Econophysics I:
Basic Concepts

Thomas Guhr

Let’s Face Chaos through Nonlinear Dynamics, Maribor 2011
Mutual Attraction between Physics and Economics

Mathematical modeling in physics and economics has always been similar, many connections exist for a long time: Bachelier, Einstein, Mandelbrot, Markowitz, Black, ...

during the last 15 to 20 years, the number of physicists working on economics problems has grown quickly, the term “econophysics” was coined

Physics $\rightarrow$ Economics: much better economical data now, general interest in complex systems

Economics $\rightarrow$ Physics: risk management, expertise in model building based on empirical data
“... Every tenth academic hired by Deutsche Bank is a natural scientist ... ”
Outline

• some economical basics: market, price, arbitrage, efficiency
• empirical results and models for price dynamics
• importance of financial derivatives and options
• portfolio and risk management
• rôle of financial correlations
Who Makes the Market Price?
Who Makes the Market Price?
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the price of an asset is made by demand and supply

\[ \begin{align*}
\text{demand up and/or supply down} & \quad \rightarrow \quad \text{price up} \\
\text{demand down and/or supply up} & \quad \rightarrow \quad \text{price down}
\end{align*} \]
Who Makes the Market Price?

The price of an asset is made by demand and supply. If demand increases and/or supply decreases, the price goes up. If demand decreases and/or supply increases, the price goes down. Therefore, the market makes the price!
Exploit Price Difference for Oranges

5000 oranges yield riskless and quick profit

\[ 5000 \cdot (22 - 20) \text{ Cents} = 100 \text{ Euros} \]

(no fees, etc)
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continue doing that, other people start doing the same →

Maribor: orange supply goes up → price goes down
Ljubljana: orange supply goes down → price goes up

→ prices in Ljubljana and Maribor equilibrate!
Arbitrage and Efficiency

risk and time characterize economical transaction (deal, trade)
bank: no risk, long time $\iff$ lottery: high risk, short time
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arbitrage: transaction that yields riskless and instantaneous profit
arbitrage  \rightarrow  price differences  \rightarrow  equilibration  \rightarrow  no arbitrage
arbitrage destroys itself!  \rightarrow  arbitrage time scale
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arbitrage $\rightarrow$ price differences $\rightarrow$ equilibration $\rightarrow$ no arbitrage
arbitrage destroys itself! $\rightarrow$ arbitrage time scale

many arbitrage opportunities in commodity markets, very few in capital markets

requires homogenous information, absence of operational obstacles $\rightarrow$ efficiency
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assumption: no arbitrage in the capital markets!
Empirical Evidence

Coca Cola

Maribor, June/July 2011
Empirical Evidence

many traders on same market we expect a “fair game”

\[ \langle dS(t) dS(t + \tau) \rangle_t \sim \delta(\tau) \]

no prediction possible!
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\[
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Maribor, June/July 2011
stochastic differential equation \[ dS(t) = \sigma \varepsilon(t) \sqrt{dt} \]

\( \varepsilon(t) \) uncorrelated random for every \( t \), volatility \( \sigma \) is a constant


notation \[ dS = \sigma \varepsilon \sqrt{dt} \]
Ballistic versus Diffusive

particles moving in liquid in cells of plants

Perrin (1948)

two–dimensional!

ballistic \( x(t) \sim t \) ←→ diffusive \( \langle x^2(t) \rangle \sim t \)
Very Long Time Scales

Dow–Jones 1974–2004

\[ S(t) \sim \exp(\mu t) \]

\[ \Rightarrow \quad dS = S\mu dt \]

\( \mu \) is the drift constant

deterministic and stochastic part in stock price dynamics!

\[ dS = S \left( \mu dt + \sigma \varepsilon \sqrt{dt} \right) \quad \Rightarrow \quad \frac{dS}{S} = \mu dt + \sigma \varepsilon \sqrt{dt} \]

geometric Brownian motion
Connection to Banks and Interest

put an amount of money $V(t)$ in a bank account
receive interest at a rate $r$ within time $dt$

$$dV = V r dt \quad \longrightarrow \quad V(t) = V(t_0) \exp(r(t - t_0))$$

also an exponential law!
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loosely speaking: the average of all stock market investors must make as much money as the average of all bank investors

\[ \longrightarrow \quad \text{“global no–arbitrage effect”} \]
Return and Logarithmic Difference

the quantities $dV/V$ and $dS/S$ are returns

logarithmic difference

$$G(t) = \ln S(t + \Delta t) - \ln S(t) = \ln \frac{S(t + \Delta t)}{S(t)}$$

the same for small time intervals $\Delta t$

$$dS = S(t + \Delta t) - S(t)$$

$$G(t) = \ln \frac{S(t) + dS}{S(t)} = \ln \left( 1 + \frac{dS}{S} \right) \approx \frac{dS}{S}$$

Maribor, June/July 2011
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generic Brownian motion
Resulting Distributions

geometric Brownian motion implies log–normal price distribution with variance $\sigma^2 \cdot (t - t_0)$ for moving window $t - t_0$ (here one day)

---

low frequency  

high frequency  

$\rightarrow$ tails are much fatter than expected!
Large Events

returns normalized to unit variance from Standard & Poor’s 500

always 850 points

clearly non–Gaussian

Gopikrishnan, Plerou, Amaral, Meyer, Stanley, PRE 60 (1999) 5305
Power Law Far in the Tails

Standard & Poor’s 500 normalized one-minute returns $g$

$\rightarrow$ far tails of distribution go with $1/|g|^4$
also for other data, very stable result

Gopikrishnan, Plerou, Amaral, Meyer, Stanley, PRE 60 (1999) 5305
Fluctuating Volatility

Standard & Poor’s 500

time window 1 min

frequency of 6.5 hours

describing models: (general) autoregressive conditional heteroskedasticity (ARCH/GARCH)
coupled stochastic processes for price and volatility

how can one find a dynamical model?
Turbulence, Earthquakes, Ising Models ...
Turbulence, Earthquakes, Ising Models ...

Loma Prieta 1989
Turbulence, Earthquakes, Ising Models ...

Loma Prieta 1989

... and many more
empirically found power laws in the far tails of distributions:

\[ g \text{ normalized returns go } \sim \frac{1}{|g|^{1+\alpha}} \text{ with } \alpha \approx 3 \]

\[ v \text{ traded volumes go } \sim \frac{1}{|v|^{1+\beta}} \text{ with } \beta \approx \frac{3}{2} \]

proposed explanation for the observation \( \alpha \approx 2\beta \)

only really big funds trade large volumes

\( v \longrightarrow \text{ fund manager wants to buy stocks that are cheaper than what he thinks is the fair price} \)
\( \longrightarrow \text{ he has to be fast, before mispricing closes and before he moves the market too much} \)
\( \longrightarrow \text{ he offers to buy the stocks at a price concession} \)
\( g \longrightarrow \text{ he wants } g \text{ to be small} \)
\( \longrightarrow \text{ to find many stocks, that is large } v, \text{ he needs a time } T \sim \frac{v}{g} \longrightarrow \text{ the larger } g, \text{ the smaller } T \longrightarrow \text{ optimization problem} \longrightarrow g \sim \sqrt{v} \)

# Order Book and Efficiency

<table>
<thead>
<tr>
<th></th>
<th>BUY</th>
<th>SELL</th>
</tr>
</thead>
<tbody>
<tr>
<td>liquidity takers (&quot;informed&quot; traders)</td>
<td>4</td>
<td>7.2</td>
</tr>
<tr>
<td>versus</td>
<td>6</td>
<td>7.0</td>
</tr>
<tr>
<td>liquidity providers (market makers)</td>
<td>7</td>
<td>6.9</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>6.7</td>
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</tbody>
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ABC–Company

liquidity takers $\rightarrow$ long–range persistence $\rightarrow$ superdiffusive
liquidity providers $\rightarrow$ anti–persistence $\rightarrow$ subdiffusive

$\rightarrow$ net–effect is diffusive $\ldots$ information, efficiency ?

Bouchaud, Gefen, Potters, Wyart, Quant. Fin. 4 (2004) 176
Financial Derivatives

Derivatives are contracts about the trading of some underlying asset at or within a specified time in the future.

forwards and futures

options

options are extremely important in the financial markets
Call Options

At time $t = 0$, person A buys from person B the right to buy a certain stock at maturity time $t = T$ at the strike price $E = S(0)$. The price at $t = 0$ for this call option is $C$.

profit diagram at maturity $t = T$
Call Option and Underlying Stock

options are themselves assets and are traded at derivative markets

option price depends on the price of the underlying stock

BMW stock

BMW call option
Idea of Black and Scholes Theory

stock with price $S(t)$ and option with price $G(S, t)$

construct a portfolio with value $V(S, t) = G(S, t) - \Delta(S, t) \cdot S$

with a function $\Delta(S, t)$ to be chosen

exact result: if $\Delta(S, t) = \frac{\partial G(S, t)}{\partial S}$ then $dV = V r dt$

$\rightarrow$ risk eliminated!  $\rightarrow$ hedge

also yields partial differential equation for option price $G(S, t)$
closed solution for price $G(S, t)$ of any option, if one assumes that $S(t)$ follows geometric Brownian motion
Black, Scholes, Merton and their Formula

The Black-Scholes-Merton model, a landmark in financial mathematics, has had a profound impact on the financial industry. Named after its creators, Fischer Black, Myron Scholes, and Robert Merton, the model was developed in the late 1970s and revolutionized the way financial derivatives are priced.

The Black-Scholes formula, expressed as:

\[ C = SN(d) - Le^{-rt}N(d - \sigma \sqrt{t}) \]

where:

- \( C \) is the price of the call option,
- \( S \) is the current market price of the underlying asset,
- \( N \) is the cumulative distribution function of the standard normal distribution,
- \( r \) is the risk-free interest rate,
- \( t \) is time to maturity,
- \( \sigma \) is the volatility of the underlying asset,
- \( L \) is the exercise price of the option.

The formula is derived from the assumption of a Black-Scholes process, which models the price of the underlying asset as a geometric Brownian motion. This process is characterized by a constant drift rate and a constant volatility rate.

The model has been widely adopted and has become a cornerstone in the field of financial engineering. It has also led to the development of many subsequent models and techniques for pricing and hedging financial derivatives.
Putting together a Portfolio

Portfolio 1
- ExxonMobil
- British Petrol
- Daimler
- Toyota
- ThyssenKrupp
- Voestalpine

Portfolio 2
- Sony
- British Petrol
- Daimler
- Coca Cola
- Novartis
- Voestalpine

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correlations $\rightarrow$ diversification lowers portfolio risk!
Correlations between Stocks

visual inspection for Coca Cola and Procter & Gamble

correlations change over time!
Diversification — Empirically

systematic risk (market) and unsystematic risk (portfolio specific)

A wise choice of $K = 20$ stocks (or risk elements) turns out sufficient to eliminate unsystematic risk.
Portfolio and Risk Management

portfolio is linear combination of stocks, options and other financial instruments

\[ V(t) = \sum_{k=1}^{K} w_k(t) S_k(t) + \sum_{l=1}^{L} w_{Cl}(t) G_{Cl}(S_l, t) + \sum_{m=1}^{M} w_{Pm}(t) G_{Pm}(S_m, t) + \ldots \]

with time–dependent weights!

portfolio or fund manager has to maximize return

- high return requires high risk: speculation
- low risk possible with hedging and diversification

find optimum for risk and return according to investors’ wishes

\[ \rightarrow \text{ risk management} \]
Literature