Econophysics II: 
Detailed Look at Stock Markets and Trading 

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Let’s Face Chaos through Nonlinear Dynamics, Maribor 2011
Outline

• stock markets and trading in reality
• two extreme model scenarios: Efficient Market Hypothesis, Zero Intelligence Trading
• large scale data analysis reveals non–Markovian features
• artificial stock market to encircle mechanisms
• trading strategies and temporal correlations
Stock Market Trading in Reality
Clearing House and Orders

trading via clearing house, buy and sell offers/orders (bids and asks)

limit order: bid or ask for a specific volume at a specific price within a certain time window,

best ask $a(t)$, best bid $b(t)$, always $a(t) \geq b(t)$

if equal $\rightarrow$ trade, price $S(t) = a(t) = b(t)$ immediately thereafter

market order: buy or sell immediately what is offered,
$S(t) = b(t)$ or $S(t) = a(t)$
Order Book

make public to provide all traders with same information

limit orders appear in the order book, market orders do not
in between trades, there is no price!

bid–ask spread \[ s(t) = b(t) - a(t) < 0 \]
the higher the trading frequency, the smaller is \( s(t) \)

midpoint \[ m(t) = \frac{a(t) + b(t)}{2} \]

immediately after a trade, define trade sign
\[
\vartheta(t) = \begin{cases} 
+1 & \text{if } S(t) \text{ is higher than the last } m(t) \\
-1 & \text{if } S(t) \text{ is lower than the last } m(t)
\end{cases}
\]
positive, if trade was triggered by a market order to buy
negative, if trade was triggered by a market order to sell
Traders and Liquidity

Market is liquid, if there are always enough shares at a “reasonable” price to ensure that every planned trade can be carried out and if the trading happens continuously.

Small bid-ask spread \( s(t) = b(t) - a(t) \) is indicator.

Limit orders make a market liquid \( \leftrightarrow \) liquidity providers.

Market orders absorb liquidity \( \leftrightarrow \) liquidity takers (“informed”).

Liquidity providers and takers are not static populations, these rôles change constantly.
Model Scenarios
First Extreme Model Scenario — EMH

Efficient Market Hypothesis

Traders always act fully rationally. Market price results from consensus between the traders about the “fair” price. It always exists and reflects quantifiable economic value of asset. Deviation of market price from “fair” price → arbitrage → disappears. Consensus comes about, because group of traders processed all available information. Price can only change if new information arrives. The new information is totally random.
Second Extreme Model Scenario — ZIT

Zero Intelligence Trading

Individual trader is irrational and acts fully randomly. The other traders do not know that and interpret the buy and sell decisions made by others as potentially information driven. Price change is not attributed to new information, it automatically follows from the fact that trading takes place \( \rightarrow \) demand and supply. There is no fair price, midpoint \( m(t) \) moves as well. Traders immediately accept the new midpoint as the new reference point about which they send out their random buy and sell orders.
Where is the Truth?

both scenarios lead to a Markovian random walk model for price!

both partly compatible with reality, but there are objections:

**EMH:** “fair” price deeply obscure $\rightarrow$ what is then rational? — high volatilities incompatible with rational pricing — time scales of trading not consistent with those of information flow

**ZIT:** irrationality not realistic either, traders use information

truth is somewhere in between $\rightarrow$ need detailed data analysis
Collecting Empirical Information
Data Analysis


- fully electronically traded French stocks 2001–2002
- intraday
- high frequency, up to 10000 trades/day
- volumes between a few and 80000 shares
- trade time instead of real time
Volatility and Diffusion

subtract drift from $S(t) \rightarrow$ detrended price $Z(t)$

diffusion function $D(\tau) = \langle (Z(t + \tau) - Z(t))^2 \rangle$

largely constant volatility function

$\sqrt{D(\tau)/\tau}$

for France–Telecom

diffusive motion!
Average Response to Trading

response function $R(\tau) = \langle (Z(t + \tau) - Z(t)) \vartheta(t) \rangle$

average impact of trading at $t$ on subsequent price changes

non–zero empirical result proves non–Markovian behavior!
Distribution of Sign Supplemented Price Changes

sign supplemented price changes $u(t, \tau) = (Z(t + \tau) - Z(t)) \vartheta(t)$
response $R(\tau) = \langle u(t, \tau) \rangle$, diffusion function $D(\tau) = \langle u^2(t, \tau) \rangle$

distribution $p(u(t, \tau))$ for $\tau = 128$

moment $R(128) > 0$

small arbitrage

truly informed traders
Power Law Autocorrelations in Trade Signs

Trade sign autocorrelation

\[ \Theta(\tau) = \langle \vartheta(t + \tau) \vartheta(t) \rangle - \langle \vartheta(t) \rangle^2 \]

Power law

\[ \Theta(\tau) \sim \frac{1}{\tau^\gamma} \]

With \( \gamma < 0 \)

Non-Markovian, outrules ZIT idea!
Modeling the Price Dynamics
Non–Markovian Model

\[ Z(t) = \sum_{t'=1}^{t} G_0(t - t') \vartheta(t') \ln V(t') + \sum_{t'=1}^{t} \varepsilon(t') \]

first term non–Markovian, second Markovian

ansatz for bare impact function \( G_0(\tau) \sim \frac{1}{(1 + \tau/\tau_0)^\beta} \)

\[ \rightarrow D(\tau) \sim \tau^{2-2\beta-\gamma}, \quad \text{critical exponent } \beta_c = \frac{1 - \gamma}{2} \]

\( \beta = \beta_c \) diffusive, \( \beta > \beta_c \) sub–diffusive, \( \beta < \beta_c \) super–diffusive

possible to reproduce empirical \( R(\tau) \) for \( \beta \approx \beta_c \) and \( \tau_0 \approx 20 \)

Liquidity Takers versus Liquidity Providers

Reality is non–Markovian, interpreted as a competition: Consider trader who is “informed” that price of a company will go up. He wants to buy shares, likely by market orders → liquidity taker. Not be wise to place big offer, because this would alert liquidity providers who emit the limit orders to sell (“knows something”). They would place their limit orders at higher price. Liquidity taker is aware → divides his market order into smaller chunks which he places one after the other → introduces temporal autocorrelations Θ(τ). Liquidity providers want to mean revert price → $R(\tau) \to 0$ for large $\tau$. They do that slowly, because they do not know whether liquidity taker’s information becomes true → maximum of $R(\tau)$. → Persistence: liquidity providers do not sufficiently mean revert the price → super–diffusive. Antipersistence: they mean revert too strongly → sub–diffusive. → Subtle balance between sub– and super–diffusive → effectively diffusive. Compares to balancing a stick on the palm.
Artifical Stock Market
Agent Based Modeling

financial markets are complex systems: many degrees of freedom, non-linear effects, basic processes and time evolution governed by probabilistic rules, not by deterministic equations

top–down approach: schematic models, stochastic processes → successes and limitations

bottom–up approach: artificial stock market on computer with virtual traders → agent based modeling
  • set up system microscopically and let evolve
  • price dynamics and all other macroscopic observables result
  • identify crucial mechanisms by encircling them

various examples in biology, social sciences, economics, one of the first is Conway’s Game of Life (1970)
Wigner’s Caveat

“It is nice to know that the computer understands the problem. But I would like to understand it too.”

Eugene Paul Wigner, 1902–1995
Impact of Trading Strategies

introduce different types of traders traders → top–down element in an otherwise bottom–up approach, not adaptive

- ZeroIntelligenceTrader
- RandomTrader
- EagerTrader
- LiquidityProvider
- RandomInformedTrader
- SerialTrader
- ExpectingTrader

let evolve and see what happens !

Berseus, Schäfer, Guhr (2007)
Zero Intelligence Trading

population of 300 ZeroIntelligenceTraders return distribution after 10000 trades

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non–Gaussian,

heavy tails !

Maribor, June/July 2011
Heavy Tails and Order Book

population of 300 traders, distributions of price differences
EagerTraders and three versions of RandomTraders

non-Gaussian
when order book becomes
more important

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Mixed Populations — Volatility Function

LiquidityProvider and three versions of RandomInformedTraders

![Graph showing volatility function](image)

- \( \lambda = 0.1 \)
- \( \lambda = 0.01 \)
- \( \lambda = 0.001 \)

largely diffusive

depends sensitively on likeliness to emit market orders

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Mixed Populations — Trade Sign Autocorrelations

Liquidity Providers, Serial Traders

Liquidity Providers, Expecting Traders

Expecting Trader waits for gap between $m(t)$ and “fair” price, Serial Trader does not $\rightarrow$ trade sign autocorrelations
Summary and Conclusions

• stock markets and trading in reality
• two extreme model scenarios: Efficient Market Hypothesis, Zero Intelligence Trading
• large scale data analysis reveals non–Markovian features
• schematic top–down stochastic model
• artificial stock market as bottom–down approach
• heavy tails are order book effect
• trading strategies sensitively determine temporal correlations