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Directed transport in a spatially periodic magnetic billiard

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As an example of a Hamiltonian ratchet in two-dimensional space but without driving, we study a spatially periodic billiard immersed in a uniform stationary magnetic field. Our billiard consists of a straight waveguide with equidistant walls attached perpendicularly to one side. The magnetic field is sufficient to break all symmetries relevant for transport, an external driving therefore is unnecessary. In the low- and high-field limits, the motion becomes respectively pseudo-integrable and integrable; in both cases no transport is possible. In the medium-field regime, surfaces of section reveal mixed phase-space structures. Chaotic components support diffusive transport only in one direction, while regular components can support ballistic transport in both directions. We analyze the dynamical mechanisms underlying directed transport in terms of stable periodic orbits.

Nonlinear dynamics of pathogenesis and prognosis factor

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A mathematical model showing the interaction of normal cells, cancer cells and food supply is shown to yield interesting results. This model is extended to a more general case of pathogenesis. From these models a prognosis factor is computed that relates to the eventual outcome of the disease.

Sparks like coral reefs (the press said...)

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The streamers of glowing, ionized gas in a lightning strike have an inevitable tendency to break up at their tips, creating a familiar tree-like pattern. This tip-splitting is similar to the same kind of process that gives rise to branching in snowflakes, river channels, tumor growth and other different natural phenomena. In the 1980s, researchers identified a process called Laplacian growth which explains why branching occurs at the boundary between two substances. We have shown that a Laplacian instability also occurs at the edge of a lightning streamer. We analyse the minimal continuum model for a negative streamer in a non-ionized and non-attaching gas with impact ionization reaction in local field approximation. Using computer simulations, we have found that a streamer undergoes spontaneous branching at the tip. The mathematical analysis of the model identifies the instabilities as Laplacian ones. Despite their complicated structure, the basic model for streamers is simple. A single free electron traveling in a strong, uniform electric field ionizes the gaseous molecules around it, generating more electrons and a chain reaction of ionization. The ionized gas creates its own electric field, and a streamer is born. Eventually its charge and field concentrate in a thin, dome-shaped layer at the tip of the growing streamer. In this state, any tiny protrusions in the charged front focus the electric field around them. Because the speed of ionization depends on the surrounding field, these bumps grow faster than the rest of the front, and branching results. A similar mechanism can operate in a coral reef, when one part pulls ahead of the rest, it gets more food and so grows even faster.

Deterministic diffusion in a gravitational billiard

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We study numerically the motion of a classical particle in a homogeneous gravitational field bouncing elastically from a boundary limiting the motion from below. The boundary is piecewise linear and has wedge-like shape which is periodically extended over the entire horizontal axis. The angle characterizing the symmetric wedge discriminates between the region with only chaotic motion, and the region with periodic, limited or extended quasiperiodic, and chaotic trajectories.

It is observed that for a random set of initial conditions on the constant energy surface, the mean-square displacement asymptotically becomes linear function of time. The diffusion coefficient is examined as a function of energy and wedge angle. For large wedge angles and intermediate values of time t, the mean-square displacement scales as a power function of time with degree which can be larger then 2.

Examination of higher order moments gives further numerical evidence for tendency toward Gaussian distribution.

Homoclinic orbits of invertible maps

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We present a numerical method for finding all homoclinic orbits of an invertible map in any dimension. The method has completely controllable accuracy and is capable of uniquely identifying and naming each homoclinic orbit.

Breathers and homoclinic orbits

Homoclinic orbits are presented as a useful tool for obtaining breather solutions (time periodic, spatially localized) of one dimensional nonlinear lattices (Klein Gordon, FPU and mixed). We present how the relationship between homoclinic orbits and breathers is formed, and advantages of using this method for obtaining breathers over the traditional method of continuation.

Physics of love

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Groups of individuals will be discussed by the proposed report. Both are described as very abstract things. The theory is applicable to a lot of individual kinds, which tend to organise themselves to groups. Some examples are cells, animals, humans, economies, nations. The property all of them have to share is the possibility of information processing, although the minimum can be one single method of comparison. Quantum physical results and a fermionic explanation of gender are issued. The theory uses mainly quasi-static description, but has at least revealed a standard thermodynamic law: dH=TdS+Vdp The two terms at the right side display the two main parts of the reports content. The properties derived from the axioms show similarity to real thermodynamic gases. Publications of social science in the past had left well-known problems unsolved. One of them is feedback of experience another are unselfish individuals, a third one is diversity. The three named problems and probably others are vanished in the models of the theory. The founding book content (3MB, 260 pages) can be downloaded at: http://www.celebran.de. One target of application for the conference is searching for a doctor father, scientific acceptation of the theory and/or controversy discussion. Learning about other contributions and stimulation of new ideas is another target.

A pedagogical survey on some aspects of martingales

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We give a pedagogical introduction to the concept of martingales: The contribution addresses especially students participating in the International Conference "Let's Face Chaos Through Nonlinear Dynamics" in Maribor 2002. We deal in particular with Doob's Upcrossing Lemma and present some applications. A recently established connection between the structure of martingales and Hermite functions by Fitzsimmons is presented. Basic properties of the classical Hermite functions are revised. There remains the question whether one is able to generalize the obtained results in context of a discrete scenario, i.e. when replacing the used continuous Hermite functions by suitable discrete ones.

Quantum manifestation of classical stochasticity in the mixed state

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A Hamilton system with a few local minima in the potential energy surface represents a model in frame of which one can describe the dynamics of transitions between different equilibrium states, including such important transitions as chemical reactions, nuclear fission and others. Systems of such type allow existence of several critical energy values even for a fixed set of potential parameters. That leads to a possibility of existence of so-called mixed state [1] for such potentials: different dynamic regimes (regular or stochastic) are realised in the same energy interval in different local minima. It gives new possibilities for studies of the quantum manifestations of classical stochasticity (QMCS), using the following objects: statistical properties of energy spectrum (nearest-neighbour spacing distribution), structure of the wave functions (nodal curves, probability density) and wave packet dynamics. We consider such possibilities in application to quadrupole surface oscillations of nuclei, described by the lowest terms of the deformation potential decomposition in deformation parameters. In that potential the mixed state is observed in that part of the parameter space, where the equilibrium shape of the nucleus can be either spherically symmetric or deformed, i.e. for the potentials with a few local minima. In the case of the potential energy surface of complicated topology numerical calculation based on matrix diagonalization becomes ineffective, but the so-called spectral method [2] can become an inspiring alternative. Since the spectral method is fundamentally based on numerical solutions of a time-dependent differential equation, its implementation is always straightforward. Neither special ad hoc selection of basis function is required, nor is it necessary for the potential to have a special analytic form. The spectral method is in principle applicable to problems involving any number of dimensions.

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b-cell repolarization in the bursting Chay-87 model

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The pancreatic b-cell is the only insulin-producing type of cell in the body and is thus crucial in the regulation of the plasma glucose concentration. Failure of the glucose regulation leads to diabetes mellitus, a medical condition of almost epidemic proportions: According to the WHO, there are presently in the order of 150 million people suffering from diabetes mellitus. A number which is believed to double by the year 2025. The insulin secretion of the b-cell is known to be associated to a distinct pattern of oscillation of the cell membrane potential called bursting. The bursting pattern contains a silent phase which alternates with bursts of rapid oscillations (spikes). Since the mid-80s, a significant number of models of the b-cell electrophysiology have been constructed. In 1987 T. R. Chay proposed a b-cell model, which is based on five ionic currents through the cell membrane: Voltage gated calcium and potassium currents generate the rapid spikes of the active phase, while ATP- and calcium-dependent potassium currents are responsible for the level of activity and the repolarizing of the membrane, respectively. Finally, a leak current stabilizes the dynamics. The model contains three variables. The time constants of the membrane potential, V, and the opening probability of the voltage gated potassium channels, n, are small compared to that of the slow variable, the intracellular free calcium concentration. This difference inspires the separation of the three-dimensional phase space into a plane of rapid dynamics and a slow direction. Using the slow variable as a parameter for the fast subsystem yields the so-called reduced system. It has been shown that the reduced system contains a stable limit cycle which disappears in a homoclinic bifurcation as the system leaves the active phase. The poster will show the homoclinic bifurcation of the reduced system and investigate the dynamics in the three-dimensional phase space in vicinity of the isoclines of the reduced system.

Quantum chaos experiments with superconducting microwave cavities

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In the last few decades, billiard systems have provided a most appropriate model for the understanding and discussion of classical chaotic Hamiltonian systems and their quantum counterparts. In two dimensions the Schrödinger equation for quantum billiards coincides with the Helmholtz equation for flat cylindrical microwave cavities of corresponding shape. This correspondence provides the basis for experimental studies of quantum manifestations of classical chaos. We present results of experiments with normal and superconducting microwave cavities, where the former are used for the measurement of wavefunctions, the latter for the measurement of energy eigenvalues.

Power laws in a noisy finite-time-singularity model

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We discuss the influence of white noise on a generic dynamical finite-time-singularity model for a single degree of freedom. We find that the noise effectively resolves the finite-time-singularity and replaces it by a first-passage-time or absorbing state distribution with a peak at the singularity and a long time tail exhibiting power law or stretched exponential behavior. The study might be of relevance in the context of hydrodynamics on a nanometer scale, in material physics, and in biophysics.

Wigner function statistics of quantum maps

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We studied statistical properties of Wigner functions $W(x)$ of 1D quantum maps on compact phase space of finite area V. For this purpose we defined a Wigner function probability distribution $P(w) = 1/V \int \delta(w - W(x))dx$, which has, by definition, fixed first and second moment. In particular, we concentrate on relaxation of time evolving quantum state in terms of $W(x)$, starting from a coherent state.

We have shown that for a classically chaotic quantum counterpart the distribution $P(w)$ becomes a Gaussian distribution that is determinated by the first two moments. The numerical studies were done on the quantum sawtooth map and the quantized kicked top. In a quantum system with Hilbert space dimension $N(\sim 1/\hbar)$ the transition of $P(w)$ to a Gaussian distribution was observed at times $t \propto \log N$. In addition, it has been shown, that the statistics of Wigner functions of propagator eigenstates is a Gaussian as well in classically fully chaotic regime.

Estimation of effective vertical diffusivity and viscosity in turbulent exchange flows by numerical experiments

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Goitsche lake complex in Central Germany comprises two sub-basins, namely Muehlbeck and Niemegk, with distinct water compositions, which are linked to one another by a channel. The channel owes its nearly constant cross section and flat design of the bottom to the mining activity, which left a void after closure of the mine in the 1990s. The importance of exchange flows through such channels or straits, which are interconnecting water bodies of different densities, lies in the fact that water mass characteristics in sub-basins are determined by these flows, extend of which depends on the exchange rate between the interconnected water bodies. The knowledge of kinematic viscosity, k, and vertical diffusivity, v, along a channel/strait is therefore crucial as this gives us an idea about the amount of heat, mass and dissolved solids transported across it and provide parameterisations for circulation models. By conducting a number of numerical experiments with varying Rayleigh number and aspect ratio, we attempted to give an estimation of effective values for k and v in this channel. From comparison with numerical results of an exchange through a channel at various values for Ra, Pr and aspect ratio, we infer the effective value for kv as it appears in the Rayleigh number.

Rule and density behavior in rule-dynamical systems

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'Rule-dynamics' is a framework for simple models of rule-changing systems using cellular automata rules. Due to the coupling between the several density-range and rules, the correlation between rule and density generates, and some rule-dynamical systems have complete correspondence of rule sequence to the density-value. We propose one of possible methods to capture the rule and density behavior in terms of the correspondence of rule sequence to small density-range using typical example system.

Wavefunction scarring in open quantum dots: A microwave-billiard analog study

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We study the wavefunctions and transmission properties of an open microwave cavity in the frequency range of 1 - 17 GHz. The transmission of microwaves between the input and output antennas of cavity is found to exhibit large fluctuations as a function of frequency, while the cavity wavefunctions are found to reveal the presence of periodically-recurring scars, in close correspondence with specific peaks in the Fourier transform of the transmission fluctuations. This work therefore provides strong support for the results of recent studies of open quantum dots [1], which have revealed the presence of measurable transport effects associated with specific wavefunction scars.

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Universal transmission fluctuations in open microwave billiards with and without time-reversal symmetry

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Random matrix theory predicts characteristic distributions for the transmission amplitude T in dependence of the number of attached wave guides, and of time-reversal symmetry as well [1,2]. In quantum dots the predictions could be verified only qualitatively [3,4]. In the present experiment the transmission of microwave billiards was measured with and without [5] reflection symmetry for different numbers of channels. Time-reversal symmetry was broken via introduced ferrite cylinders. The obtained transmission intensity distributions are compared with predictions from random matrix theory. Because of the strong absorption caused by the ferrites, the existing statistical scattering theories had to be modified by incorporating a number of additional absorbing scattering channels. A better characteristic for the systems can be obtained from the distribution of dT/dE.

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Periodic phase synchronization in coupled chaotic oscillators

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Recently synchronization behavior in mutually coupled chaotic oscillators with slight parameter mismatch has been extensively studied because of its potentiality in application to many different disciplines of science and technology. What has been known about it that non-synchronization state transits to lag synchronization through phase jumps and next phase synchronization (PS) as the coupling strength increases. One of the most intriguing subjects among those is PS which is characterized by the coincidence of the phases of two chaotic oscillators within 2π, while their amplitudes remain chaos and uncorrelated each other. A typical behavior here is that one of the vanishing Lyapunov exponents becomes negative. However, before phase synchronization, one can usually observe a negative dip in the Lyapunov exponent. So the aim of our study is to investigate synchronization behavior in this dip and to manifest the full route from non-synchronization state to lag synchronization in coupled chaotic oscillators. When two Rössler oscillators with slight parameter mismatch are coupled with each other, the phase difference of them can be classified into two kinds of dynamics, fast and slow. When the fast dynamics is removed after being averaged over during the time segment of the half period of the slow dynamics, we can observe that the slow dynamic is synchronized periodically at the dip. We call this synchronization phenomenon periodic phase synchronization. In this region, the phase difference of the two oscillators jumps periodically, so that the derivative of it touches the zero line periodically while their phases are synchronized temporally. Before periodic phase synchronization, the two chaotic oscillators are de-synchronized. So what we clarify about the synchronization transition in coupled chaotic oscillators is that non-synchronization state develops to lag synchronization through periodic phase synchronization, next intermittent phase synchronization, and then PS.

Chaotic properties of some billiards with singular boundaries

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Several types of lemon-shaped billiards and some symmetrical billiards with discontinuities in the curvature radius are investigated classically and quantally. Special attention is paid to orbits in which the particle is reflected from the boundary at the singular points. The orbit stability is investigated in dependence on the shape parameter and compared for different boundary types. Further analysis of the presented classical billiards include the Poincare surface sections. It can be observed that for certain boundary shapes and for some ranges of the shape parameter the chaotic part of the phase plane separates into several distinct chaotic segments. In the quantal calculation the chaotic fraction of the phase space is computed. The obtained classical values are compared with the corresponding results obtained from the statistical analysis of the energy level densities. The method used is the Nearest Neighbour Spacing Distribution (NNSD), and in fitting the calculated histograms the Brody, Berry-Robnik and Berry-Robnik-Brody distributions are used. Some properties of the calculated wave functions will also be discussed.

Limiting level spacing distribution of integrable quantum systems

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We report our recent studies on the energy level statistics of integrable quantum systems. Based on the assumption proposed by Berry and Robnik, level spacing distribution of a system consisting of infinitely many independent components is derived as a weak limit and its deviations from the Poisson distribution is discussed. The limiting level spacing distribution is specified by a single monotonically increasing function $m(S)$ of the level spacing S. Three cases are distinguished by using $m = m(\infty)$: (i) Poissonian if $m = 0$, (ii) Poissonian for large S, but possibly not for small S if $0 < m(\infty) < 1$, and (iii) sub-Poissonian if $m(\infty) = 1$. This implies that, even when energy level distributions of individual components are statistically independent, the Berry-Robnik approach would provide level spacing distributions other than the Poissonian distribution.

On Marcus approach to quadratic ODE and stability of origin

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A possible hypothesis for algebraic properties for stability of the origin of quadratic systems of ODE's is given. Marcus classification of all commutative algebras in the plane is used for proving the absolute validity of given hypothesis for quadratic systems in dimension two. For inducing the hypothesis a large family of quadratic systems in 3D was treated.

Anomalous diffusion in a Hamiltonian system

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We numerically study transport property inside the stochastic layer of 2 degrees of freedom Hamiltonian system where the trajectories are chaotic because of Homoclinic tangles. We found that the diffusion is anomalous and density profile is not Gaussian. Furthermore, we show that, with a random sampling method, this system exhibits Levy flight like diffusion. We discuss the relation between regular and random sampling in terms of scaling exponents.

Equivalence between isospectrality and iso-length spectrality for a certain class of planar billiard domains

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Isospectrality of the planar domains which are obtained by successive unfolding of a fundamental building block is studied in relation to iso-length spectrality of the corresponding domains. Although an explicit and exact trace formula such as Poisson's summation formula or Selberg's trace formula is not known to exist for such planar domains, equivalence between isospectrality and iso-length spectrality in a certain setting can be proved by employing the matrix representation of "transplantation of eigenfunctions" [1]. As an application of the equivalence, transplantable pairs of domains, which are all isospectral pair of planar domains and therefore counter examples of Kac's question "can one hear the shape of a drum?", are numerically enumerated and it is found at least up to the domain composed of 13 building blocks transplantable pairs coincide with those constructed by the method due to Sunada [1,2]

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Can one determine the shape of a drum through the spectrum of the interior Dirichlet problem and the cross sections of the exterior Neumann scattering?

The quantum billiard problem, that is the Dirichlet problem for the Helmholtz equation, can be rewritten as a Fredholm integral equation of the second kind with the aid of the Green's theorem, and then the eigenenergies of the quantum billiard can be characterized as the zeros of the Fredholm determinant on the real axis [1,2]. From this view point, we pose a new question "can one determine the shape of billiard table through the Fredholm determinant?" instead of the famous Kac's question "can one determine the shape through the eigenenergies i.e. the zeros of the Fredholm determinant?" [3], which was solved negatively [4]. Our numerical tests reveal that the answer to our question addressed above is "yes", that is, the shapes of the isospectral pair of billiards are distinguishable from the eyes of the Fredholm determinants although they have exactly the same eigenenegies [5]. Via "the interior-exterior factorization of the Fredholm determinant" [2], the difference of the Fredholm determinants between the isospectral pair can be interpreted as the difference of the scattering phase shifts when the billiard table are regarded as a scatterer against the exterior wave function.

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Noise induced phase synchronization in diode lasers

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Chaos synchronization by external common noise is an intriguing phenomenon in chaotic dynamical systems in connection with application to secure communication and generalized synchronization. When common arbitrary noise is added to identical chaotic systems externally, the systems are synchronized at a certain region of the noise amplitude. In apart from these studies, phase synchronization is also extensively studied. When two slightly different chaotic systems are coupled with each other, their phases begin to be synchronized when the coupling strength exceeds the critical value. Here, we study phase synchronization in diode lasers by applying a common noise-like signal simultaneously. In the experiment, when two diode lasers generate chaotic outputs, we feed a common noise-like signal to the laser drivers simultaneously. Then, each laser pumped by dc current is modulated by the noise-like signal. We observe the transition to phase synchronization as the amplitude of the noise-like signal increases. When the amplitude exceeds a critical voltage, the phases of the two lasers are synchronized with a time delay. Before phase synchronization, the dynamics shows intermittent phase jumps, which is similar to that appearing in coupled chaotic oscillators. This transition due to the common noise-like signal is a new phenomenon that has not been reported yet. We study the characteristics of the transition to phase synchronization in the phase spaces. And phase synchronization and intermittent phase jumps are analyzed by the dynamics of the phase difference of the two lasers according to the amplitude of the noise-like signal.

Random matrix analysis of human EEG data

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The correllation analysis of the human EEG and NIRS data is performed. We show that the spectral statistics of the underlying correlation matrix ensemble can be linked to the medical diagnosis of the given patient. Similar analysis is done also for data with chanels shifted by varying time lag. In this case the the analysis of (nonsymmetric) correlation matrix uncovers certain causal processes in the brain that participate in the signal generation.

Stochastic auto-oscillation of temperature and defect density in crystals under fluctuating conditions of irradiation

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The influence of external noise (that is fluctuation of defect production rate and heating under irradiation) on the dynamics of defect density and irradiated-crystal temperature change is examined. Time-dependence of crystal temperature and defect density according to irradiation conditions and crystal properties is one of the various stochastic processes. This is a stationary stochastic process or process which looks like auto-oscillation and Wiener's process. It is shown that regular auto-oscillation of mean values of defect density and temperature intermit with periods of irregular behavior. Characteristics of the auto-oscillations are not dependent on selection of initial conditions. The oscillation period is situated in the range 20 - 50 s. Changes of the second moments look like positive pulses, their values increase from zero up to amplitude value and then decrease again. With increase of intensity of external noise, the amplitude value begin to grow slowly, behavior of mean values do not change. When dispersion reaches the critical value, that is comparable to corresponding mean values, breakdown of the periodic regime takes place. It is characterized by sharp irregular change of all variables and significant increase of dispersion. Then the regular periodic regime is restored, amplitudes of dispersion become small and the whole process repeats again. The duration of the irregular behavior in various cases can be different. It is equal to nearly ten periods of the regular movement that continues during hundreds and thousands of its own regular periods. Regular periodic change of mean values is near to auto-oscillations of deterministic model. On the basis of these results one can conclude that the role of fluctuations of irradiation conditions becomes apparent mainly in the dynamic behavior of a system.

Stability of low-dimensional solutions in the Fermi-Pasta-Ulam system

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The Fermi-Pasta-Ulam (FPU) system represents a one-dimensional array of equi-mass particles coupled by nonlinear springs. Since the pioneering work of FPU [1], a large amount of numerical experiments have been carried out with this system in order to elucidate how nonlinearity leads to ergodicity and the equipartition law of energy. One of the striking phenomena revealed in these experiments is the "induction phenomenon" in the energy exchange process among normal modes, i.e., initially excited modes retain energy over a certain period before violent energy exchange process sets in [2]. The FPU system possesses low-dimensional solutions whose dynamics can be completely described by reduced Hamiltonians. Since the growth of initially non-excited modes can be attributed to the instability of the low-dimensional solutions, identification of them is important for precise understanding of the induction phenomenon. One can systematically construct the reduced Hamiltonians by introducing in the mode number space the "type I subsets" defined by Poggi and Ruffo [3]. In this presentation, we first present general expressions for the type I subsets, which, as far as we numerically checked, cover all of the possible type I subsets in the FPU system [4]. Using these expressions, we next construct the reduced Hamiltonians and investigate the stability of their solutions under the full system's dynamics, where the linear stability of these solutions is described by coupled Hill's equations. We report the independence of the stability properties on the phase space structure of the reduced Hamiltonian systems. Finally, we discuss the conditions for the occurrence of the induction phenomenon in Hamiltonian systems.

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On some difference equations in context of q-Fourier analysis

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A discrete Fourier transform on a q-linear grid is presented. Several of its analytic properties are discussed and compared with the continuum situation. We recognize that a special invariant of the related Fourier operator is closely connected to discretizations of the Hermite functions. This result is similar to the continuum scenario and reveals a key role of the q-Fourier transform to the understanding of difference equations. The continuum limit $q \to 1$ in the sense of strong L2-convergence is investigated for the derived q-Fourier invariant generalization of the Gauss curve. Applications to Schrödinger difference equations are briefly mentioned.

Finding periodic orbits in area preserving maps by topological degree theory

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We consider methods based on the topological degree theory to compute periodic orbits of area preserving maps. Numerical approximations to the Kronecker integral give the number of fixed points of the map provided that the integration step is small "enough". Since in any neighborhood of a fixed point the map gets four different combinations of its algebraic signs we use points on a lattice to detect the candidate fixed points by selecting boxes whose corners show all combinations of sign. This method and the Kronecker integral can be applied to bounded continuous maps such as the beam-beam map. On the other hand they cannot be applied to maps defined on the torus, such as the standard map, which has discontinuity lines propagating by iteration, or unbounded maps such as the Henon map. However, the systematic use of the bisection method initialized on the lattice, even though unable to detect all fixed points of a given order, allows us to find a sufficient number of them to provide a clear picture of the dynamics, even for maps of the torus because the discontinuity lines have measure zero.

Constrained random matrices

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The study of random matrices [1] has provided insight into many physical problems both in the quantum as well as in the classical domain. For example, random matrices have been very successfully used to model statistical properties of disordered conductors and of highly excited classically chaotic quantum systems. In the classical domain, random matrices arise in the context of diffusion in random, directed environments (see [2] and references therein). In all of these cases the random matrix elements obey symmetry requirements where appropriate, and are otherwise taken to be independently distributed random variables.

There are many cases, however, where constrained matrices must be considered: examples are electron hopping in amorphous semiconductors (see [3] and references therein), random reactance networks [4], and random Master equations.

We calculate the density of eigenvalues of constrained random $N \times N$ matrices in the limit of large N using a perturbative expansion [5]. The results are compared to the results of numerical simulations for finite values of N.

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Bifurcations in sixth-order PDE modelling spinning reaction fronts

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Spinning solid flames, so named because reacting solid compounds produce solid reaction products, and spinning detonation waves may be phenomenologically described by the sixth-order evolution-type PDE (Strunin, IMA J. Appl. Math., 63, 1999, 163). The equation is formulated in terms of the location of combustion front defined as the surface separating cold fresh mixture from hot reaction products. The equation contains nonlinear energy source term, nonlinear term facilitating energy flux towards smaller scales, and linear dissipation term expressed by sixth-order derivative. The model yields stable kink-type solutions travelling along spiral trajectories on cylindrical sample. Only large initial distortions of plane front will develop into nontrivial regimes, namely the spinning waves; small distortions will decay. Qualitatively, such dynamics well addresses the real phenomena. In terms of derivative of the front profile with respect to transverse coordinate the kinks correspond to pulses, or auto-solitons. The auto-soliton manifests itself as fundamental solution of the equation. We demonstrate numerically that arbitrary initial condition eventually breaks up into a set of identical pulses. Stress that the travelling waves (pulses) here are produced by purely dissipative mechanism. We present and analyse new results of numerical simulations of the model, focusing on bifurcations of regimes with various wave numbers. Of particular interest is the question whether chaotic regimes occur in large domains. We give some preliminary evidences in favour of positive answer to the question. We also discuss mathematical links of the equation to the generalized nonlinear phase diffusion equation of Kuramoto.

Multiplicity of limit cycle attractors in a coupled heteroclinic cycle system

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We study a new type of coupled oscillators system in which each oscillator has a heteroclinic cycle attractor instead of a limit cycle or a phase oscillator. Oscillators are distributed on a two-dimensional square lattice and coupled with the nearest neighbors diffusively. If an orbit approaches a heteroclinic cycle attractor, the period of the oscillation gets exponentially slower for each oscillation and the system has no characteristic time scale. We employ a replicator system with 4 components as the heteroclinic oscillator.

A replicator system is a model for an ecological system or a chemical reaction network of self-catalyzing molecules. Therefore the situation of our system can occur in a spotted ecological system with diffusion (ex. a system on trees of an orchard) or a population dynamics of self-catalyzing proteins in cells.

In this poster, we report a novel class of patterns that are spatially disordered but periodic in time. These patterns are found in the range of the small diffusion constant. We investigate the patterns and show that they are limit cycle attractors in the ambient phase space (i.e. not chaotic) and many limit cycles exist dividing the phase space as their basins. In addition, these patterns are constructed with a local law of difference of phases among the oscillators. The number of patterns grows exponentially with increasing of the number of oscillators.

Self-similar magneto-conductance fluctuations in general-shaped soft-wall quantum billiards

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Recently, using semiclassical Kubo formula, Budiyono and Nakamura showed [1] that the self-similar conductance is caused by the self-similar periodic orbits generated by pitchfork bifurcations of a straight-line liberating orbit oscillating toward saddle point in Henon-Heiles potential. This potential has three saddle points that give rise to the bifurcations, and is suitable to model a real triangle billiard with three leads [2]. They argue that the existence of such harmonic saddles is essential and that the self-similar conductance phenomena should be observed in systems with non-Henon-Heiles potential. To support their idea, using their methods, we show theoretically the emergence of the self-similar magneto-conductance fluctuations in general-shaped soft-wall quantum billiards. First we calculate the monodromy matrices of the periodic orbits in the Barbanis potential with two harmonic saddles that give rise to the bifurcations. And using the above result, we show the self-similar conductance. Furthermore we will show the self-similar conductance in real systems with more general potentials. Finally we expect that our results confirm the experimental evidence observed by Micolich et al. [3], that the global shapes of billiards are not the relevant issue to discuss the dimension of the fractal fluctuation.

References

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Geometric and statistical properties induced by separatrix crossings in volume-preserving systems

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We consider a 3D volume preserving system inside the unit sphere. The system is a small perturbation of an integrable one. Almost all phase trajectories of the integrable system are closed curves. Under an arbitrarily small non-zero perturbation all the interior of the sphere, up to a residue of a small measure, is apparently a domain of chaotic motion. The phenomenon is described as a result of jumps in an adiabatic invariant of the system occurring when a phase trajectory crosses the 2D separatrix of the unperturbed (integrable) system. The dynamical properties can be understood as a consequence of splitting of invariant manifolds under the perturbation. A 2D return map generated by the system possesses strong stretching properties. This makes the dynamics of the system close to hyperbolic. Numerical investigation of the statistical properties of the system demonstrates a good agreement with theoretical predictions made under the assumption that the system has a big ergodic component whose measure tends to the measure of the sphere's interior as the perturbation tends to zero. However, the system is not ergodic, and stable periodic solutions are found, surrounded by stability islands of measure exponentially small with the perturbation.

Parametrically driven dark solitons

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It is well known that bright parametrically driven, damped nonlinear Schrödinger solitons are prone to oscillatory instabilities which set in above a certain driving threshold. In this communication I shall demonstrate that the parametrically driven, damped dark solitons are immune from instabilities throughout their existence region. Furthermore, I shall show that for the undamped case there are two different stable solutions, which are also capable of forming stable complexes. I shall mention some noteworthy characteristics of the interaction of these different solutions.

Chaotic outputs of an operational amplifier

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Electronic circuits exhibiting chaotic behaviors have attracted much attention for their availability to simple experimental verification of chaos theory. Until now, such nonlinear electronic circuits have been developed as an inductor-resistor-diode circuit, a Chua circuit, and a double scroll circuit, and many nonlinear phenomena studied in theory have been investigated in these circuits. So to find out a new circuit that exhibits rich chaotic behavior is very important not only for experimental verification of chaos theory but also for application of chaos to practical fields like secure communication. For this, we introduce an element, operational amplifier (OP Amp), which generates chaos for itself. An OP Amp is used as one of the most important electronic elements that performs a wide variety of linear functions. In an OP amp, when the inverting terminal is connected with the output terminal through a resistor, and positive input terminal is grounded, the output signal has a phase shift due to the resistance-capacitance lag network within the amplifier. At this time when the total lag network phase shift equals or exceeds 180 degrees, the system is unstable and can generate a periodic signal. Here, when the absolute voltages of the negative and the positive power supplies are much less than 15 Volts and the resistor is zero, a fast OP Amp generates chaotic behaviors. We carry out an experiment by using LF357, and observe various chaotic behaviors according to the power supplies voltages. The bifurcation diagram and the phase diagram exhibit that the system develops to chaos through period doubling bifurcation. And the return maps show the origins of the chaotic behaviors that are type- I and II intermittencies, periodic windows, crisis, etc. All the experimental results well agree with the chaotic behaviors of the logistic map.

Wave propagation in random media with nonlocal coupling

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The purpose of this poster is to elucidate how noise intensity effects wave propagation in nonlocally coupled oscillators. It was shown that in the case of globally coupled oscillators, wave cannot propagate if the intensity of white Gaussian noise becomes above a critical point and that this transition occurs via a Hopf Bifurcation. We expect from this that the same fact is true for nonlocally coupled oscillators and show it by both a theoretical approach and computer simulation. The phase of nonlocally coupled oscillators obeys a stochastic equation. We can lead the Fokker-Planck equation equivalent to it. Near a point of the critical point, the Fokker-Planck equation can be reduced to the Ginzburg-Landau equation. The coefficients of the GL equation depend on a coupling function. When it is the pulse coupling function, the GL equation has an instability. We investigate the chaotic behavior. This model is thought to represent the real neurons and has an important meaning.

Many-body symbolic dynamics of a classical oscillator chain

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We study a certain type of the celebrated Fermi-Pasta-Ulam particle chain, namely the inverted FPU model, where the inter-particle potential has a form of a quartic double well. Numerical evidence is given in support of a simple symbolic description of dynamics (in the regime of sufficiently high potential barrier between the wells) in terms of an (approximate) Markov process. The corresponding transition matrix is formally identical to a ferromagnetic Heisenberg quantum spin-1/2 chain with long range coupling, whose diagonalization yields accurate estimates for a class of time correlation functions of the model.