CHAOS AND THE DYNAMICAL EVOLUTION OF BARRED GALAXIES

T. MANOS* and E. ATHANASSOULA†
* Observatoire Astronomique de Marseille-Provence (OAMP), FRANCE.
† Centre for Research on Applied Nonlinear Systems (CRANS),
Department of Mathematics, University of Patras, GREECE.

ABSTRACT

The dynamical evolution of barred galaxies depends crucially on the fraction and distribution of chaotic orbits in them. In order to distinguish between ordered and chaotic motion, we use the Smaller Alignment Index (SALI) method, a very powerful method which can be applied to problems of galactic dynamics. Using model potentials, and taking into account the full 3D distribution of matter, we discuss how the distribution of chaotic orbits depends on the main model parameters, in particular the mass and thickness of the bar.

1 INTRODUCTION - ORDERED AND CHAOTIC MOTION

The distinction between ordered and chaotic motion in dynamical systems is fundamental in many areas of applied sciences. This distinction is particularly difficult in systems with many degrees of freedom (dof), basically because it is not feasible to visualize their phase space. We thus, need fast and accurate tools to give us information about the chaotic or ordered character of orbits, especially for conservative systems.

In this paper we focus our attention on the method of the Smaller Alignment Index (SALI) [1 - 8] or Alignment Index ([9,10]) and we present some applications of the index in Ferrers barred galaxy potentials of 2 and 3 dof [11,16,17].

In order to compute the SALI for a given orbit one has to follow the time evolution of the orbit itself and also of two deviation vectors \( v_1 \) and \( v_2 \), which initially point in two different directions. At every time step the two deviation vectors are normalized and the SALI is then computed as:

\[
SALI(t) = \min \left( \frac{v_1(t)}{\|v_1(t)\|} + \frac{v_2(t)}{\|v_2(t)\|} \right).
\]


2 APPLICATIONS IN THE FERRERS BARRED GALAXY POTENTIAL 2 DOF

This model consists of the superposition of the potentials of an axisymmetric and a bar component. The axisymmetric part makes up both the bulge and the disk, while the bar is a strongly triaxial, centrally condensed body [18].

Axial symmetric component:

\[
V(x,y,z) = -\frac{GM_1}{\sqrt{x^2 + y^2 + z^2}} - \frac{GM_2}{\sqrt{(A + \sqrt{B^2 + z^2})^2}}
\]

Bar component:

\[
\rho = \begin{cases} 
\rho_0, & \text{for } m \leq 1 \\
0, & \text{for } m > 1
\end{cases}
\] where \( m = \frac{x^2 + y^2}{a^2} - \frac{z^2}{c^2} \)

The total mass of the bar is related to the central density:

\[
M_{\text{bar}} = 1.05 \frac{GM_0}{2 \pi} a b c.
\]

The Hamiltonian function is:

\[
H = \frac{1}{2} (p_x^2 + p_y^2 + p_z^2) + V(x,y,z) - \Omega(z)p_z.
\]

STUDY OF THE POTENTIAL IN 2 DEGREES OF FREEDOM, FOR DIFFERENT VALUES OF ENERGY BY Poincaré SURFACE OF SECTION AND SALI

We choose initial conditions on a line on the surface of the Poincaré Surface of Sections [15] and we calculate the SALI of each orbit. We can see that, as the energy increases, the amount of chaotic orbits also increases competing the results of the two different methods of detecting chaotic and ordered motion in 2 dof. In 3 (or more in general) degrees of freedom, there is no way to have plots of Poincaré Surface of section but the SALI method can still be used.

3 APPLICATIONS IN THE FERRERS BARRED GALAXY POTENTIAL 3 DOF

Here we study the 3 dof case for 3 models and 2 different ways of choosing the initial conditions (CASE A - B). We can thus investigate how some model parameters affect the relative fraction of ordered or chaotic orbits.

CASE A: 27000 initial conditions on \((x, y, z)\) and \((y, z, px) = (0,0,0)\).

CASE B: 27000 initial conditions on \((x, y, z)\) and \((x, z, px) = (0,0,0)\).

By comparing models A1 - A3 and B1 - B3 we can see that, as the mass of the bar \( M_{\text{bar}} \) increases, the behaviour of the system becomes more chaotic. This is in good agreement with the results found for 2 dof [11]. In particular, in the CASE B - B3 (where we vary the momentum \( p_z \)) the percentage of the chaotic orbits is almost 2.5 times larger than in the initial model B1, while in model A3 it is roughly 2 times larger. There is also a transfer of a considerable number of ordered orbits to chaotic orbits with near border line - value for the SALI \( \rightarrow 10^{-5} \) [3,4], i.e. a value beyond which the trajectories are characterized as non-regular. This effect is stronger in model A1, where we vary the initial position \( z \).

On the other hand, when we increase the parameter \( c \) of the bar (models A2 - B2), the system presents more regular behaviour than the initial basic models (A1 - B1). This is due to the fact that we keep the bar mass constant, so that its volume density drops and thus creates a smaller perturbation.

4 ACKNOWLEDGEMENTS

This work was supported by the French-Greek graduate fellowship No 13935 of the French Ministry of Education and by "Marie Curie" fellowship.

REFERENCES