Magnetic domain patterns under an oscillating fields

Kazue Kudo

Ochadai Academic Production, Ochanomizu University

Collabolator: Katsuhiro Nakamura

(Osaka City University, Uzbek Academy of Sciences)

Domain Patterns

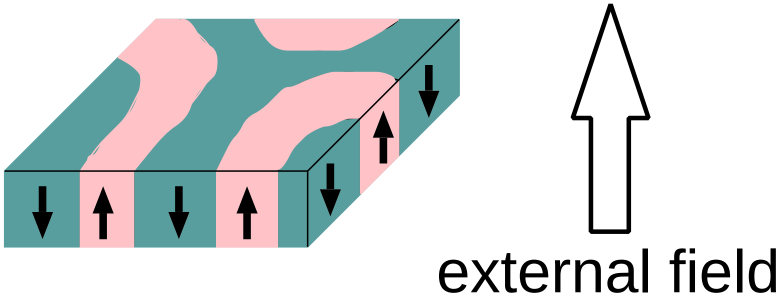
A wide variety of physical and chemical systems display domain patterns: for example,

- Thermal convection in fluids
- Chemical reaction systems
- Ferromagnetic thin films
  Ferrofluids
- Superconductors
- Biological media

etc.

Magnetic Domain Patterns

Let us consider a ferromagnetic thin film like the schematic picture.



- It has strong uniaxial magnetic anisotropy.
- Its easy axis is perpendicular to the film.
- Because of interactions between spins, up and down spins form clusters (domains).

Outline

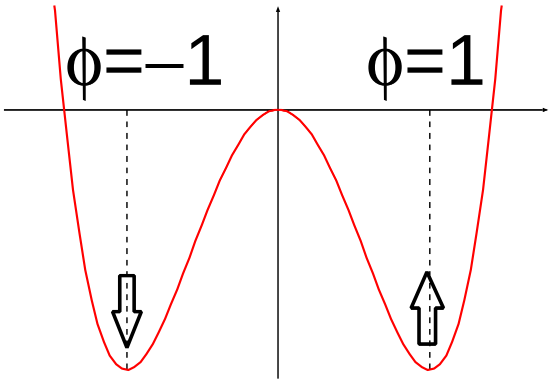
1. Model and Method
   for numerical simulations
2. Labyrinth $\longrightarrow$ Stripes $\longrightarrow$ Lattice
   typical domain patterns under an oscillating field
3. Traveling pattern
   equation for slow motion
4. Concentric circles, Spiral pattern
   some interesting patterns
5. Summary

Model & Equation

Simple two-dimensional Ising-like model.
The Hamiltonian consists of 4 terms written by using a scalar field $\phi(\boldsymbol{r})$.

1. **uni-axial anisotropy:**

$$H_{\mathrm{ani}} = \alpha \int \mathrm{d}\boldsymbol{r} \left(-\frac{\phi(\boldsymbol{r})^2}{2} + \frac{\phi(\boldsymbol{r})^4}{4} \right)$$



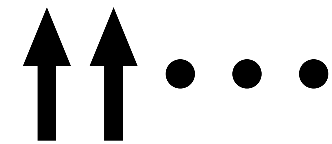
2. **external field:**

$$H_{\mathrm{ex}} = -h(t) \int \mathrm{d}\boldsymbol{r} \phi(\boldsymbol{r})$$

Model & Equation

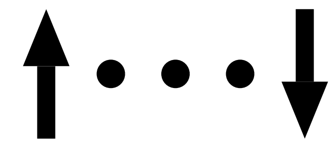
3. **exchange interactions:**

$$H_J = \beta \int \mathrm{d}\boldsymbol{r} \frac{|\nabla \phi(\boldsymbol{r})|^2}{2}$$



4. **dipolar interactions:**

$$H_{\mathrm{di}} = \gamma \int \mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{r}' \phi(\boldsymbol{r})\phi(\boldsymbol{r}')G(\boldsymbol{r},\boldsymbol{r}')$$



$G(\boldsymbol{r},\boldsymbol{r}') = 1/|\boldsymbol{r}-\boldsymbol{r}'|^3$ at long distances.

Then the dynamical equation is described by

$$\frac{\partial \phi(\boldsymbol{r})}{\partial t} = -\frac{\delta(H_{\mathrm{ani}} + H_J + H_{\mathrm{di}} + H_{\mathrm{ex}})}{\delta \phi(\boldsymbol{r})}$$

Equation in Fourier Space

The equation in Fourier space

$$\frac{\partial \phi_{\boldsymbol{k}}}{\partial t} = \underbrace{\left(\alpha - \beta k^2 - \gamma G_{\boldsymbol{k}}\right)}_{\eta_{\boldsymbol{k}}} \phi_{\boldsymbol{k}} + h(t)\delta_{\boldsymbol{k},0} - \phi^3\big|_{\boldsymbol{k}}$$

Here, $\cdot\big|_{\boldsymbol{k}}$ means the convolution sum, and

$$G_{\boldsymbol{k}} = a_0 - a_1 k, \quad (k = |\boldsymbol{k}|)$$

$$a_0 = 2\pi \int_d^\infty r \mathrm{d} r G(r) = 2\pi/d, \quad a_1 = 2\pi$$

d: cutoff length, which is fixed as $d = \pi/2$ below.

Linear Growth Rate

Let us consider only linear terms in the equation:

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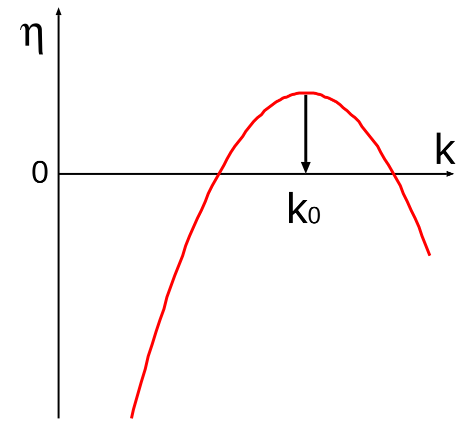
(But the nonlinear term prevents $\phi_{\boldsymbol{k}}$'s growing too much.)

$$\begin{aligned} \eta_{\boldsymbol{k}} &= -(\beta k^2 - \gamma a_1 k + \gamma a_0) + \alpha \\ &= -\beta \Bigg(k - \underbrace{\frac{a_1 \gamma}{2\beta}}_{k_0} \Bigg)^2 + \frac{a_1^2 \gamma^2}{4\beta} - \gamma a_0 + \alpha \end{aligned}$$



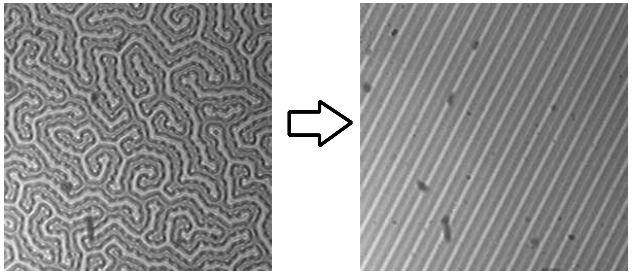
The characteristic length of domain patterns should be $2\pi/k_0$.

Here, we set $\beta = 2.0$, $\gamma = 2/\pi \Rightarrow k_0 = 1$.

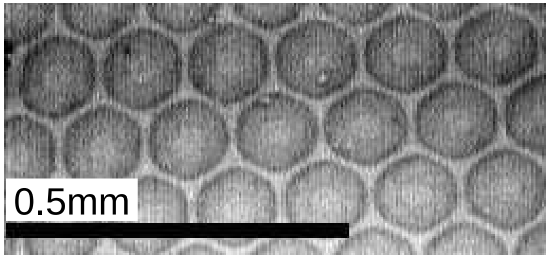
Experiments

Examples of experimentally observed domain patterns under oscillating fields

- The labyrinth structure changes into parallel-stripes when the field is not very strong.



- When the field amplitude is increased, a lattice structure appears.

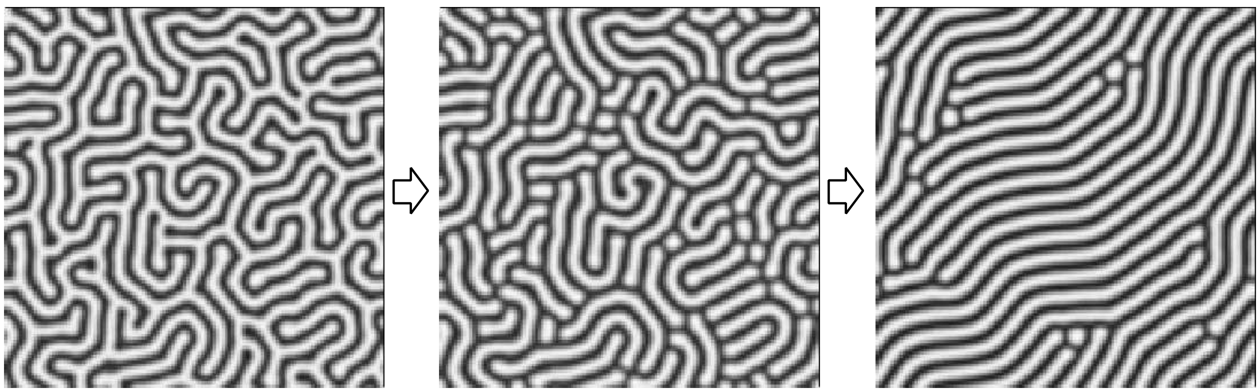


[Courtesy of Prof. Mino (Okayama Univ.): Experiments in iron garnet films.]

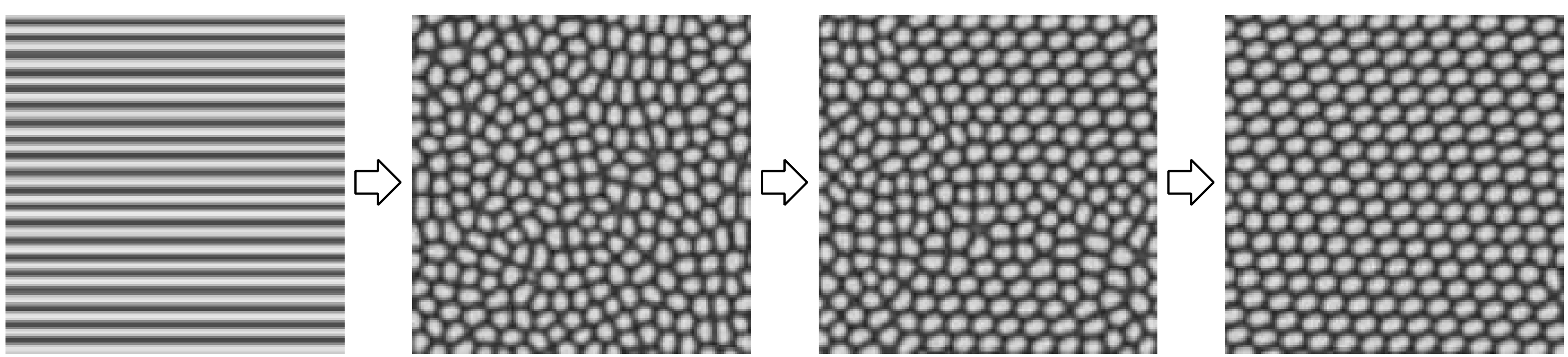
Numerical Simulations

External field: $h(t) = h_0 \sin \omega t; \quad \omega = 2\pi \times 10^{-2}$

- h_0 is not large; $h_0 = 0.72$. $(\alpha = 2.0)$



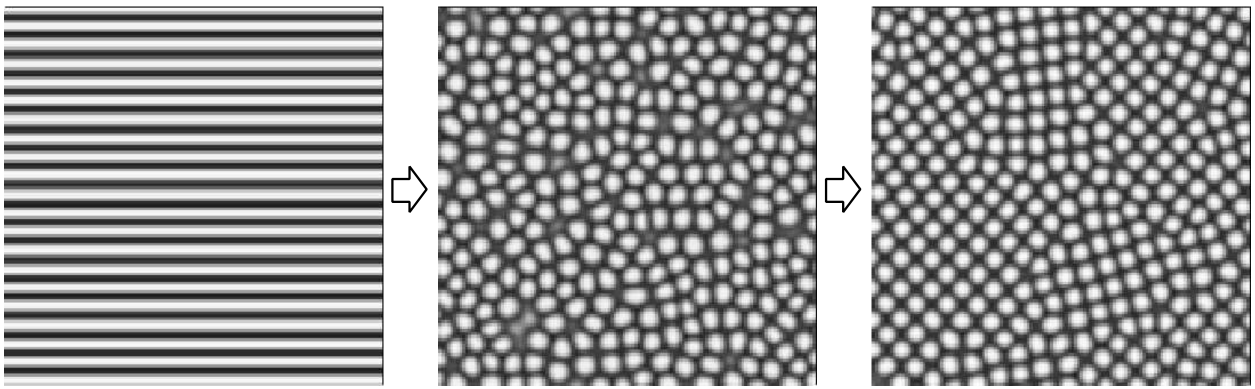
- h_0 is large; $h_0 = 1.15$. $(\alpha = 2.0)$



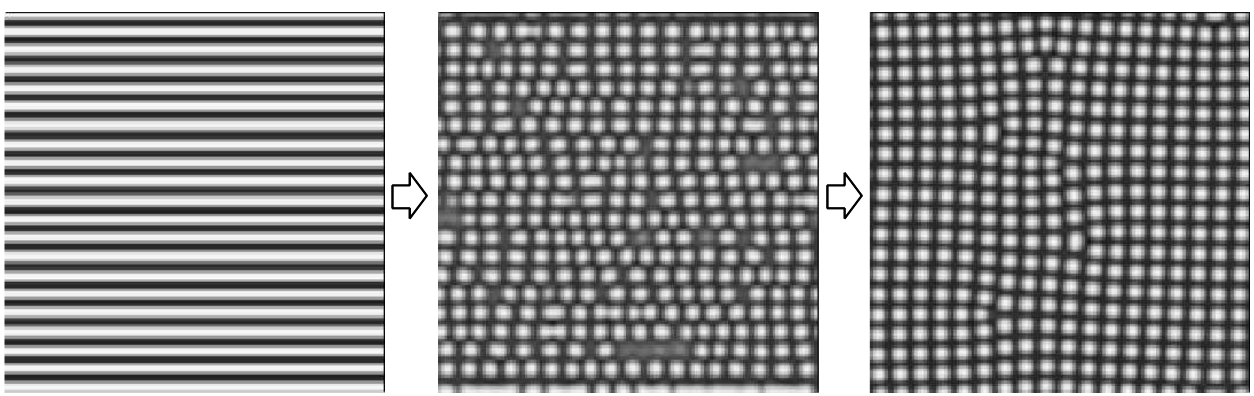
ω-dependence of Lattice Formation

The lattice structure depends on the frequency ω.

- $\omega = 2\pi \times 2 \times 10^{-2} \quad (\alpha = 2.0,\ h_0 = 1.15)$



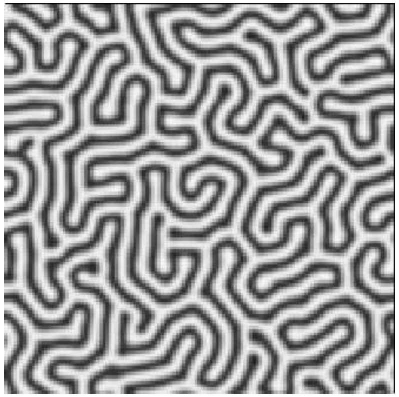
- $\omega = 2\pi \times 5 \times 10^{-2} \quad (\alpha = 2.0,\ h_0 = 1.15)$



Traveling Pattern

The whole pattern moves much more slowly than the field frequency.

$$\alpha = 2.0,$$
$$\omega = 2\pi \times 5 \times 10^{-2}$$



Ex. 1: $h_0 = 0.80$

Ex. 2: $h_0 = 0.95$

Traveling Pattern

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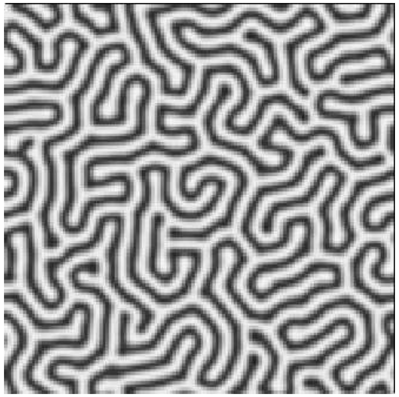
Basic mechanism: **drift bifurcation (parity-breaking bifurcation)** [1,2]

— a periodic pattern begins to drift when its second spatial harmonic is not damped strongly (k-$2k$ interaction).

$\alpha = 2.0,$
$\omega = 2\pi \times 5 \times 10^{-2}$



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[1] B.A. Malomed & M.I. Tribelsky, Physica **14D** (1984) 67.
[2] P. Coullet *et.al.*, Phys. Rev. Lett. **63** (1989) 1954; S. Fauve *et.al.*, Phys. Rev. Lett. **65** (1990) 385.

Dynamical Equation for Slow Motion

The patterns travel very slowly compared with the time scale of the field frequency.

$\boxed{\text{How shall we analyze the traveling pattern theoretically?}}$

Dynamical Equation for Slow Motion

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How shall we analyze the traveling pattern theoretically?

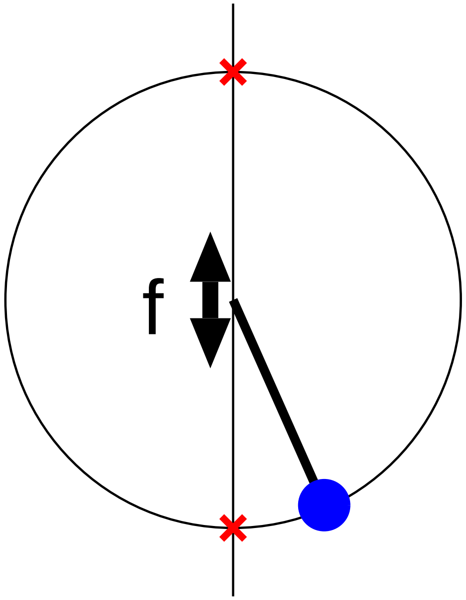
The dynamics under a rapidly oscillating field can be separated into a rapidly oscillating part and a slowly varying part.

— Kapitza's inverted pendulum [3]

[3] Landau & Lifshitz, *Mechanics* (Pergamon, Oxford, 1960).

Kapitza's Inverted Pendulum

When a rapidly oscillating force is applied to a pendulum, the unstable stationary point can turn to a stable point.



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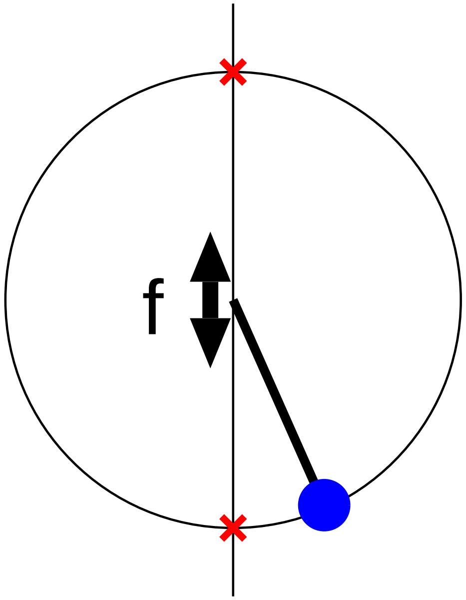

The equation of motion is

$$m\ddot{x} = -\frac{\mathrm{d}U}{\mathrm{d}x} + f.$$

f: a force oscillating rapidly (frequency: ω).
Let us separate $x(t)$ into a slowly varying part $X(t) = \overline{x}$ and a small rapidly oscillating part $\xi(t)$:

$$x(t) = X(t) + \xi(t).$$

Effective Potential

Expanding in powers of ξ as far as the first order terms, we obtain

$$m\ddot{X} + m\ddot{\xi} = -\frac{\mathrm{d}U}{\mathrm{d}x} - \xi\frac{\mathrm{d}^2U}{\mathrm{d}x^2} + f(X,t) + \xi\frac{\partial f}{\partial X}. \quad \text{—}(*)$$

For the oscillating terms,

$$m\ddot{\xi} = f(X,t) \quad \longrightarrow \quad \xi = -f/m\omega^2$$

We average Eq. $(*)$ with respect to time:

$$m\ddot{X} = -\frac{\mathrm{d}U}{\mathrm{d}X} + \overline{\xi\frac{\partial f}{\partial X}} = -\frac{\mathrm{d}U}{\mathrm{d}X} - \frac{1}{m\omega^2}\overline{f\frac{\partial f}{\partial X}}$$

We may rewrite it as

$$m\ddot{X} = -\frac{\mathrm{d}U_{\text{eff}}}{\mathrm{d}X}; \qquad U_{\text{eff}} = U + \frac{\overline{f^2}}{2m\omega^2}.$$

Equation for Fast Motion

The original equation:

$$\frac{\partial \phi(\boldsymbol{r})}{\partial t} = \alpha[\phi(\boldsymbol{r}) - \phi(\boldsymbol{r})^3] + \beta \nabla^2 \phi(\boldsymbol{r}) - \gamma \int \mathrm{d}\boldsymbol{r}' \phi(\boldsymbol{r}') G(\boldsymbol{r}, \boldsymbol{r}') + h(t)$$

Assumption: $\phi(\boldsymbol{r}, t) = \Phi(\boldsymbol{r}, t) + \phi_0(t)$

$\Phi(\boldsymbol{r}, t)$: slowly varying term (space-dependent)

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The rapidly oscillating part:

$$\dot{\phi}_0 = \alpha(\phi_0 - \phi_0^3) - \gamma \phi_0 \int \mathrm{d}\boldsymbol{r}' G(\boldsymbol{r}', 0) + h_0 \sin \omega t$$

$\longrightarrow \boxed{\phi_0 = \rho_0 \sin(\omega t + \delta)}$; ρ_0 and δ can be enumerated.

Approximation Methods

We propose two approximation methods to obtain the equation for slow motion [4].

1. The rapidly oscillating part is averaged out (on the basis of Kapitza's idea).
   $\Longrightarrow$ **Time-averaged model**
2. The delay of the response to the oscillating field is considered (instead of taking a time average).
   $\Longrightarrow$ **Phase-shifted model**

[4] K. Kudo & K. Nakamura, Phys. Rev. E **76**, 036201 (2007).

Equation for Slow Motion

Dynamical equation for the slowly varying part:

1. Time-averaged model

$$\frac{\partial \Phi(\boldsymbol{r})}{\partial t} = \alpha\left(\Phi(\boldsymbol{r}) - \Phi(\boldsymbol{r})^3\right) + \beta\nabla^2\Phi(\boldsymbol{r}) - \gamma\int \mathrm{d}\boldsymbol{r}' G(\boldsymbol{r},\boldsymbol{r}') + \frac{3}{2}\alpha\rho_0^2\Phi(\boldsymbol{r})$$

2. Phase-shifted model

$$\begin{aligned}\frac{\partial \Phi(\boldsymbol{r})}{\partial t} &= \alpha\left(\Phi(\boldsymbol{r}) - \Phi(\boldsymbol{r})^3\right) + \beta\nabla^2\Phi(\boldsymbol{r}) - \gamma\int \mathrm{d}\boldsymbol{r}' G(\boldsymbol{r},\boldsymbol{r}') \\ &\quad -\alpha\Phi(\boldsymbol{r})\left(\Phi(\boldsymbol{r})^2 + 3\Phi(\boldsymbol{r})\rho_0\sin\delta + 3\rho_0^2\sin^2\delta\right) + C \\ C &= \eta_0\rho_0\sin\delta - \alpha\rho_0^3\sin^3\delta - \omega\rho_0\cos\delta\end{aligned}$$

How to Discuss a Traveling Pattern

1. We consider a parallel-stripe-type solution including second harmonics:

$$\Phi(\boldsymbol{r}, t) = A_0(t) + A_1(t)\sin(kx + b(t))$$

$$+A_{21}(t)\cos[2(kx + b(t))] + A_{22}(t)\sin[2(kx + b(t))]$$

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If $\dot{b} \neq 0$ at a stable SP, the pattern travels.
4. The pattern can also travel if $\dot{b} = 0$ at an unstable SP.

Is a Traveling Pattern Possible?

1. **Time-averaged model — impossible**

$$\dot{b} = -3\alpha A_0 A_{22}$$

   There are only SPs with $A_0 = A_{21} = A_{22} = 0$, and they are always stable along A_0-axis. But we can estimate the max h_0 to observe a non-uniform pattern.

2. **Phase-shifted model — possible**

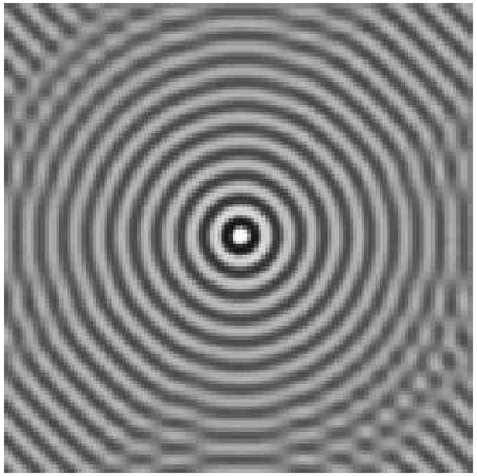
$$\dot{b} = -3\alpha(A_0 + \rho_0 \sin\delta)A_{22}$$

   There are SPs where $A_0 + \rho_0 \sin\delta \neq 0$ but $A_{22} = 0$, and they can be unstable along A_{22} in some region of h_0.

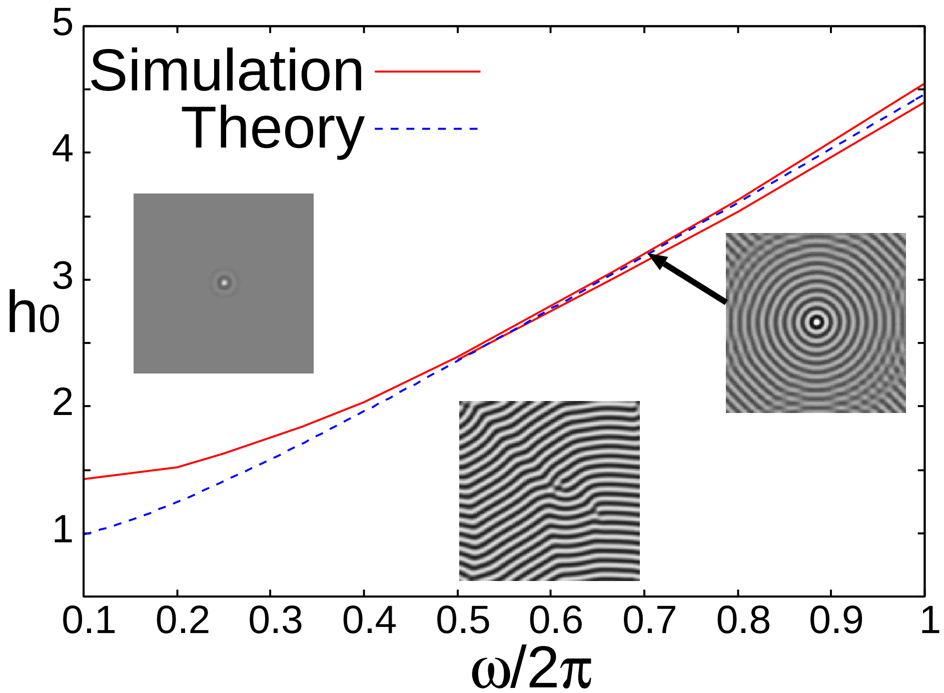
Concentric Circles

Concentric circles can appear in some cases.

- The field is very strong and the frequency is very high.
- (Assume) a strong defect at the center — The spin at the center is always up.



Diagram



Above the upper red line: homogeneous pattern except for the vicinity of center.

Below the lower red line: maze or lattice patterns

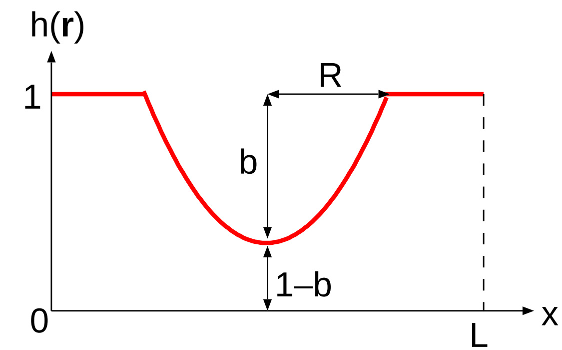
Between the upper and lower red lines

— Concentric circles appear.

The theoretical line above which no pattern but a homogeneous pattern appears is obtained from the time-averaged model.

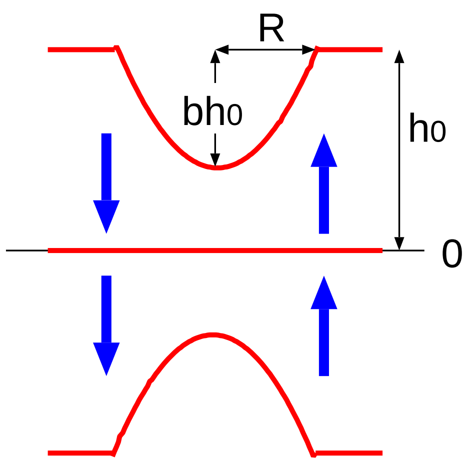
Spiral Pattern under a particular field

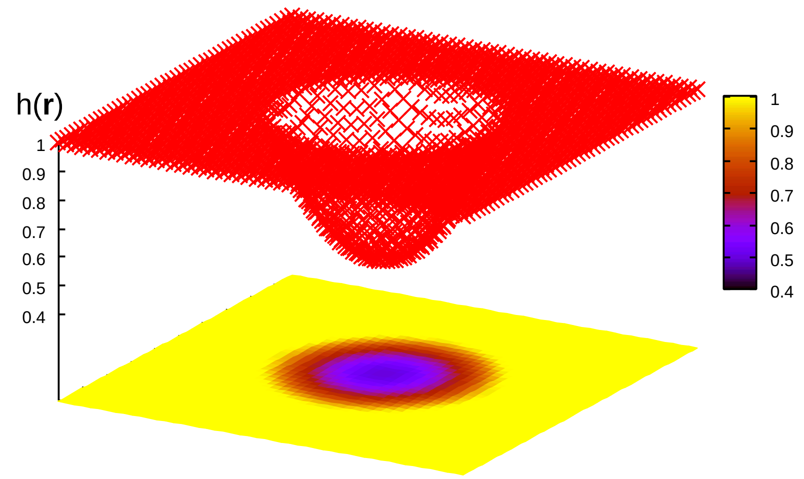
Numerical simulations show interesting patterns under a time-periodic and spatially inhomogeneous field.



Here, we redefine the magnetic field as $h(\boldsymbol{r})h_0 \sin \omega t$, and

$$h(\boldsymbol{r}) = \begin{cases} b(x^2+y^2)/R^2 + (1-b) & \text{when} \quad x^2+y^2 < R^2 \\ 0 & \text{when} \quad x^2+y^2 > R^2 \end{cases}$$





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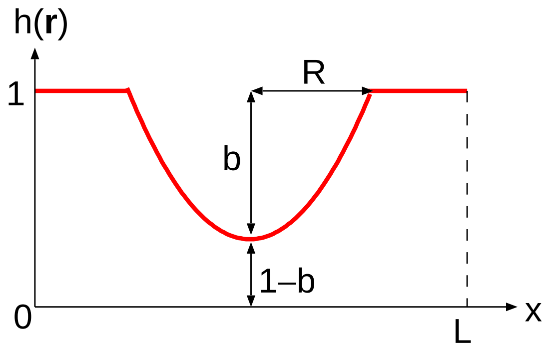

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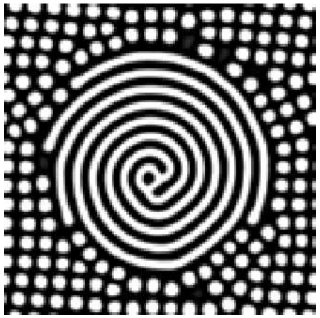
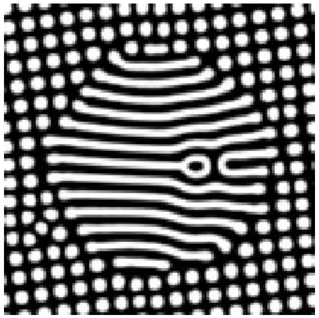
$R = L/4$
$b = 0.5$



$R = 3L/8$
$b = 0.8$



$R = 3L/4$
$b = 0.5$

$L = 128$

Summary

- Under oscillating fields, a labyrinth structure changes into a parallel-stripe or lattice structure depending on the field strength and frequency.
- In some cases, we can see traveling patterns, which move very slowly compared with the time scale of the field frequency.
- Two methods were proposed to study the effects of the oscillating filed.
- Phase-shifted model explains the existence of the traveling pattern.
- Time-averaged model explains the existence of the threshold of the homogeneous pattern.