FAKULTÄT FÜR PHYSIK

UNIVERSITÄT DUISBURG ESSEN

Econophysics I: Basic Concepts

Thomas Guhr

Let's Face Chaos through Nonlinear Dynamics, Maribor 2011

Mutual Attraction between Physics and Economics

mathematical modeling in physics and economics has always been similar, many connections exist for a long time: Bachelier, Einstein, Mandelbrot, Markowitz, Black, ...

during the last 15 to 20 years, the number of physicists working on economics problems has grown quickly, the term “econophysics” was coined

physics $\longrightarrow$ economics: much better economical data now, general interest in complex systems

economics $\longrightarrow$ physics: risk managment, expertise in model building based on empirical data

Einstein’s Heirs at the Banks

Frankfurter Allgemeine Zeitung on Friday, May 20th, 2005

Frankfurter Allgemeine Zeitung — Finanzmärkte und Geldanlage — Freitag, 20. Mai 2005, Nr. 115 / Seite 27

Einsteins Erben in den Banken

Die Mathematik der „Brownschen Bewegung“ ist Grundlage sowohl der Atomphysik als auch der modernen Finanzmathematik / Von Benedikt Fehr

Vor 100 Jahren, im Mai 1905, lieferte Albert Einstein mit seiner „Theorie der Brownschen Bewegung“ einen Beweis für die moderne Vorstellung vom Atom. Unbeabsichtigt, indirekt und auf verschlungenen Wegen trug Einsteins Geniestreich ein dreiviertel Jahrhundert später dazu bei, eine weitere Revolution zu zünden – auf den Finanzmärkten. Denn auch Börsenkurse lassen sich als „Brownsche Bewegung“ deuten. Physikern und Mathematikern sind deshalb die komplizierten Formeln geläufig, ohne die im modernen Bank- und Finanzgeschäft nichts mehr geht. Eine erste Anwendung fand die hochgezüchtete Mathematik in der Optionspreistheorie, die in den siebziger Jahren entwickelt wurde. Inzwischen geht die Wirkung weit darüber hinaus. So schätzen Finanzmathematiker zum Beispiel auch die Ausfallwahrscheinlichkeiten von Krediten und die Risikoprämien für Kreditausfallversicherungen mit diesen hochabstrakten Modellen. Die neuen Instrumente tragen dazu bei, unternehmerische Risiken besser beherrschbar zu machen – was das rasche Wachstum der Märkte für diese Finanzinnovationen erklärt. Doch falsch angewendet, können sie selbst zu einem Risiko für die Stabilität des Finanzsystems werden.

Überraschende Karrieren: Goetz Giese hat 1996 über die „Dynamik granularer Teilchen“ seinen Doktor in Physik gemacht, heute arbeitet er im Risiko-Controlling der Commerzbank. Roland … gilt er als der frühe Begründer der modernen Finanzmathematik.

Bacheliers später Siegeszug beginnt in den fünfziger Jahren. Inzwischen hatte John M. Keynes, der selbst ein leiden- …

… Deutschland einstellt, von der Ausbildung her Naturwissenschaftler. Im Derivate-Eigenhandel der Hypo-Vereinsbank sind fünf von sechs Mitarbeitern Mathematiker und Physiker.

Risikokontrolle ist deshalb zu einem wichtigen Wettbewerbsparameter, zu einer Art Produktionsfaktor geworden.

Die Optionspreismodelle wiederum erlauben es den Banken, die Derivate, die …

… rance“, die Mitte der achtziger Jahre in Wall Street mit viel Marketinggetöse an Investoren verkauft wurde. Diese „synthetische Option“ sollte Anleger gegen einen Verfall der Aktienkurse schützen. Das …

… ker Ernst Eberlein wiederum, Generalsekretär der „Bachelier Finance Society“, tüftelt an hochgezüchteten „Lévy-Modellen“, in denen die Brownsche Bewegung nur ein Spezialfall unter vielen ist.

“ ... Every tenth academic hired by Deutsche Bank is a natural scientist ... ”

Outline

- some economical basics: market, price, arbitrage, efficiency
- empirical results and models for price dynamics
- importance of financial derivatives and options
- portfolio and risk management
- rôle of financial correlations

Who Makes the Market Price?

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the price of an asset is made by demand and supply

demand up and/or supply down $\longrightarrow$ price up
demand down and/or supply up $\longrightarrow$ price down

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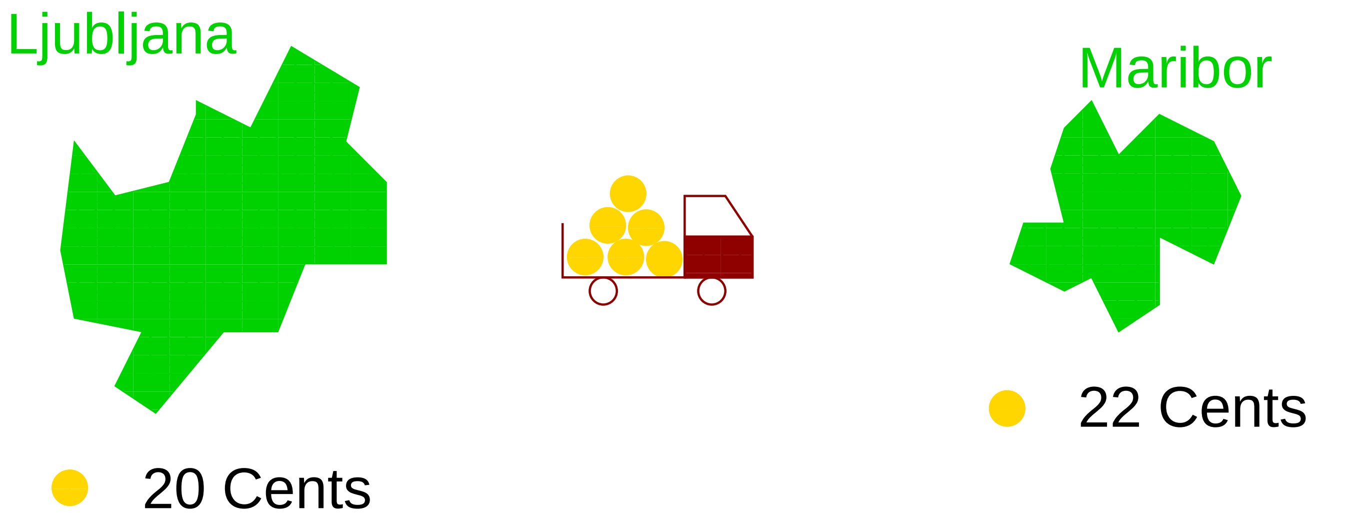
$\longrightarrow$ THE MARKET MAKES THE PRICE!

Exploit Price Difference for Oranges



5000 oranges yield riskless and quick profit

$5000 \cdot (22 - 20)$ Cents $=$ 100 Euros (no fees, etc)

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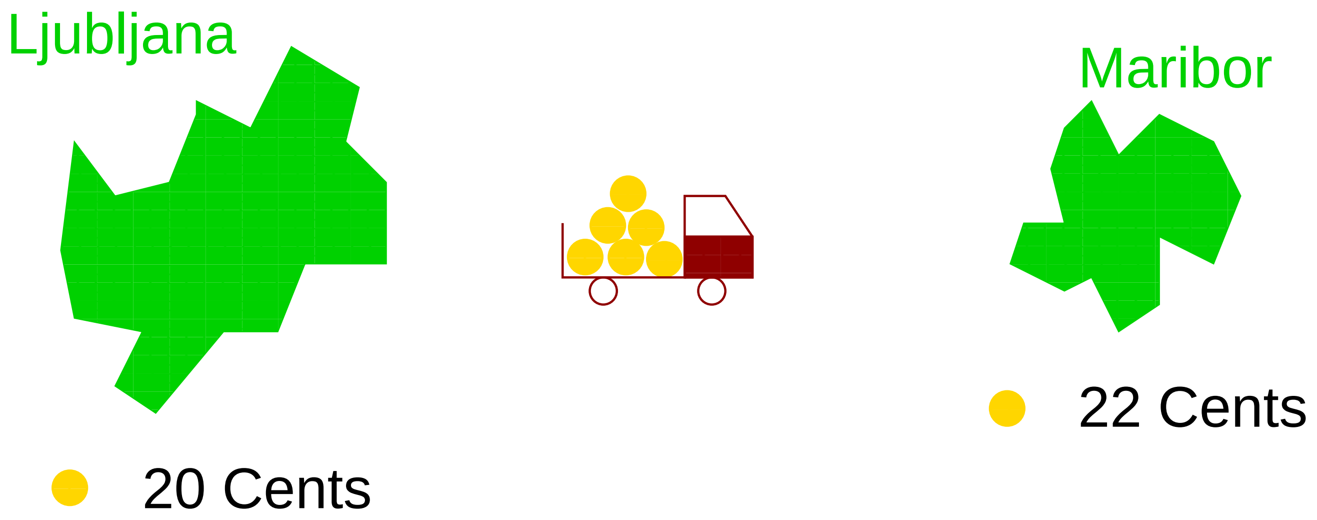

5000 oranges yield riskless and quick profit

$5000 \cdot (22 - 20)\ \text{Cents} \ = \ 100\ \text{Euros}$ (no fees, etc)

continue doing that, other people start doing the same $\longrightarrow$

Maribor: orange supply goes up $\longrightarrow$ price goes down
Ljubljana: orange supply goes down $\longrightarrow$ price goes up

$\longrightarrow$ prices in Ljubljana and Maribor equilibrate!

Arbitrage and Efficiency

risk and time characterize economical transaction (deal, trade)

bank: no risk, long time $\longleftrightarrow$ lottery: high risk, short time

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arbitrage destroys itself! $\longrightarrow$ arbitrage time scale

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requires homogenous information, absence of operational obstacles $\longrightarrow$ efficiency

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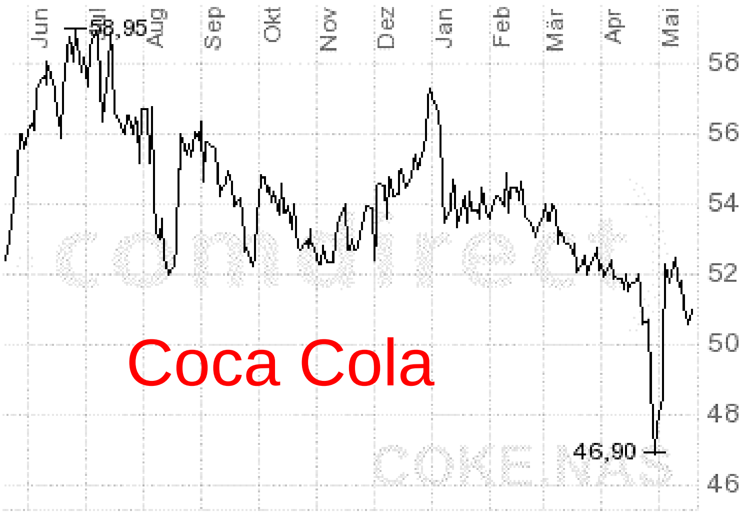
assumption: no arbitrage in the capital markets!

Empirical Evidence



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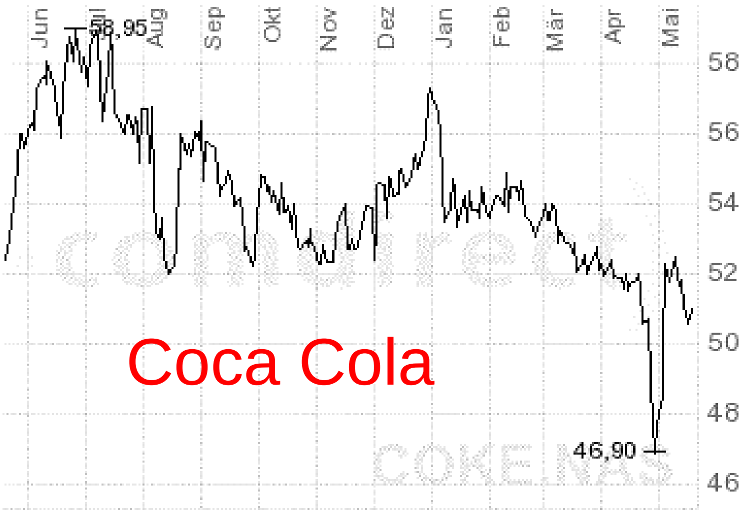

many traders on same market

we expect a “fair game”

$$\langle dS(t) dS(t+\tau)\rangle_t \sim \delta(\tau)$$

no prediction possible!

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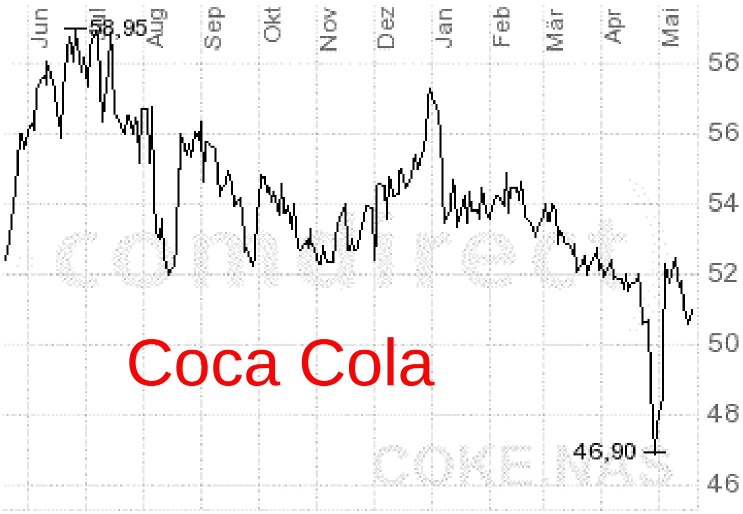

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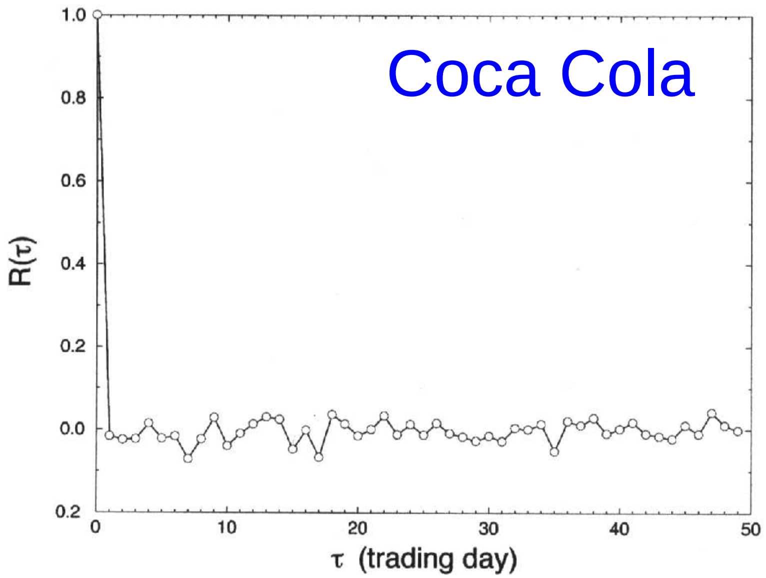
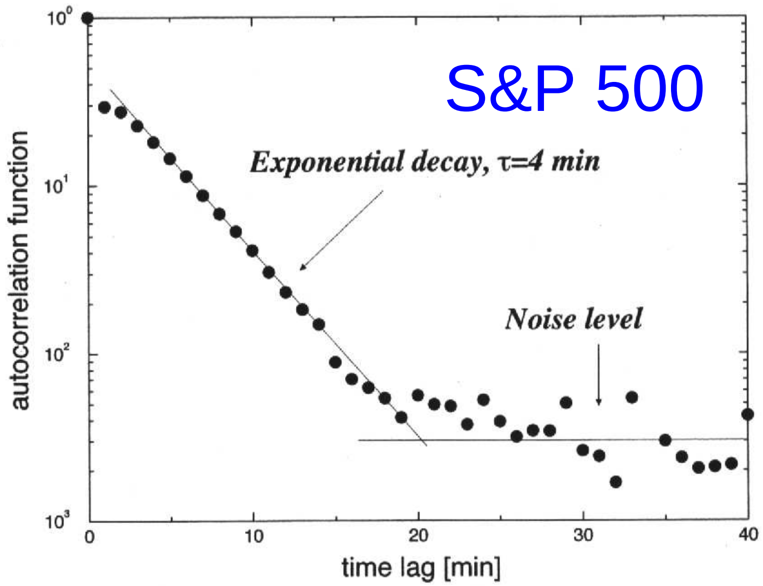
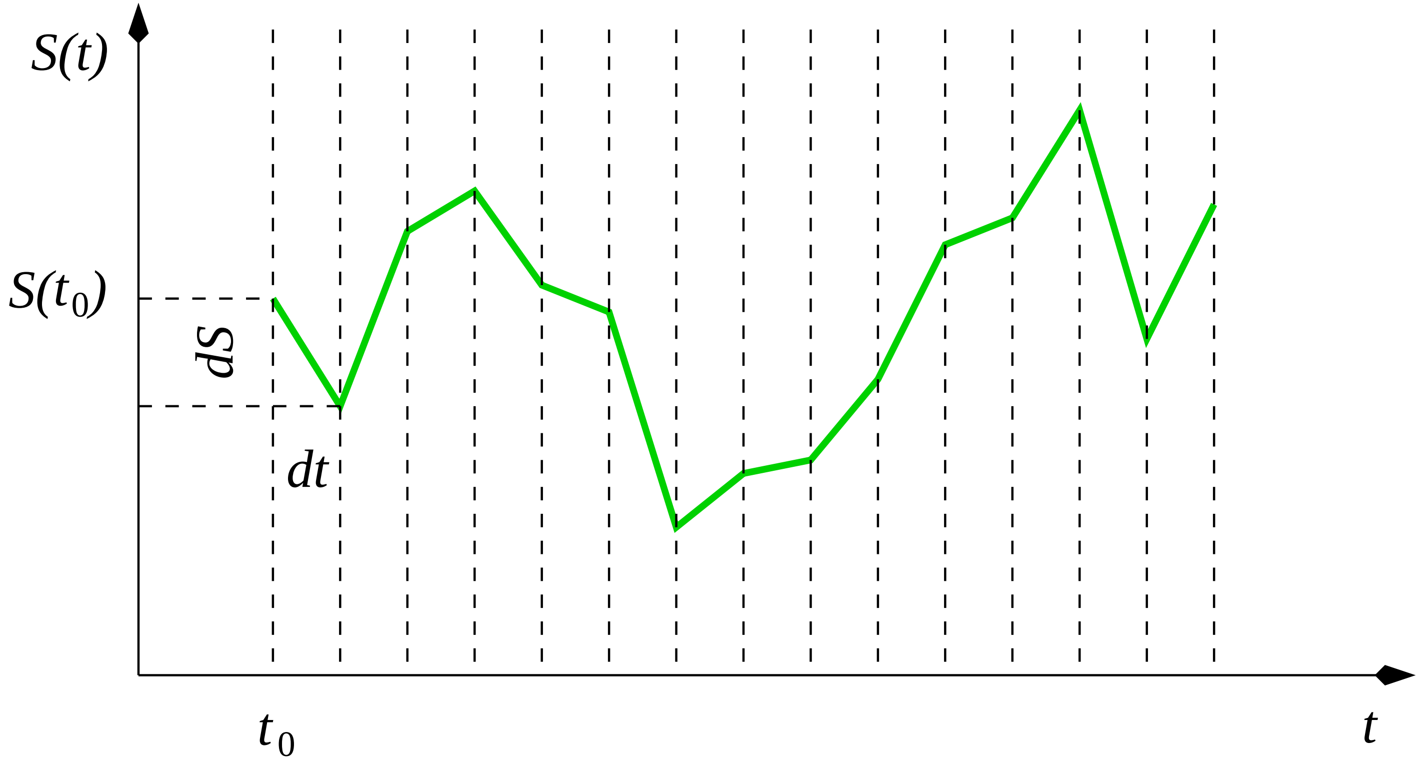




Price Dynamics and Brownian Motion

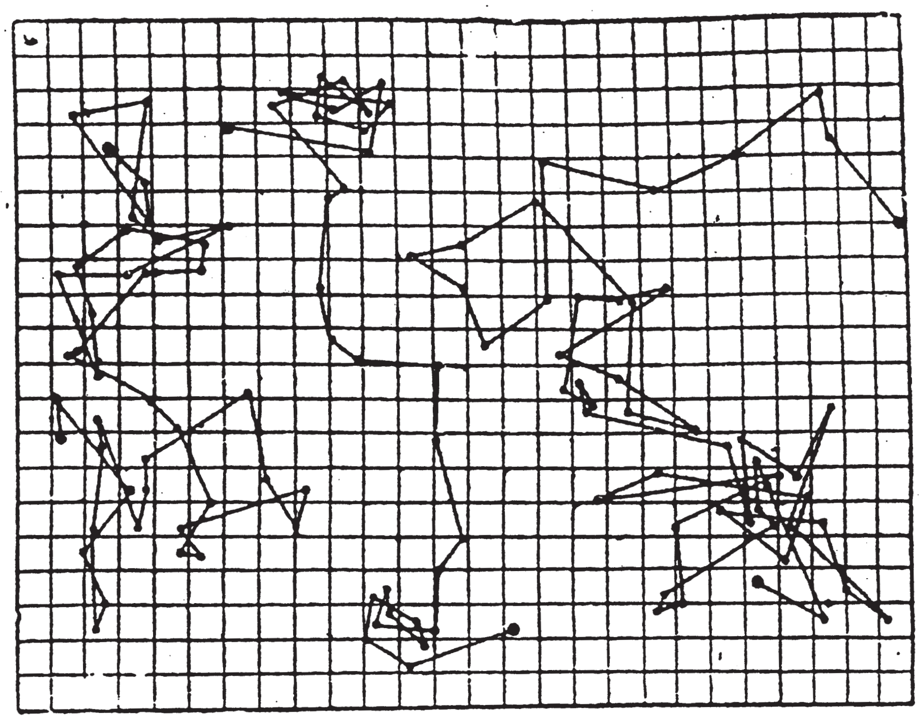
stochastic differential equation $dS(t) = \sigma\varepsilon(t)\sqrt{dt}$

$\varepsilon(t)$ uncorrelated random for every t, volatility σ is a constant



notation $dS = \sigma\varepsilon\sqrt{dt}$

Ballistic versus Diffusive



particles moving in liquid
in cells of plants

Perrin (1948)

two–dimensional!

ballistic $x(t) \sim t$ $\longleftrightarrow$ diffusive $\langle x^2(t) \rangle \sim t$

Very Long Time Scales

Dow–Jones 1974–2004

$$S(t) \sim \exp(\mu t)$$

$$\longrightarrow \quad dS = S\mu dt$$

μ is the drift constant

deterministic and stochastic part in stock price dynamics!

$$dS = S\left(\mu dt + \sigma\varepsilon\sqrt{dt}\right) \quad \longrightarrow \quad \frac{dS}{S} = \mu dt + \sigma\varepsilon\sqrt{dt}$$

geometric Brownian motion

Connection to Banks and Interest

put an amount of money $V(t)$ in a bank account

receive interest at a rate r within time dt

$$dV = V r dt \quad \longrightarrow \quad V(t) = V(t_0) \exp(r(t - t_0))$$

also an exponential law!

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losely speaking: the average of all stock market investors must make as much money as the average of all bank investors

$\longrightarrow$ “global no–arbitrage effect”

Return and Logarithmic Difference

the quantities dV/V and dS/S are returns

logarithmic difference

$$G(t) = \ln S(t + \Delta t) - \ln S(t) = \ln \frac{S(t + \Delta t)}{S(t)}$$

the same for small time intervals Δt

$$dS = S(t + \Delta t) - S(t)$$

$$G(t) = \ln \frac{S(t) + dS}{S(t)} = \ln \left(1 + \frac{dS}{S}\right) \approx \frac{dS}{S}$$

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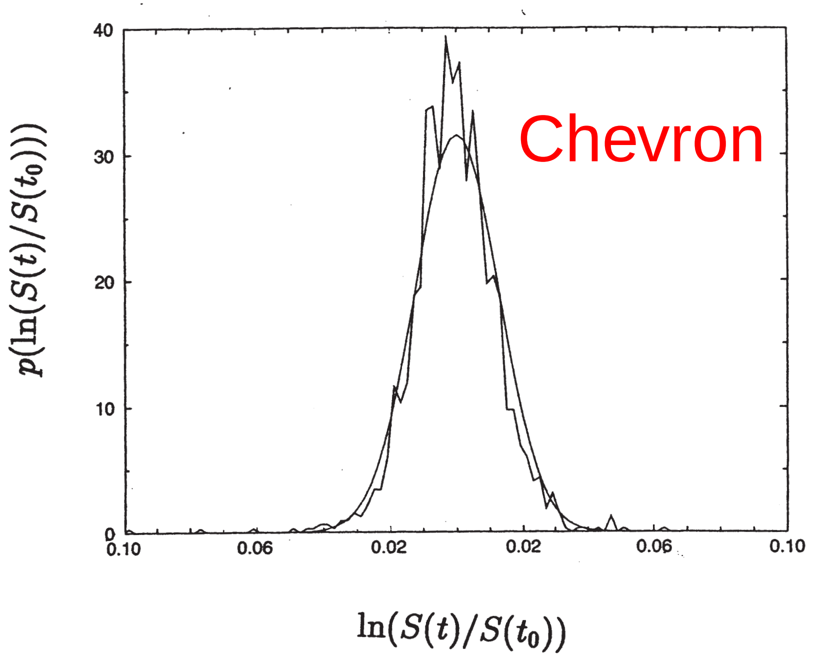
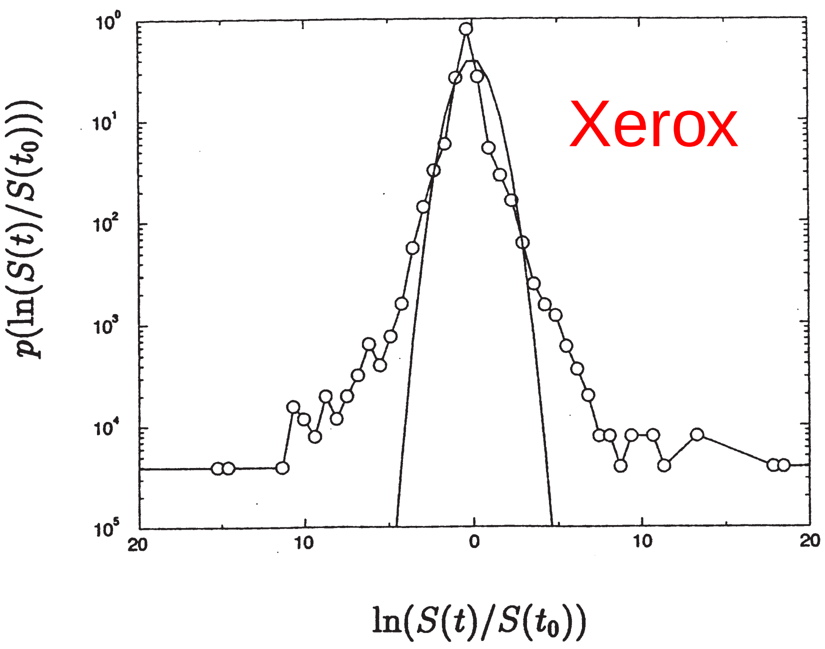
Resulting Distributions

geometric Brownian motion implies log–normal price distribution with variance $\sigma^2 \cdot (t - t_0)$ for moving window $t - t_0$ (here one day)



low frequency

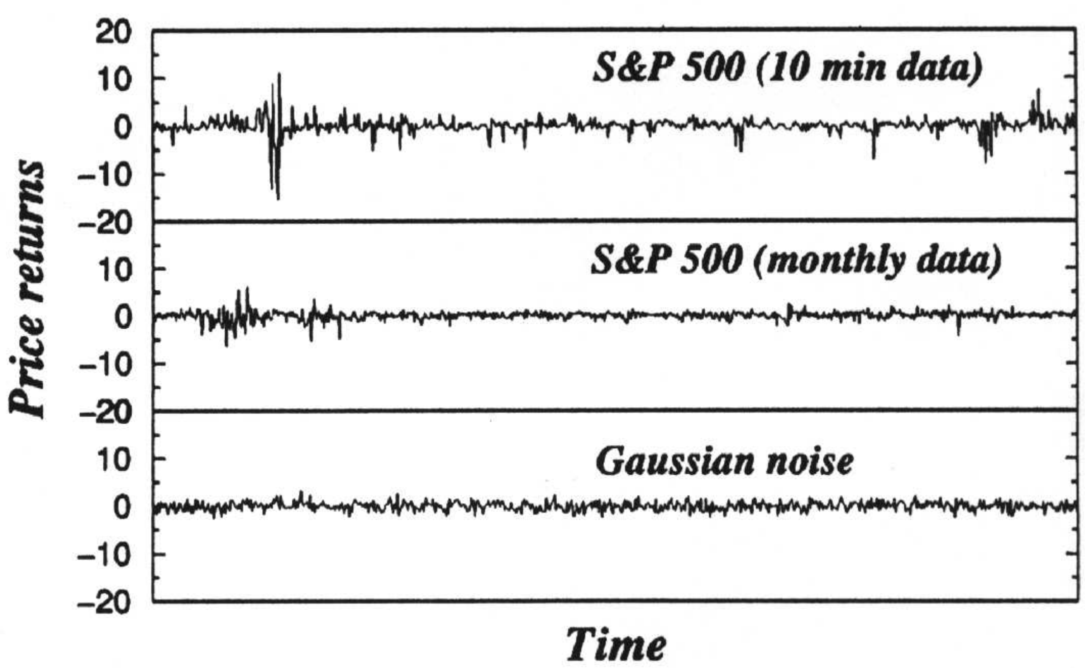


high frequency

$\longrightarrow$ tails are much fatter than expected!

Large Events

returns normalized to unit variance from Standard & Poor's 500



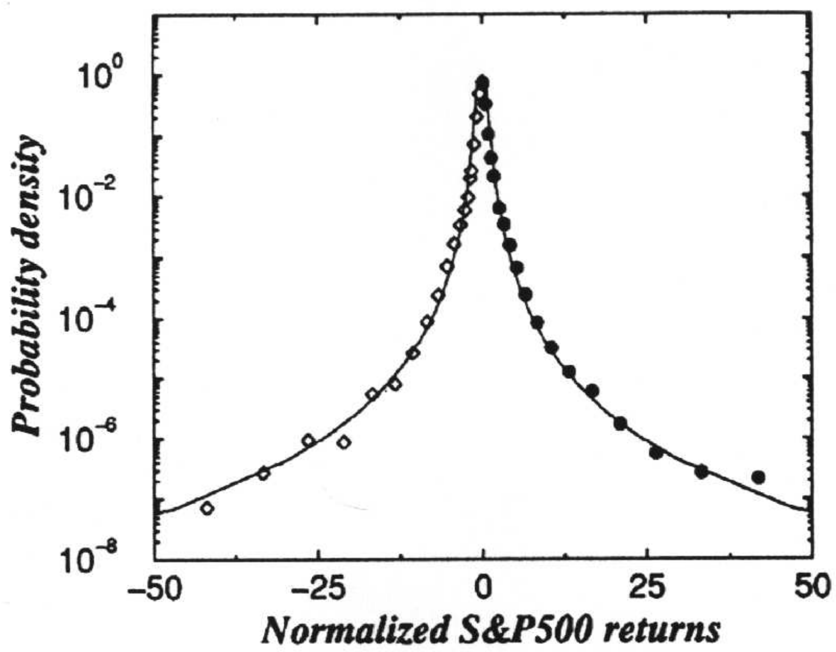
always 850 points

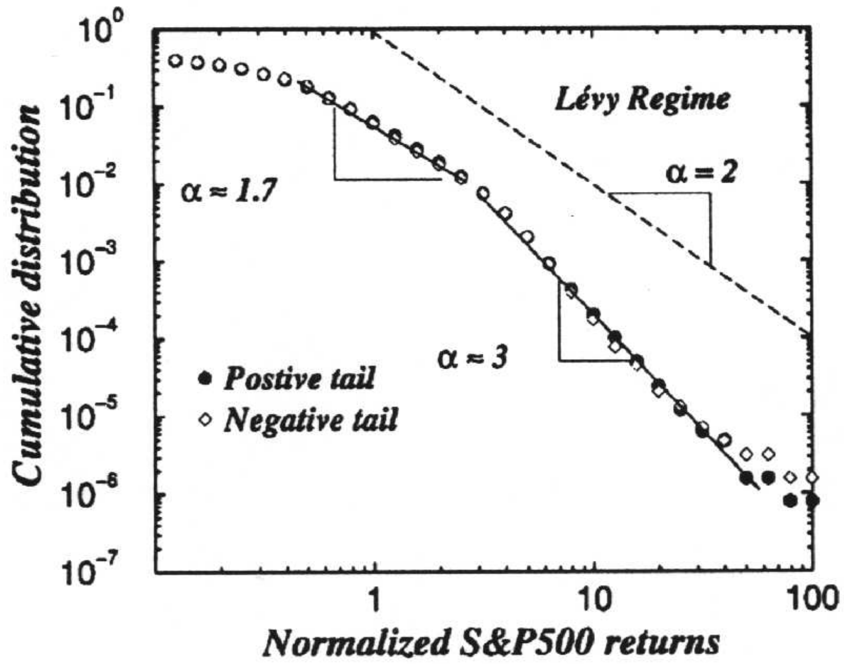
$\longrightarrow$ clearly non–Gaussian

Gopikrishnan, Plerou, Amaral, Meyer, Stanley, PRE 60 (1999) 5305

Power Law Far in the Tails

Standard & Poor's 500 normalized one–minute returns g



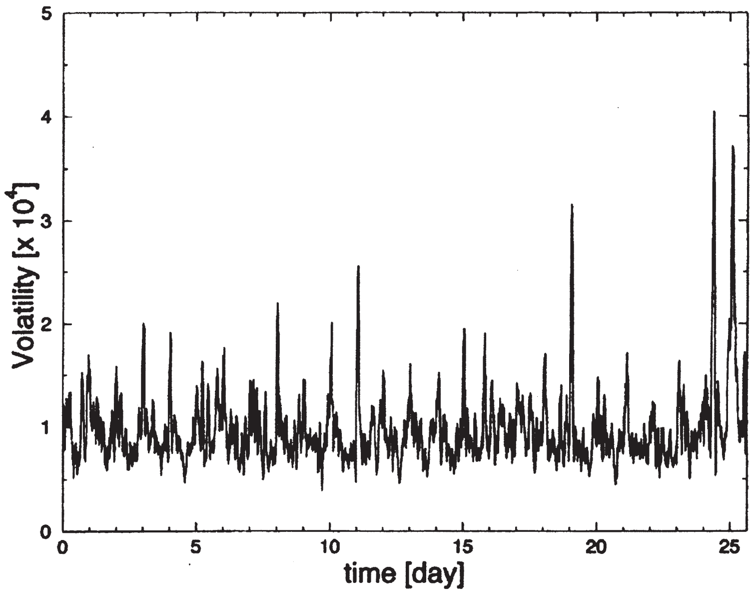


$\longrightarrow$ far tails of distribution go with $1/|g|^4$

also for other data, very stable result

Gopikrishnan, Plerou, Amaral, Meyer, Stanley, PRE 60 (1999) 5305

Fluctuating Volatility



Standard & Poor's 500

time window 1 min

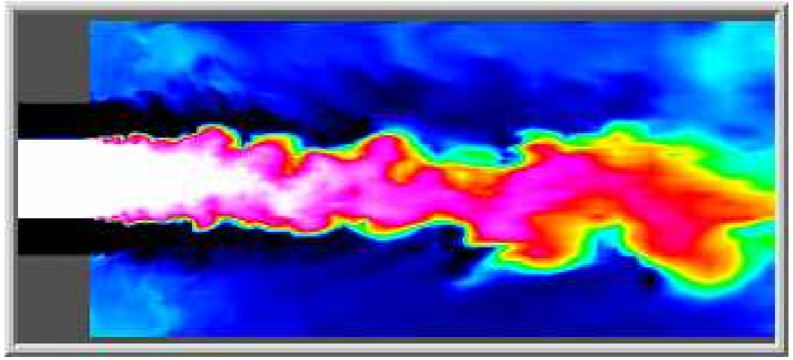
frequency of 6.5 hours

describing models: (general) autoregressive conditional heteroskedasticity (ARCH/GARCH)

coupled stochastic processes for price and volatility

how can one find a dynamical model?

Turbulence, Earthquakes, Ising Models ...



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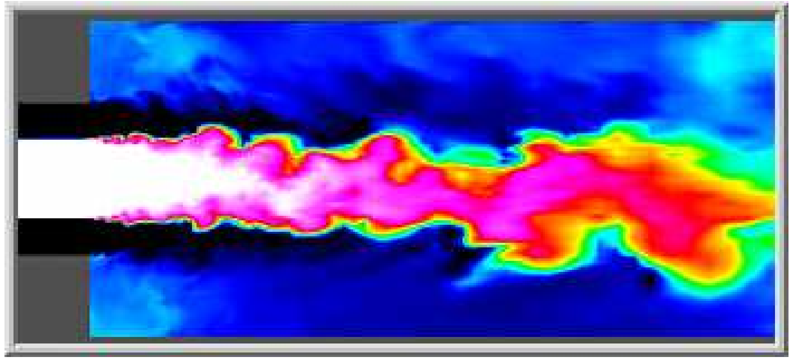

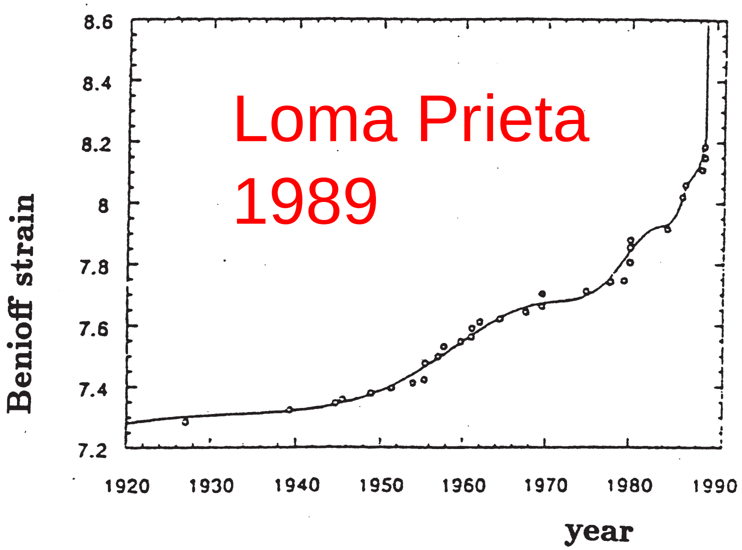

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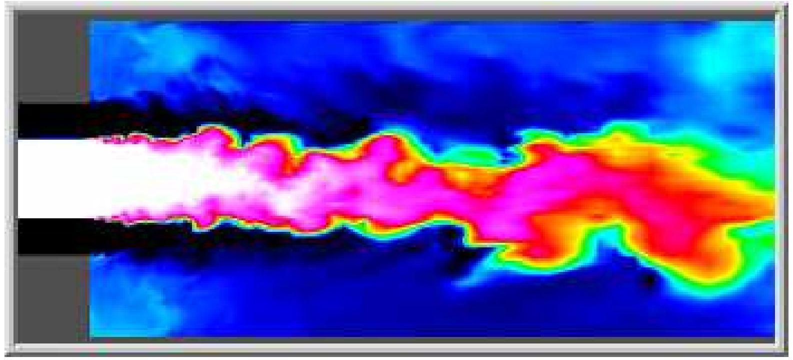
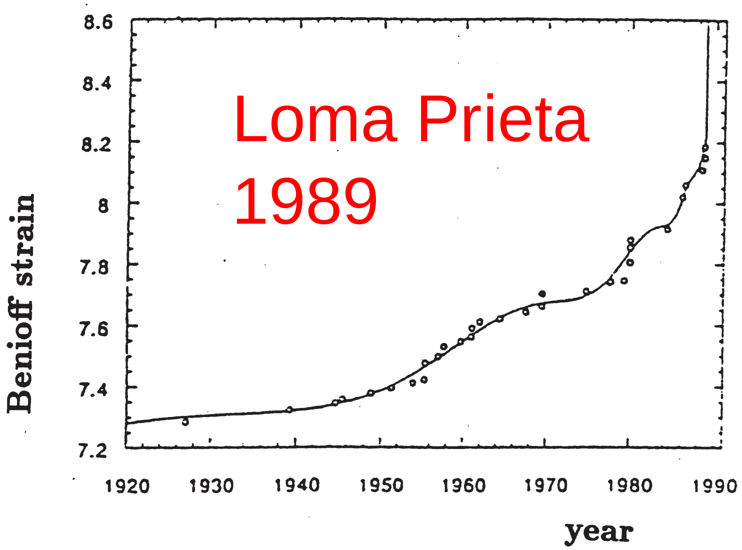
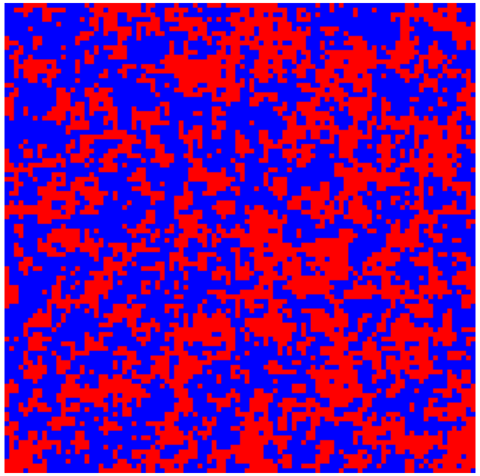






... and many more

Power Laws and Large Volumes

empirically found power laws in the far tails of distributions:

g normalized returns go $\sim 1/|g|^{1+\alpha}$ with $\alpha \approx 3$

v traded volumes go $\sim 1/|v|^{1+\beta}$ with $\beta \approx 3/2$

proposed explanation for the observation $\alpha \approx 2\beta$

only really big funds trade large volumes $v \longrightarrow$ fund manager wants to buy stocks that are cheaper than what he thinks is the fair price $\longrightarrow$ he has to be fast, before mispricing closes and before he moves the market too much $\longrightarrow$ he offers to buy the stocks at a price concession $g \longrightarrow$ he wants g to be small $\longrightarrow$ to find many stocks, that is large v, he needs a time $T \sim v/g \longrightarrow$ the larger g, the smaller $T \longrightarrow$ optimization problem $\longrightarrow g \sim \sqrt{v}$

Gabaix, Gopikrishnan, Plerou, Stanley, Nature (London) 423 (2003) 267

Order Book and Efficiency

ABC−Company

BUY		SELL	
4	7.2	2	7.3
6	7.0	3	7.5
7	6.9	6	7.6
9	6.7	7	7.7

liquidity takers
(“informed” traders)

versus

liquidity providers
(market makers)

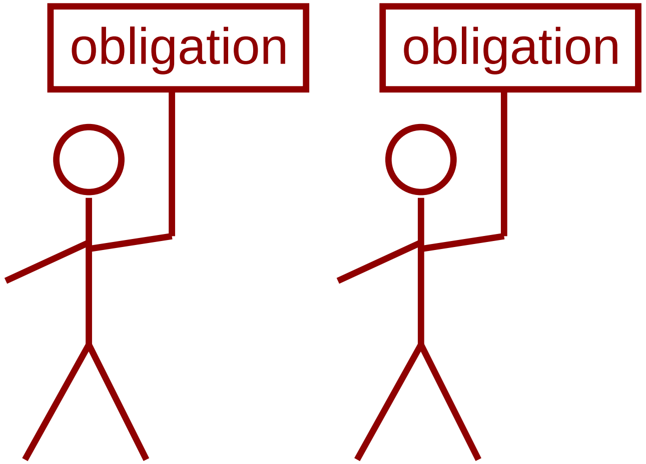
liquidity takers ⟶ long–range persistence ⟶ superdiffusive
liquidity providers ⟶ anti–persistence ⟶ subdiffusive

⟶ net–effect is diffusive ... information, efficiency ?

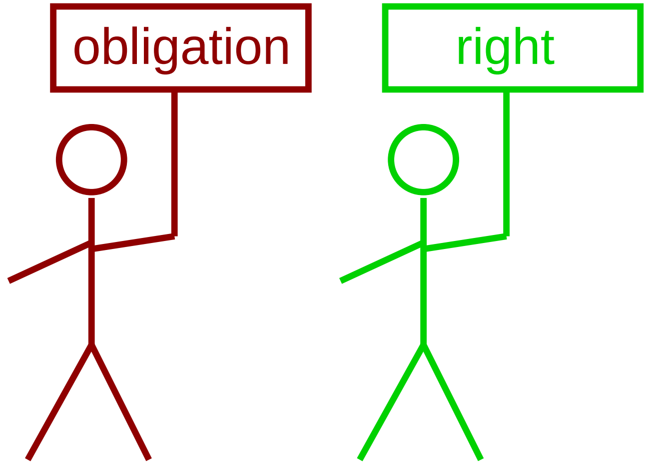
Bouchaud, Gefen, Potters, Wyart, Quant. Fin. 4 (2004) 176

Financial Derivatives

Derivatives are contracts about the trading of some underlying asset at or within a specified time in the future.



forwards and futures

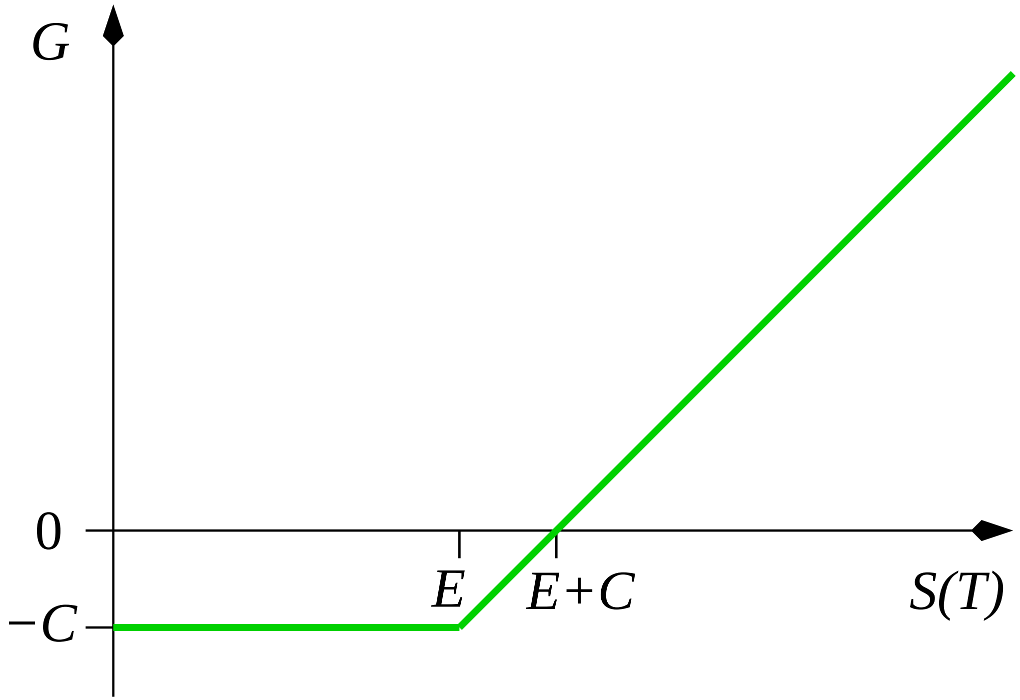


options

options are extremely important in the financial markets

Call Options

At time $t = 0$, person A buys from person B the right to buy a certain stock at maturity time $t = T$ at the strike price $E = S(0)$.
The price at $t = 0$ for this call option is C .



profit diagram

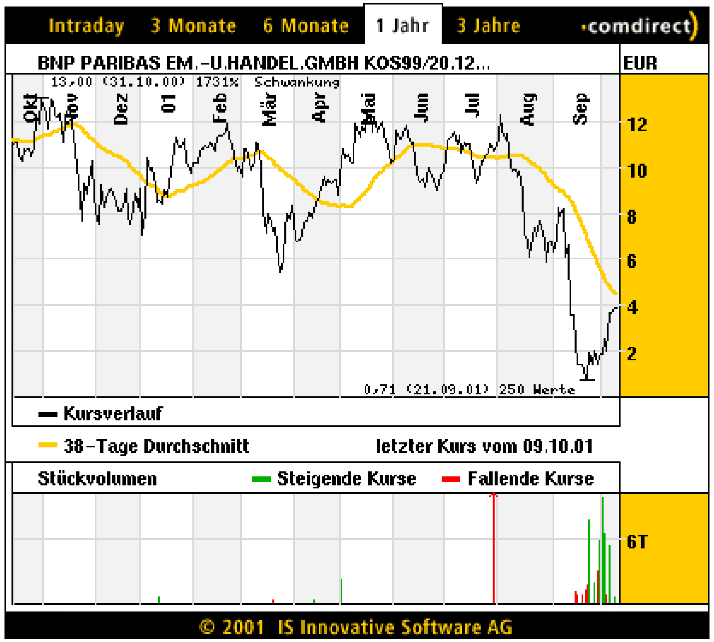
at maturity $t = T$

Call Option and Underlying Stock

options are themselves assets and are traded at derivative markets

option price depends on the price of the underlying stock

BMW stock



BMW call option

Idea of Black and Scholes Theory

stock with price $S(t)$ and option with price $G(S,t)$

construct a portfolio with value $\quad V(S,t) = G(S,t) - \Delta(S,t) \cdot S$

with a function $\Delta(S,t)$ to be chosen

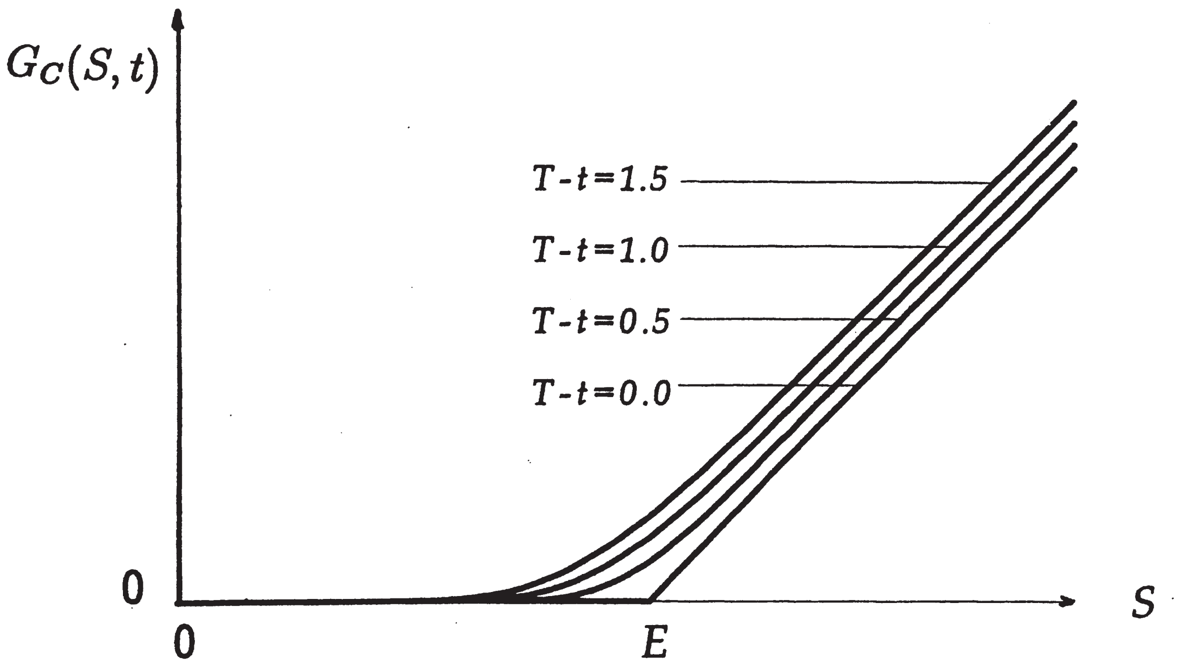
exact result: if $\quad \Delta(S,t) = \dfrac{\partial G(S,t)}{\partial S} \quad$ then $\quad dV = Vr\,dt$

$\longrightarrow$ risk eliminated! $\longrightarrow$ hedge

also yields partial differential equation for option price $G(S,t)$

Option Price

closed solution for price $G(S, t)$ of any option, if one assumes that $S(t)$ follows geometric Brownian motion



call option

Black, Scholes, Merton and their Formula

wicklung der Ak-
)as brachte den
'entlichten Black
nach ihnen be-
l für die Bewer-
ton folgte wenig
lls bahnbrechen-
s Eigenkapital ei-
ufoption für des-
oles und Merton
-Gedächtnispreis
; Black hätte ihn
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verstorben.
ormel war ein
einerten und er-
cker und andere
matiker das zu-
: Bewertungsmo-
nanzmathematik
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kt". Gleichzeitig
Vall Street daran,
in Geschäftsmo-
es dabei an ein-
omen fehlte, heu-
nken zahlreiche
er an. So hielten
ers – Erben Ein-
er Banken. Noch
der Entdeckung
l, zählen sie dort
ist jeder zehnte
eutsche Bank in

liches Geschäft, zum Beispiel die Maschinenproduktion. Eine hochgezüchtete

haben, herbe Verluste. Zu den größten „Unfällen" zählte die „Portfolio Insu-

Fisher Black, Myron Scholes und Robert Merton

Fotos Archiv

$$C = SN(d) - Le^{-rt} N(d - \sigma\sqrt{t})$$

$$d = \frac{\ln \frac{S}{L} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

Die Black-Scholes-Formel für die Bewertung von Optionen ist das kleine Einmaleins des boomenden Wirtschaftszweigs Financial Engineering.

das globale Fina
sen. Sie wurde e
amerikanische N
Auch die heu
bergen solche I
zung. Viele Fors
ten deshalb dara
delle immer weit
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re, allerdings auch
tische Formeln e
ist die sogenan
Mit dem Zunger
leute eine Kennzi
ten beschreiben
sind, sondern im
bert Engle erhiel
den Wirtschafts-
scher setzen Gr
stündlich erzeug
den Finanzmärk
auszuwerten. Sie
durch mit „besse
Wieder ander
Konzepten. De
Mandelbrot zun
heute gängigen
zur Abschätzun
auf den Formeln
gung aufbauen, s
Um diese Mänge
von ihm erfund
tik" für die Fina
machen. Der Fre

Frankfurter Allgemeine Zeitung, May 20th, 2005

Putting together a Portfolio

Portfolio 1	Portfolio 2
ExxonMobil	Sony
British Petrol	British Petrol
Daimler	Daimler
Toyota	Coca Cola
ThyssenKrupp	Novartis
Voestalpine	Voestalpine

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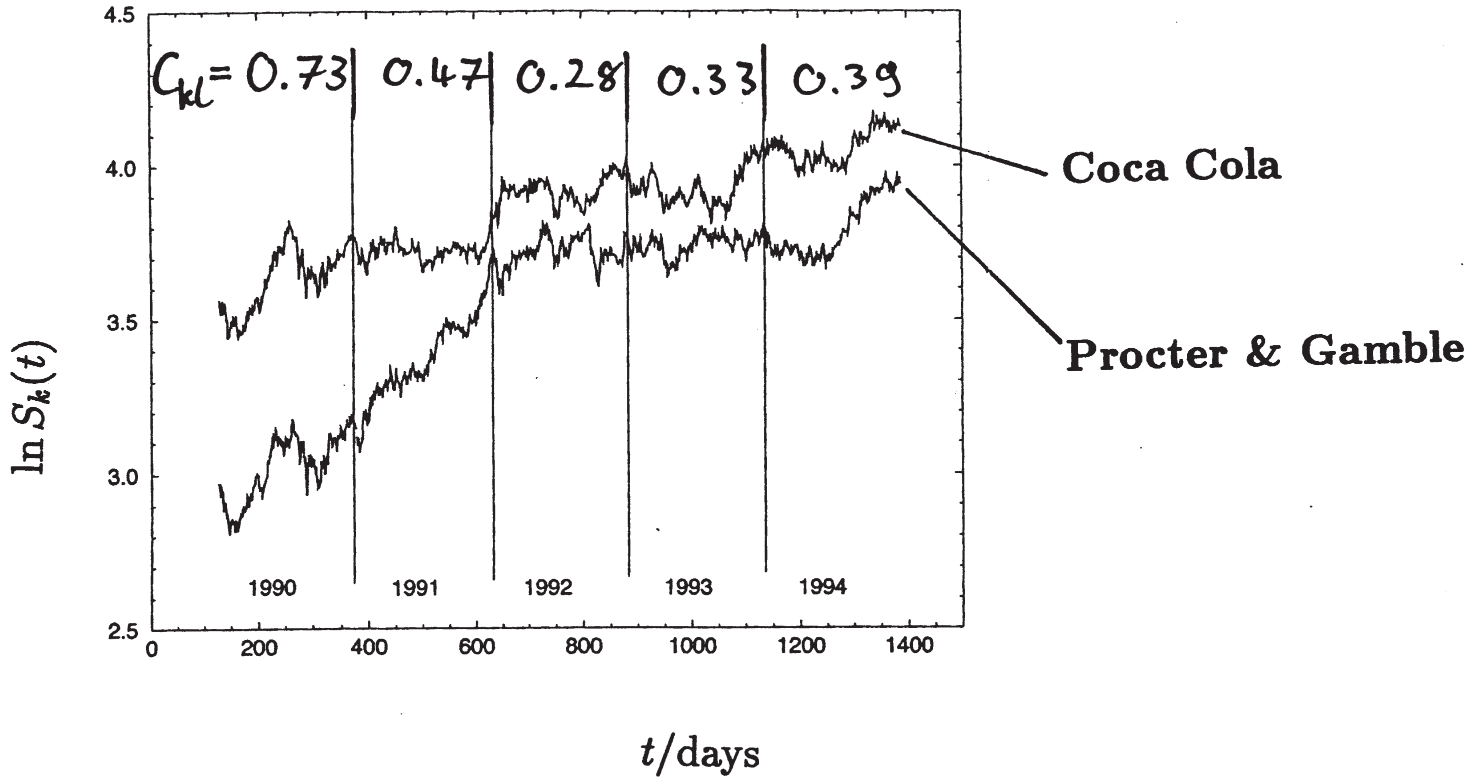
correlations $\longrightarrow$ diversification lowers portfolio risk!

Correlations between Stocks

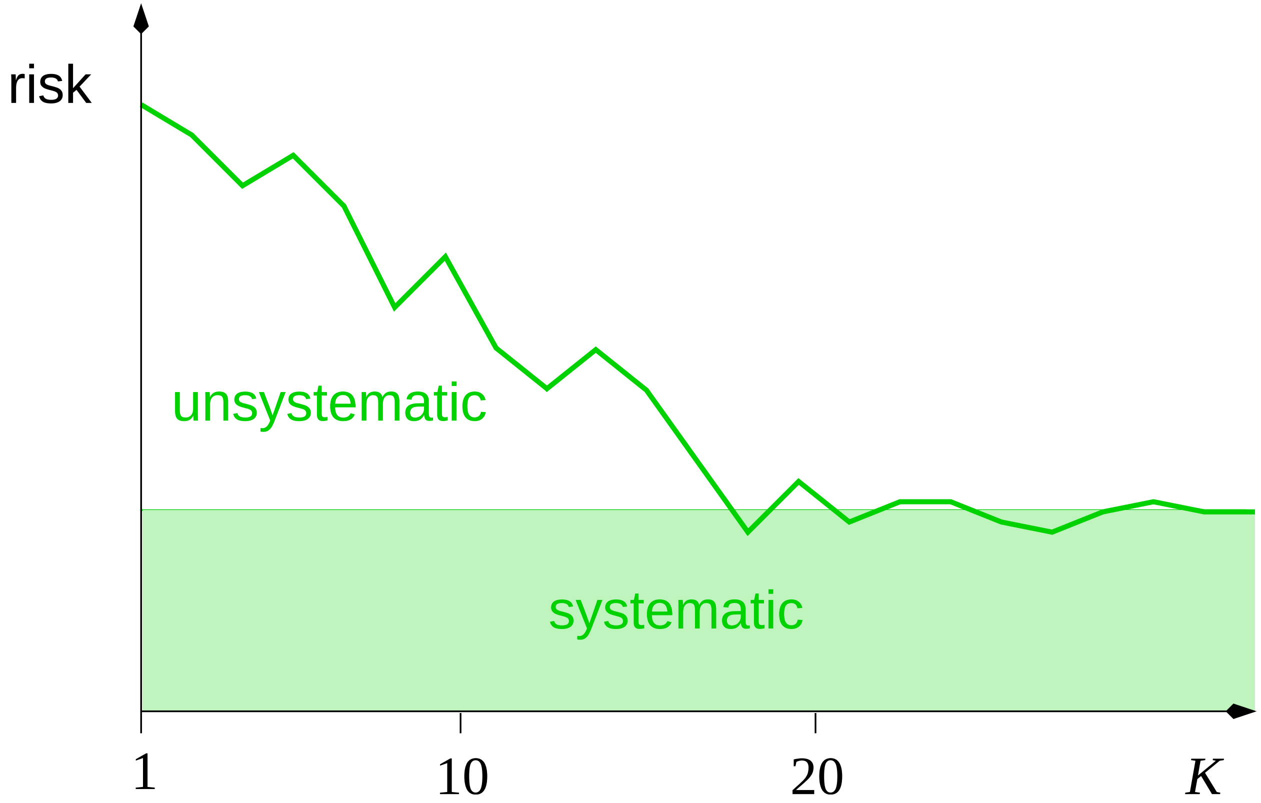
visual inspection for Coca Cola and Procter & Gamble



correlations change over time!

Diversification — Empirically

systematic risk (market) and unsystematic risk (portfolio specific)



a wise choice of $K = 20$ stocks (or risk elements) turns out sufficient to eliminate unsystematic risk

Portfolio and Risk Management

portfolio is linear combination of stocks, options and other financial instruments

$$V(t) = \sum_{k=1}^{K} w_k(t) S_k(t) + \sum_{l=1}^{L} w_{Cl}(t) G_{Cl}(S_l, t) + \sum_{m=1}^{M} w_{Pm}(t) G_{Pm}(S_m, t) + \ldots$$

with time–dependent weights!

portfolio or fund manager has to maximize return

- high return requires high risk: speculation
- low risk possible with hedging and diversification

find optimum for risk and return according to investors' wishes

$\longrightarrow$ risk management

Literature

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