non-linear Mech. **30** *191*