CAMTP

SYMBOLIC COMPUTATION AND ITS APPLICATIONS

Hotel PIRAMIDA, Maribor, Slovenia
30 June 2010 - 2 July 2010

PROGRAMME

SYMBOLIC COMPUTATION AND ITS APPLICATIONS

Hotel PIRAMIDA, Maribor, Slovenia
30 June 2010 - 2 July 2010

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SCHEDULE

Wednesday 30 June	
9:00–9:20	**Opening**
Chairman	**Paule**
09:20–10:10	**Wilf**
10:10–11:00	**Shafer**
11:00–11:30	Tea & Coffee
11:30–12:10	**Sakata**
12:10–12:50	**Makhnev**
12:50–13:30	**Logar**
13:30–15:00	Lunch
Chairman	**Han**
15:00–15:40	**Levandovskyy**
15:40–16:20	**Romanovski**
16:20–16:50	Tea & Coffee
16:50–17:30	**Kutzler**
17:30–18:00	**Yilmaz**
18:00–18:30	**Ruffing**
18:40–19:30	**Excursion to Wine Cellar**
19:45	Welcome Dinner

Thursday 1 July	
Chairman	**Wilf**
09:00–09:50	**Paule**
09:50–10:40	**Han**
10:40–11:10	Tea & Coffee
11:10–11:50	**Moravec**
11:50–12:30	**Edneral**
12:30–13:10	**Ziv-Av**
13:10–15:00	Lunch
Chairman	**Gerdt**
15:00–15:40	**Vassiliev**
15:40–16:10	**Raab**
16:10–16:40	Tea & Cofee
16:40–17:20	**Klep**
17:20–18:00	**Chen**
18:00–18:30	**Kavitha**
19:15	Reception, Concert & Conference Dinner

Friday 2 July	
Chairman	**Pagon**
09:00–09:50	**Pisanski**
09:50–10:40	**Gerdt**
10:40–11:20	**Petkovšek**
11:20–11:50	Tea & Coffee
11:50–12:30	**Jurišić**
12:30–13:10	**Schneider**
13:10–13:40	**Marlewski**
13:40–15:15	Lunch
Chairman	**Petkovšek**
15:15–15:55	**Pagon**
15:55–16:35	**Cozma**
16:35–17:05	**Niu**
17:05–17:30	Tea & Cofee
17:30–18:00	**Papamikos**
18:00–18:30	**Ferčec**
19:00	Dinner

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Abstracts

Isochronous center problem for time-reversible cubic systems

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Problems of center and isochronicity are important for investigation of qualitative properties of planar differential systems. Some nice results were obtained in the 1960's and 1970's and at present these problems are once again attracting considerable interet(see [1] and references therein). Many works have been done for quadratic, cubic, quartic or quintic systems with homogeneous nonlinearities, Kolmogorov systems, Liénard type systems and Hamiltonian systems.
For time-reversible cubic systems some sufficient conditons for the origin to be an isochronous center were obtained in [2-5]. Using the Darboux linearization method in [6], we give necessary and sufficient conditions for the complexfied system to be linearizable and, hence, obtain necessary and sufficient conditions for the origin of time-reversible cubic systems to be isochronous. Therefore, the isochronous center problem of time-reversible cubic systems is solved completely.

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Integrability of cubic differential systems with algebraic invariant curves

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Consider the cubic system of differential equations

$$\begin{aligned} \dot{x} &= y + ax^2 + cxy + fy^2 + kx^3 + mx^2y + pxy^2 + ry^3 \equiv P(x,y), \\ \dot{y} &= -(x + gx^2 + dxy + by^2 + sx^3 + qx^2y + nxy^2 + ly^3) \equiv Q(x,y), \end{aligned} \tag{1}$$

where $P(x,y)$, $Q(x,y) \in \mathbb{R}[x,y]$ are co-prime polynomials. The origin $O(0,0)$ is a singular point of a center or a focus type for (1), i.e. a weak focus. The problem arises of distinguishing between a centre and a focus (the problem of the centre).

In this talk we discuss the difficulty of this problem and present the results concerning the relation between integrability, invariant algebraic curves and Liapunov quantities. The problem of the centre is investigated for (1) with lines and conics as the invariants.

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Integrability of a planar multiparameter system of ODEs near a degenerated stationary point

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We consider an autonomous system of ordinary differential equations, which is resolved with respect to derivatives

$$\begin{aligned}\dot{x} &= -\,y^3 - b\,x^3\,y + (a_0\,x^5 + a_1\,x^2y^2),\\ \dot{y} &= (1/b)\,x^2\,y^2 + x^5 + (b_0\,x^4y + b_1\,x\,y^3).\end{aligned} \tag{1}$$

This system is degenerated at the stationary point $x = y = 0$.

We say that the system (1) is *locally integrable* in a neighborhood U of the stationary point $X = X^0$ if it has in U the sufficient number m of first integrals of the form

$$a_j(X)/b_j(X), \quad j = 1, \ldots, m,$$

where functions $a_j(X)$ and $b_j(X)$ are analytic in U. Otherwise we call the system (1) *locally non-integrable* in this neighborhood. It is said that a planar $(n = 2)$ system is locally analytically integrable if it admits in U a first integral .

For studying the local integrability of equation (1) near a degenerate stationary point we use an approach based on Power Geometry and on the computation of the resonant normal form [1-3]. We found the complete set of necessary conditions on parameters of the system for which the system is locally integrable near a degenerate stationary point [4,5]. The set of parameters satisfying the conditions consists of four two-parameter subsets in the 5-parameter space. For all such subsets, we found global first integrals of the system. So we found also the sufficient conditions of integrability (1).

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Integrability conditions for complex systems with homogeneous quintic nonlinearities

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The problem of integrability of systems of differential equations is one of central problems in the theory of ODE's. Although integrability is a rare phenomena and a generic system is not integrable, integrable systems are important in studying various mathematical models, since often perturbations of integrable systems exhibit rich picture of bifurcations.

If we try to study the local integrability (the so-called center problem [1,3]) for the system

$$\begin{aligned} \dot{x} &= x - a_{40}x^5 - a_{31}x^4y - a_{22}x^3y^2 - a_{13}x^2y^3 - a_{04}xy^4 - a_{-15}y^5, \\ \dot{y} &= -y + b_{5,-1}x^5 + b_{40}x^4y + b_{31}x^3y^2 + b_{22}x^2y^3 + b_{13}xy^4 + b_{04}y^5, \end{aligned} \tag{1}$$

where $x, y, a_{ij}, b_{ji} \in \mathbb{C}$, then it turns out the computations involved to the determination of the necessary conditions of integrability for the full family (1) are so heavy that they cannot be completed even using powerful computers and computer algebra systems. Thus, it is reasonable to study some subfamilies of system (1). Recently the center conditions for the subfamily of (1), with $a_{-15} = b_{5,-1} = 0$, called the Lotka-Volterra system, have been obtained in [2].

We study the local integrability of the system

$$\begin{aligned} \dot{x} &= x - a_{40}x^5 - a_{31}x^4y - a_{22}x^3y^2 - a_{04}xy^4 - y^5, \\ \dot{y} &= -y + x^5 + b_{40}x^4y + b_{22}x^2y^3 + b_{13}xy^4 + b_{04}y^5. \end{aligned}$$

The necessary conditions for local integrability of the system are obtained. The sufficiency of some of these conditions is proved.

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Consistency of Finite-Difference Approximations to Systems of PDEs and Related Symbolic Computation

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In this talk we present the results obtained in collaboration with Yu.A.Blinkov and D.Robertz on computer algebra application to study consistency of finite-difference approximations (FDAs) to systems of partial differential equations (PDEs) of the form $f_1 = \cdots = f_p = 0$. Here $F := \{f_1, \ldots, f_p\}$ is a set of partial differential polynomials over the field of rational functions with rational coefficients. For orthogonal and uniform solution grids by combining the finite volume method with the difference elimination one can algorithmically generate [1] a FDA $\tilde{f}_1 = \cdots = \tilde{f}_p = 0$ to the initial differential system. Provided with a discrete version of the boundary or/and initial value conditions, the FDA yields a finite difference scheme. To provide convergence of a numerical solution of the finite difference scheme to the exact solution to the differential equations when the grid steps go to zero the scheme has to be consistent [2].

We strengthen the generally accepted concept of equation-wise consistency (e-consistency) of the difference equations as approximation to the differential ones. Instead, we suggest a notion of s-consistency (strong consistency) as the consistency of any difference consequence of the polynomial set $\tilde{F} := \{\tilde{f}, \ldots, \tilde{f}_p\}$ with a differential consequence of F. In the case of linear PDEs s-consistency admits algorithmic verification [3] via a Gröbner or involutive basis of the difference ideal $\langle \tilde{F} \rangle$. In doing so, the consistency verification algorithm uses also a differential Gröbner or involutive basis of the ideal generated by the initial PDE system. In the last case the involutive basis is obtained by completion of system to involution. By applying the Maple packages Janet and LDA (abbreviation for Linear Difference Algebra) [4] implementing author's involutive algorithm for constructing Janet bases for ideals generated by linear differential and difference polynomials we analyze some examples of finite difference approximations to linear PDEs, including those which are e-consistent and s-inconsistent. We also found an example of e-consistent and s-inconsistent nonlinear finite-difference scheme for the Navier-Stokes equation system [5] describing fluid dynamics.

In accordance to the brilliant Lax-Richtmyer equivalence theorem [2] proved first for scalar linear PDEs and extended to some scalar nonlinear equations, a consistent FDA to a PDE, when the last admits a well-posed initial value (Cauchy) problem, converges if and only if it is stable. Thus, in practice, the consistency check has to be applied to the discrete form of the differential equations admitting well-posedness of the Cauchy problem. Unlike scalar PDE for a system of PDEs this requires its completion to involution [6] prior to discretizing. For linear PDEs the completion to involution is fully algorithmic. However, by a simple example of overdetermined linear PDE system we show that it can be nontrivial to find an s-consistent finite-difference scheme for the involutive form of the input differential system.

Generally, a nonlinear PDE system does not admit its completion to involution. Instead, one can split the system into a finitely many involutive subsystems with disjoint set of solutions. Being involutive every subsystem admits well-posedness of Cauchy problem [7], and can be discretized together with the corresponding boundary or/and initial value conditions. The detailed description of the combined algorithm for the splitting and completion to involution given in [8]. Its application will be illustrated in the talk by a simple example.

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On Hopf bifurcations of piecewise planar Hamiltonian systems

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In this paper we consider a piecewise Hamiltonian system of the form

$$\dot{x} = H_y,\ \dot{y} = -H_x,\ x \neq 0, \tag{1}$$

where

$$H(x,y) = \begin{cases} H^+(x,y), & x > 0, \\ H^-(x,y), & x < 0, \end{cases}$$

and $H^{\pm}(x,y) \in C^{\omega}$ with $H^{\pm}(0,0) = 0$. Let $H^{\pm}(0,y) = \lambda^{\pm}\left(y^2 + H_1^{\pm}(y) + H_2^{\pm}(y)\right)$ where $\lambda^{\pm} > 0$, and

$$H_1^{\pm}(y) = \sum_{j=1}^{k^{\pm}} h_j^{\pm} y^{2j+1},\ \ H_2^{\pm}(y) = \sum_{j=1}^{l^{\pm}} r_j^{\pm} y^{2j+2},\ \ l^{\pm} = k^{\pm} \text{ or } k^{\pm} - 1.$$

One of our main results can be stated as follows.
Theorem Let $k_0 = \min\{k^+, k^-\},\ l = \max\{l^+, l^-\}$. Then for any $\varepsilon_0 > 0$ there are $r_j^{\pm} \in (-\varepsilon_0, \varepsilon_0)$ for $1 \leq j \leq l^{\pm}$ and $h_j^{\pm} \in (-\varepsilon_0, \varepsilon_0)$ for $1 \leq j \leq k^{\pm}$ such that the system (1) has at least $k_0 + l - 1$ small amplitude limit cycles near the origin surrounding the focus at the origin. Further, there are at most $[(\deg H_0^+ - 1)(\deg H_0^- - 1) - 2]/2$ limit cycles which surround the origin.

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Use of orthogonal polynomials in discrete mathematics

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Orthogonal polynomials were developed in the late 19th century from a study of continued fractions by Chebyshev and were pursued by Markov, Stieltjes and by a few other mathematicians. Since then, applications have been developed in many areas of mathematics and physics. In our talk we will concentrate on their applications in discrete mathematics. For example, we show how to use them to improve efficient implementations of cryptosystems based on finite fields and in particular on elliptic curves. Most finite objects of sufficient regularity are closely related to certain distance-regular graphs, which can be in turn treated as combinatorial interpretations of certain orthogonal polynomials. We will exploit these connections and finally, we show that the determinant of a Töplitz matrix can be written as a product of two determinants of approximately half the size of the original one.

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Symbolic computation and shape changing soliton solutions of certain higher order nonlinear Schrödinger equations

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In magnetic materials, nonlinearity can possibly support intrinsic localized wave modes that occur due to interaction between nearest neighbour atoms. Particularly, the dynamics of one-dimensional Heisenberg ferromagnetic spin chain with higher order physically significant and mathematically complicated magnetic interactions can be mapped to the nonlinear partial differential equations (NLPDE) namely higher order nonlinear Schrödinger (NLS) equations. We invoke the modified extended tangent hyperbolic function method to solve the integro-differential inhomogeneous higher order NLS equations with the aid of symbolic computation for a variety of competing nonlinear inhomogeneities. We construct a series of exact travelling wave solutions with distinct structure successfully for each type of nonlinear inhomogeneity using symbolic computation. The obtained solution is in the form of soliton, solitary and periodic solutions and some of them exhibit shape changing property which gives useful insight into the physical aspect of magnetization reversal.

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On Positivity of Polynomials in Noncommuting Variables

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Many problems in applied mathematics (e.g. control theory, mathematical physics, optimization, etc.) concern various type of inequalities involving polynomials in noncommuting variables. Two most frequently used notions of positivity are given by the Löwner order (i.e., via positive semidefiniteness) or via the trace. In this talk we shall briefly explain the theoretical background and present NCSOS-tools, our Matlab toolbox for

(1) symbolic computation; and

(2) constructing and solving sum of squares programs

for polynomials in noncommuting variables.

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Technology and the Yin&Yang of Teaching and Learning Mathematics

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We develop a model comprising six teaching and learning archetypes and use this model to look at the various roles that technology, in particular computer algebra systems (CAS), can play for each.

Computational D-module theory

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Let R be a commutative ring $K[x_1, \ldots, x_n]$ over a field $K = \mathbb{C}$, and $D = D(R)$ be the n-th Weyl algebra, that is an associative K-algebra, generated by $\{x_1, \ldots, x_n, \partial_1, \ldots, \partial_n\}$ subject to relations $\partial_j x_i = x_i \partial_j + \delta_{ij} \ \ \forall\, 1 \le i, j \le n$. A short overview of the properties of Weyl algebras and a sketch on Gröbner bases theory for them will be given. Indeed, Weyl algebra is the algebra of linear partial differential operators with polynomial coefficients.

How to compute a (possibly smallest) system of PDE's with polynomial coefficients, such that $f \in R$ is a solution of such system? Since R is finitely presented $D(R)$-module with the natural action $x_i \bullet p = x_i \cdot p$, $\partial_i \bullet p = \frac{\partial p}{\partial x_i}$, we get the answer by computing (using Gröbner bases) a left ideal $\mathrm{Ann}_{D(R)} f = \{a \in D(R) \mid a \bullet f = 0\}$.

We can compute the annihilator of f^α for any **concrete** $\alpha \in \mathbb{C}$ as before. D-module theory allows us to compute the annihilator of f^s for **symbolic** s and, moreover, s itself appears in the annihilator $\mathrm{Ann}_{D(R)[s]} f^s \subset D(R)[s] = D(R) \otimes K[s]$ polynomially.

As an application, an algorithm to compute the explicit $D(R)$-module structure of the localization $K[x]_F$ for $F = \{f^i \mid i \ge 0\} \subset R$ will be demonstrated.

J. Bernstein proved in 1972, that for a polynomial $f \in R$ there exist an operator $P(s) \in D(R)[s]$ and a monic polynomial $b(s) \in K[s]$, such that for any s the equality

$$P_f(s) \bullet f^{s+1} = b_f(s) \cdot f^s$$

holds. $b_f(s)$ is called the Bernstein-Sato polynomial of f. The famous theorem of Kashiwara states, that all roots of $b_f(s)$ are rational numbers. Moreover, -1 is always a root. The integer roots of Bernstein-Sato polynomial are of big importance in many applications. For instance, if the hypersurface, defined by f is smooth, one can easily show that $b_f(s) = s + 1$. Otherwise $b_f(s)$ might be very nontrivial and its computation very challenging. We show, how to compute $\mathrm{Ann}_{D(R)[s]} f^s, b_f(s)$ and $P_f(s)$ effectively. In practice, these computations are quite challenging to any computer algebra system. Some important applications of D-modules will be discussed and accompanied by nontrivial live examples, computed with the SINGULAR:PLURAL's package for D-modules. In particular, we sketch the construction of a generalization of a Bernstein-Sato polynomial to the case of an affine variety.

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Gröbner bases for submodules of $\mathbb{Z}^n$

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A binomial ideal I is an ideal of the polynomial ring $K[x_1, \ldots, x_n]$ which is generated by binomials. In [2], following the results of [5], we gave a correspondence between saturated binomial ideals of $K[x_1, \ldots, x_n]$ and submodules of $\mathbb{Z}^n$ and we showed that it is possible to construct a theory of Gröbner bases for submodules of $\mathbb{Z}^n$. As a consequence, we see that it is possible to follow alternative strategies for the effective computation of Gröbner bases of submodules of $\mathbb{Z}^n$ (and hence of binomial ideals) which avoid the use of Buchberger algorithm. In this talk we want to analyze in more details the possible techniques which allows to compute Gröbner bases and in particular we show that the problem is reconducted to the problem of computing minimal elements in suitable subsets of partially ordered lattices.

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Locally $GQ(t,t)$-graphs

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Generalized quadrangle $GQ(s,t)$ is a geometry of points and lines such that every line has $s+1$ points, every point is on $t+1$ lines (with $s>0,\ t>0$) and for any antiflag (P,y) there is the unique line z containing P and intersecting y. A hyperoval of $GQ(s,t)$ is regular subgraph of degree $t+1$ without triangles.

Consider locally $GQ(s,t)$-graphs. Locally $GQ(2,t)$-graphs are classified in [1]. The classification of locally $GQ(3,t)$-graphs is finished in [2].

The case $s>3$ is very difficult. Amply regular locally $GQ(4,2)$-graphs are classified in [3]. In this talk we discuss the problem of description of amply regular locally $GQ(t,t)$-graphs for $t\in\{4,5\}$. Every $GQ(4,4)$ is isomorphic to the ckassical quadrangle $W(4)$. Every known $GQ(5,5)$ is isomorphic to the $W(4)$ or other ckassical quadrangle $Q_4(5)$.

Let Γ be a locally $GQ(s,t)$-graphs. Then for every two vertices u,w at distance 2 a subgraph $\Gamma(u)\cap\Gamma(w)$ is a hyperoval. A classification of hyperovals in known ckassical quadrangle $W(4)$, $W(5)$ and $Q_4(5)$ was obtained by computer calculations in GAP.

Theorem 1. *Amply regular locally $GQ(4,4)$-graph does not exist.*

Theorem 2. *Let Γ be a connected amply regular locally $GQ(5,5)$-graph. Then $\mu\in\{20,26,30,52\}$ and one of the following holds:*
(1) $d(\Gamma)=2$, Γ *has parameters* $(532,156,30,52)$ *and eigenvalues* 4, -26 *of multiplicities* 455, 76*;*
(2) $d(\Gamma)=4$ *and* $\mu=20$*;*
(3) $d(\Gamma)=3$.

Corollary. *Let Γ be a connected amply regular locally $GQ(5,5)$-graph, in which for every vertex a the subgraph $\Gamma(a)$ is $W(5)$ or $Q_4(5)$. Then $d(\Gamma)=2$ and Γ is locally $W(5)$-graph with parameters $(532,156,30,52)$.*

For the vertex set S of the graph Γ we set $\Gamma(S) = \cap_{a\in S}(\Gamma(a) - S)$. A graph Γ is called t-izoregular, if for every $i \leq t$ and for every i-vertex subset S the number $|\Gamma(S)|$ is depend only from isomorphic type of subgraph induced by S. A graph on v vertices is called absolute izoregular, if it is $(v-1)$-izoregular. t-izoregular graph Γ is called exactly t-izoregular, if it is not $(t+1)$-izoregular. Cameron [4] proved that every 5-izoregular graph Γ is absolute izoregular and is isomorphic pentagon, 3×3-grid, complete multipartite graph $K_{n\times n}$ or its complement. Further every exactly 4-izoregular graph is pseudogeometric for $pG_r(2r, 2r^3+3r^2-1)$ or its complement. Let $Izo(r)$ be a pseudogeometric graph for $pG_r(2r, 2r^3+3r^2-1)$. For $r = 1$ we have the point graph of $GQ(2,4)$, and for $r = 2$ we have MacLaughlin graph.

Hypothesis A. *A graph $Izo(r)$ for $r > 2$ does not exist.*

For every vertex a of a graph $Izo(r)$ the subgraph $\Gamma(a)$ is pseudogeometric for $pG_{r-1}(2r-1, r^3+r^2-r-1)$. Makhnev [5] proved that pseudogeometric graph for $pG_{r-1}(2r-1, r^3+r^2-r-1)$ does not exist for $r = 3$, so Hypothesis A is valid for $r = 3$. In this talk we consider $Izo(4)$.

Theorem 3. *The following hold:*
(1) *if Γ is strongly regular graph with parameters $(3159, 1408, 532, 704)$, a is a vertex of Γ and $\Sigma = \Gamma(a)$, then every 6-clique of Σ contains in some 8-clique of Σ;*
(2) *if Σ is strongly regular graph with parameters $(1408, 532, 156, 228)$, in which every 6-every 6-clique contains in some 8-clique, then for every two adjacent vertices b, c of Σ the subgraph $[b] \cap [c]$ is a point graph of $GQ(5,5)$.*

Proposition. *Let Λ be a strongly regular locally $GQ(5,5)$-graph with parameters $(532, 156, 30, 52)$. Then Λ has not vertex a with $\Lambda(a)$ isomorphic to the point graph of $W(5)$ or $Q_4(5)$.*

Corollary. *A strongly regular graph with parameters $(3159, 1408, 532, 704)$ (graph $Izo(4)$) does not exist, if fore some 3-clique $\{a, b, c\}$ of Γ the subgraph $\Gamma(\{a, b, c\})$ is isomorphic to $W(5)$ or $Q_4(5)$.*

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On some image of a straight line mapped via the harmonic cross-ratio

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1. Basic notions on three-dimensional projective space

The study of line congruences has been very popular in the turn of two last centuries, it is still investigated all over the world (see, e.g., [1]-[7]). In this paper we deal with congruences in a real projective space of dimension 3, $\mathbb{P}^3$. The natural identification of a point, b, in this space is the sequence of four real numbers, at least one of them different from 0, and its natural identifier is written as $(b_1 : b_2 : b_3 : b_4)$, the numbers b_1, b_2, b_3, b_4 are called the **homogeneous coordinates** of b. We say that $b = (b_1 : b_2 : b_3 : b_4)$ is a **regular** (or **common**, **usual**) **point**, or simply a **point**, if $b_4 \neq 0$; otherwise we say that b is a **point in the infinity**. The **natural representation** of a point b in the standard Cartesian space $\mathbb{R}^3$, is the point

$$\left(\frac{b_1}{b_4}, \frac{b_2}{b_4}, \frac{b_3}{b_4}\right).$$

The natural representation of a point $(b_1 : b_2 : b_3 : 0)$ in $\mathbb{R}^3$ is the vector $[b_1, b_2, b_3]^T$. This representation makes that a point in the infinity is also called a **direction**.

In $\mathbb{P}^3$ any two point different by a scalar are identical; for any $b, c \in \mathbb{P}^3$ we write $b \equiv c$ iff there exists a nonzero number λ such that $b = \lambda \cdot c$, and λ is called an **inhomogenity multiplier**.

Any curve in $\mathbb{P}^3$ can be described by the set of four equations in one variable. The formula $p(t) = (1 : t : t^2 : t^3)$, where t is the parameter running from ∞ to $+\infty$, defines the spatial curve in $\mathbb{P}^3$. In appropriately chosen coordinate system this formula describes the **cubic curve**. We work within such coordinate system, we denote this curve by the letter K and we refer to K as to the **standard curve**.

A **line congruence** is defined as a two-parameter family of lines in $\mathbb{P}^3$. In this paper we deal with the (1, 3)-congruence in $\mathbb{P}^3$, i.e. the line congruence of order

1 and of class 3; it says that we focus on any line congruence such that 1) there is exactly one line that passes through an arbitrary point of $\mathbb{P}^3$, 2) in any plane $\mathbb{P}^2$ there are exactly 3 lines belonging to the line congruence at hand.

A line cutting the given curve at exactly two points is called a **bisecant** of this curve.

Lets take two distinct values t_1, t_2. They produce two distinct points $T_1 := K(t_1)$, $T_2 := K(t_2)$ sitting on the curve K. It is well-known (see, e.g. [3]) that the straight line T_1T_2, i.e., the line passing through points T_1 and T_2, is the bisecant of K.

2. Cross-ratio of four numbers and four points

Let a, b, c, d be real numbers. The **cross-ratio** of the four (a, b, c, d) is defined to be the number

$$(a, b\,; c, d) := \frac{a-c}{a-d} \cdot \frac{b-d}{b-c},$$

if all numbers are different. Otherwise, the cross-ratio of the four (a, b, c, d) is defined as follows: if $a = c$ or $b = d$, then $(a, b\,; c, d) := 0$; if $a = b$ or $c = d$, then $(a, b; c, d) := 1$; if $a = d$ or $b = c$, then $(a, b\,; c, d) := \infty$.

Given four points can produce at most six different values of their cross-ratios, namely λ, $1/\lambda$, $1-\lambda$, $1/(1-\lambda)$, $(1-\lambda)/\lambda$ and $\lambda/(1-\lambda)$. The cross-ratio $\lambda = (a, b\,; c, d) = -1$ is called **harmonic**. Now the number d is said to be a **harmonic conjugate** of c with respect to the pair (a, b). All cross-ratios different form -1 are called **anharmonic**.

A **cross-ratio** of four collinear points, A, B, C, D, is defined by the same formula as that of four numbers, but now a, b, c and d stand for the number identifying these points in the local coordinate system; usually, $a - b$ is the signed distance between points A and B. As in the number case, it is denoted by $(A, B\,; C, D)$.

3. Coordinates of the point M

Let's take a point on K, namely the point $A = K(a) = (1 : a : a^2 : a^3)$, and a point $B = (b_1 : b_2 : b_3 : b_4) \notin K$ such that the line AB is not a bisecant of K. Next, let's take an arbitrary point $M = M(t) = (m_1 : m_2 : m_3 : m_4)$ on AB. We discuss the line congruence of order 1, so there exists [5] exactly one bisecant of K passing by M. Lets denote the points, at which this bisecant crosses K, by $T_1 := K(t_1)$, $T_2 := K(t_2)$. There exist reals α, β such that $M \equiv \alpha \cdot T_1 + \beta \cdot T_2$ and $\alpha^2 + \beta^2 > 0$. Therefore $\varrho \cdot M = \alpha \cdot T_1 + \beta \cdot T_2$, where ϱ is the inhomogenity multiplier. It can be taken $\varrho = 1$ and from the system of equations we obtain $t_1 = (s + \sqrt{\Delta_1})/2$, $t_2 = (s - \sqrt{\Delta_1})/2$, where the sum $s := t_1 + t_2$ and the product $p := t_1 \cdot t_2$ express by formulas

$$s = \frac{m_1 \cdot m_4 - m_2 \cdot m_3}{m_1 \cdot m_3 - m_2^2},\ p = \frac{m_2 \cdot m_4 - m_3^2}{m_1 \cdot m_3 - m_2^2}$$

and $\Delta_1 := s^2 - 4p$.

4. Finding the fourth point completing given three points to a given cross-ratio

The point M we obtained above is the point where the bisecant T_1T_2 and the straight line $AB \neq T_1T_2$ meet. Now we look for a point X which lays on T_1T_2 and completes the triple (T_1, T_2, M) in such a way that the four points, T_1, T_2, M, X, have their cross-ratio equal to a given real $\lambda = -1$. Since there is no forced the order of this four, so, in general, there can be two such points. For X such that $(T_1, T_2; M, X) = \lambda$, as well as in the case $(T_1, T_2; X, M) = 1/\lambda$, we have $\varrho \cdot X = -2 \cdot \psi$, where the vector $\psi = \psi(M)$ depends on the linear combination of the products of the form $m_i \cdot m_j \cdot m_k$, where $i, j, k \in \{1, 2, 3, 4\}$.

Taking into account the parametric representation of the line AB, $M = A \cdot u + B$, we state that the components of the vector $\psi = \psi(A, B)$ are polynomials of second degree in the variable u.

The curve governed by the equation $\varrho \cdot X = -2 \cdot \psi$ is called **(a, b)-curve** (adjoint to K) and its natural representation in $\mathbb{R}^3$ is

$$(x, y, z) = \left(\frac{r_1(u)}{r_4(u)}, \frac{r_2(u)}{r_4(u)}, \frac{r_3(u)}{r_4(u)} \right),$$

where $r_j(u)$ are the polynomials mentioned above.

5. Image of a straight line in $\mathbb{P}^3$

The formula $\varrho \cdot X = -2 \cdot \psi$ reveals that the set of all points X produced when M runs the line AB is an algebraic curve in $\mathbb{P}^3$ of the second degree. In consequence, in $\mathbb{R}^3$ it is a rational curve: each parametric relation in its natural representation is a quotient of polynomials of the degree at most 2.

Using Dervive 5, the computer algebra system from Texas Instruments, Inc.(USA), we find, by the analysis of the projections of concrete curve on basic planes Oxy, Oxz and Oyz in $\mathbb{R}^3$, that all X's form a conic: if $r_4(u) > 0$ for all u, then we have an ellipse; if $r_4(u)$ has one zero, then we have a parabola; if $r_4(u) = 0$ for two distinct values of u, then we have a hyperbola.

6. Examples of the visualization

Using Dervive 5 we can visualizate considered curves in all three possible situations: elliptic, parabolic and hyperbolic ones. Moreover, we obtain the exact formulas for the projections at hand. For instance, with $a = 1$ and $b = (1 : 2 : -3 : 1)$ we obtain $r_1(u) = -2 \cdot (15u^2 + 48u + 35)$, $r_2(u) = -2 \cdot (15u^2 - 9u - 28)$, $r_3(u) = -2 \cdot (15u^2 + 24u - 7)$, $r_4(u) = 2 \cdot (9u^2 + 30u + 29) > 0$, so the (a, b)-curve is the ellipse. The standard equation of its projection on the plane $z = 0$ is $9140x^2 + 1508x \cdot y + 458y^2 + 14277x + 4674y + 735 = 0$.

7. Conclusions and final remarks

We derived general formulas for the point M laying in the cross of two distinct lines AB and T_1T_2 in the projective space $\mathbb{P}^3$. For a straight line AB passes through the point A laying on the cubic line K, and the point $B \notin K$, the line T_1T_2 is the bisecant of K which passes through M. For every M there is exactly one such bisecant and there is a point X on this bisecant such that the $(T_1, T_2; M, X) = -1$.

The analytical transformations and the visualization were effectuated in Derive 5, without any use of a computer algebra system these tasks are extremely time-consuming, the calculations in the approximate mode (as well as in any NOS, i.e., numerically oriented systems) fail (and it can be easily check, e.g., if one would like to derive a formula covering the ellipse described in the example above). Authors intend to continue their research to find, e.g., some general issues on the image at hand with an arbitrary value of the cross-ratio λ.

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Computing unramified Brauer groups of finite groups

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The Bogomolov multiplier is a group theoretical invariant isomorphic to the unramified Brauer group of a given quotient space, and represents an obstruction to the problem of stable rationality of fixed fields. We derive a homological version of the Bogomolov multiplier, prove a Hopf-type formula, find a five term exact sequence corresponding to this invariant, and describe the role of the Bogomolov multiplier in the theory of central extensions. An algorithm for computing the Bogomolov multiplier is developed. This enables computations of the unramified Brauer groups of groups of large order. A new description of the Bogomolov multiplier of a group of class two is obtained. We define the Bogomolov multiplier within K-theory and show that proving its triviality is equivalent to solving a long-standing problem posed by Bass.

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Algebraic Analysis of Stability, Bifurcation and Limit Cycles for Biological Models

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Many biological phenomena may be modeled mathematically by continuous or discrete dynamical systems. In order to understand the phenomenon described by a complex dynamical system, it is necessary to study its behaviors such as stability, bifurcations, and limit cycles qualitatively. For nonlinear dynamical systems, it is a crucial and challenging task to analyze their qualitative behaviors, and in the literature of experimental biology, this analysis is often performed by purely numerical simulation. The rigorous analysis of dynamical systems with exact symbolic and algebraic computation is an important problem.

In this talk we will show how to use algebraic methods based on triangular decomposition, Gröbner bases, discriminate varieties, quantifier elimination and real solution classification to detect the real steady states and analyze the stability of each steady state, as well as analyze bifurcations and limit cycles for dynamical systems. Some experimental results for several biological models by using our approach will be presented.

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Open-source symbolic computation in group theory

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During the last years several active and very promising open-source mathematical software projects were started. One of them - Sagemath - is a free project with many international contributions (`http://www.sagemath.org/`). This software is specially actively developing and has acquired a property that can be described as critical moment - it's increasing attracting attention and upgrading from many who might instead work on independent projects of lesser scope. The basic goal of Sagemath is to provide for mathematics what Linux has provided for operating systems - a medium for free, creative expression, not suppressed by extreme proprietary interests and restrictions. That is why Sagemath most happily runs on Linux, though it can be easily used with Windows as well.

Sagemath includes many special packages, dedicated to different mathematical areas: calculus, linear algebra, number theory, theory of groups, theory of rings and fields, etc. A good portion of Sagemath support for group theory is based on routines from well known GAP package (Groups, Algorithms and Programming) - see `http://www.gap-system.org/`. Therefore, groups can be in Sagemath described in many different ways, such as: sets of permutations, sets of matrices or just sets of general symbols, subject to some defining relations.

For permutations Sagemath uses the unique "disjoint cycle notation" and the product (composition) operation works "from left to right". It recognizes many popular classical groups as sets of permutations:

Notation	Description
`SymmetricGroup(n)`	full symmetric group of order n
`AlternatingGroup(n)`	the alternating group of order n
`DihedralGroup(n)`	symmetries of a regular n-gon
`direct_product_permgroups([G_1,G_2])`	a direct product of groups
`PermutationGroup(["(1,2,3)(4,6)(5,7)", "(1,2)(4,5,6,7)"])`	a semi-direct product of $Z_3 \rtimes Z_4$, and nonisomorphic to A_4, D_6

For each permutation group G with a command of the form G.is_... () we can find out whether it is abelian, cyclic, nilpotent, solvable, simple or transitive. There are also simple commands to check whether it is a normal subgroup and to find its classes of conjugated elements or subgroups. We can also search for Sylow subgroups or normal subgroups of a given permutation group, find its lower and upper derived series. Sagemath creates a Cayley table of a group and draws its Caley graph. There are also several built in functions dealing with the action of a group on a final set and giving us the characters of the irreducible representations of a permutation group.

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An introduction to the theory of symmetries of differential equations

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The mathematical research field known as Lie group analysis was created in the second half of the 19th century by Sophus Lie (1842-1899). Lie was influenced and inspired by the work of Abel in what is known today as Galois theory. One of Lie's greatest achievements was the creation of a unified theory of integration for both ordinary and partial differential equations. He also proved (among other things) that if a differential equation is invariant with respect to the action of a continuous group of point transformations (he also considered contact transformations) the order of the differential equation can be reduced by one. While the general theory of Lie groups is quite well and widely known, Lie's original ideas, in the context of differential equations, are now known only to specialists.

The aim of this talk is to introduce (from a modern point of view) some group theoretic methods of integration of differential equations due to S. Lie. I will discuss some basic concepts of this theory; namely: continuous groups of transformations and their infinitesimal generators, the concept of a symmetry of a differential equation, simple methods of finding those symmetries (using Lie's algorithm) and methods of using them; namely: how to find specific solutions (invariant under the action of the symmetry group) called similarity solutions, general solutions, construct new solutions from old solutions, reduce the order of the equation, integrate equations produced by a variational principle (systems that play an important role in physics and geometry).

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Symbolic Combinatorics in Physics, Special Functions, and Number Theory

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In this talk I present recent developments achieved in my working group at RISC. Primary focus is put on applications of symbolic computation methods. A major part of the talk is devoted to the application of holonomic tools to problems related to Coulomb integrals in quantum physics (joint work with S. Suslov). If time remains I will report on new computer algebra methods in connection with special function inequalities (joint work with V. Pillwein) and modular forms (joint work with S. Radu). Papers related to my talk can be found at `http://www.risc.jku.at/research/combinat/publications`.

On subholonomic sequences in combinatorial enumeration

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Substantial progress has been made in the past in design of algorithms for the class of holonomic sequences, which has very nice closure properties (cf. [1 – 5]). Although large, this class still fails to include several important and relatively simple sequences, encountered in combinatorial enumeration, such as ordinary powers, Stirling numbers of the first and second kind, and Eulerian numbers. So it seems necessary to turn attention to "subholonomic sequences" which retain some, but not all of the nice properties of holonomic ones. The hope is that this may help us in tackling certain easy-to-state but difficult-to-solve combinatorial problems, such as enumeration of lattice paths in restricted regions, and enumeration of restricted permutations.

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Enumeration and generation of trivalent graphs in several variations

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It turns out that there exist numerous useful variants of trivalent graphs. Some are needed in connection with maps, hypermaps, configurations, polytopes, benzenoid systems, or covering graphs. In this talk we briefly explore these connections and give motivation why some decorated trivalent graphs should be enumerated and generated.

Symbolic computation of linear relations of parameter integrals

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The problem of finding antiderivatives of given functions is very old. For rational integrands algorithms are known already for a long time. In 1969 Robert Risch published an algorithm that finds an elementary integral of an elementary function provided such an indefinite integral exists. In this talk an extended version of Risch's algorithm for Liouvillian integrands is presented. It will also be discussed how this can be used for obtaining linear differential, recurrence, or mixed relations for definite integrals involving parameters, e.g. special functions given by an integral. Illustrating examples will be given.

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Time-reversibility, invariants and periodic solutions in polynomial systems of ODE

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Consider a dynamical system described by ODE's of the form

$$\frac{d\mathbf{z}}{dt} = F(\mathbf{z}) \quad (\mathbf{z} \in \Omega), \tag{1}$$

where $F : \Omega \mapsto T\Omega$ is a vector field and Ω is a manifold. A time-reversible symmetry of (1) is an invertible map $R : \Omega \mapsto \Omega$, such that

$$\frac{d(R\mathbf{z})}{dt} = -F(R\mathbf{z}). \tag{2}$$

We investigate the case of two-dimensional polynomial systems, that is, $\mathbf{z} = (x, y)$ is from $\in \mathbb{C}^2$ or $\mathbb{R}^2$, and F is a polynomial vector-function. We also assume that R is a linear transformation.

If the coefficients of F are parameters, then the action of the group $SL(2, \mathbb{C})$ (or $SL(2, \mathbb{R})$) on $\mathbf{z}$ induces a transformation of the coefficients of (1). These transformations also form a group which we denote by U. We call polynomial invariants of U the Sibirsky invariants of system (1). In the talk we describe an efficient algorithm to compute a generating set of these invariants. Using methods of computational algebra we show an interconnection of the physically important phenomena of time-reversibility, involution and the Sibirsky invariants in dynamical systems (1). Furthermore, we characterize the set of all time-reversible systems within a particular family of complex two-dimensional polynomial differential systems and give an efficient computational algorithm for finding this set.

We then discuss application of the invariants to studying periodic oscillations and limit cycles bifurcations in polynomial systems of ODEs. The computations are performed using the computer algebra system SINGULAR.

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Algebraic Structures in Logical Networks

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The algebraic treatment of logical networks via Boolean Algebra is well known. In this scenario, the value of entries or switches is just zero or one, thus a binary information processing. One may however also consider networks in which switches or gates are triggered with a certain probability and ask for the signal transferring function in terms of these different probabilities. A key to the generalization is provided through the third Kolmogorow axiom and leads to a computation scheme for a given network. The signal transferring function is calculated as a more or less complex polynomial in the different probabilities which occur. Imposing certain boundary conditions for the entrances, one may calculate the probability of how "hot" or how "cold" a specific wire in the network is, of what is the "hottest" or most active part of the network and so on. The question arises: Can one find clever algorithms for computing the wire probabilities and the signal transferring term in more complex networks with huge numbers of gates?

Decoding of multipoint codes from algebraic curves

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Multipoint codes are a broad class of algebraic geometry codes derived from algebraic functions which have multiple poles on defining curves. Thus, they are more general than one-point codes which are an important class of algebraic geometry codes in the sense that they can be decoded efficiently by using the BMS algorithm [4]. Furthermore, some multipoint codes have better performance than comparable one-point codes from the same curves. In this paper we present a fast decoding method of multipoint codes from algebraic curves. Since any algebraic geometry codes from algebraic curves are essentially the same as multipoint codes, this means that almost all algebraic geometry codes can be decoded efficiently. The method is based on the observation that any multipoint codes are subcodes of one-point codes [2], and that such codes can be decoded by a variation of the original BMS algorithm called *vectorial BMS algorithm* [3].

A one-point code from an algebraic curve is defined by a symbol locator set $\mathcal{P} := \{P_i \mid 1 \leq i \leq n\}$ and a function space $\mathcal{L}(mP_\infty)$, where $\mathcal{P}$ is a set of $\mathbf{F}_q$-rational points on an irreducible and non-singular algebraic curve $\mathcal{X}$ (the infinity point $P_\infty \notin \mathcal{P}$), and $n (= \#\mathcal{P})$ is the code length. Further, $\mathcal{L}(mP_\infty)$ is the linear space of algebraic functions f on the defining curve $\mathcal{X}$ having the infinity point P_∞ as a single pole with pole order $o(f)$ ($:= -v_{P_\infty}(f)$; $v_{P_\infty}(f)$ is the valuation of function f at P_∞.) less than or equal to a given positive integer m ($< n$). Then, we have two kinds of codes, called *primal code* (or $\mathcal{L}$-code) and *dual code* (or Ω-code).

$$C(mP_\infty) := \{\underline{c} = \mathrm{eval}(f) \mid f \in \mathcal{L}(mP_\infty)\};$$
$$C^\perp(mP_\infty) := \{\underline{c} \in \mathbf{F}_q^n \mid \underline{c} \cdot \mathrm{eval}(f)(:= \textstyle\sum_{1\leq i\leq n} c_i f(P_i)) = 0,\ f \in \mathcal{L}(mP_\infty)\},$$

where $\mathrm{eval}(f) := (f(P_i))_{1\leq i\leq n} (\in \mathbf{F}_q^n)$. While a one-point code is defined from a divisor mP_∞ with a single pole P_∞ as its support, a (general) algebraic geometry code $C(G)$ from a curve is defined from any divisor $G := \sum_{1\leq i\leq a} m_i Q_i - \sum_{1\leq j\leq b} n_j R_j$, where $m_i > 0$, $1 \leq i \leq a$; $n_j > 0$, $1 \leq j \leq b$, and $\{Q_i \mid 1 \leq i \leq a\} \cap \{R_j \mid 1 \leq j \leq b\} = \emptyset$. Since there exists an algebraic function h with divisor $(h) = \sum_{1\leq i\leq a} m'_i Q_i - m' P_\infty$, where $m'_i \geq m_i$, $1 \leq i \leq a$ and $m' = \sum_{1\leq i\leq a} m'_i$, multiplication by h induces an isomorphism

$$\begin{array}{rcl} \mathcal{L}(G) & \to & \mathcal{L}(m'P_\infty - G') \\ f & \mapsto & hf \end{array}$$

where $G' := \sum_{1\leq j\leq a}(m'_i - m_i)Q_i + \sum_{1\leq j\leq b} n_j R_j$ is a positive divisor. Since $\mathcal{L}(m'P_\infty - G')$ is a linear subspace of $\mathcal{L}(m'P_\infty)$, the primal code defined from $\mathcal{L}(m'P_\infty - G')$ is a subcode of the one-point code $C(m'P_\infty)$. Therefore, we have only to consider codes $C(G)$ defined from a divisor of the form $G := mP_\infty - \sum_{1\leq j\leq b} n_j Q_j$, where $n_j > 0$, $1 \leq i \leq b$. In this paper, we consider such codes defined from a plane curve $\mathcal{X}$, particularly from a Hermitian curve: $y^{q_1} - x^{q_1+1} + y = 0$ over the finite field $\mathbf{F}_q$, where $q = q_1^2$. On discussing decoding such codes $C(G)$, the following observation is important:
Lemma 1: $\mathcal{L}(\cup_{i\geq 0} iP_\infty)$ is just the ring $R := \mathbf{F}_q[\underline{x}] := \mathbf{F}_q[x,y]$ of bivariate polynomials, where $\underline{x} = (x_1, x_2) = (x, y)$, and $\mathcal{L}(\cup_{i\geq 0} iP_\infty - \sum_{1\leq i\leq b} n_i Q_i)$ is an ideal I of the ring R, which is composed of bivariate polynomials $f = f(\underline{x}) = f(x_1, x_2) = f(x, y)$ having Q_j as zero with order $\geq n_j$, $1 \leq j \leq b$.
Beelen and Hoeholdt [1] have presented a basic decoding algorithm of (general) primal algebraic geometry codes $C(G)$ defined from general divisor G and shown
Lemma 2 (Proposition 2.10 of [1]): Let $\underline{c} = \mathrm{eval}(f) \in C(G)$ be a codeword and $\underline{e} \in \mathbf{F}_q^n$ be an error vector of weight $t < (n - \mathrm{deg}G - g)/2$, where g is the genus of the defining curve $\mathcal{X}$. For the received word $\underline{r} = \underline{c} + \underline{e}$, there exists a polynomial $Q(z) = Q_0 + Q_1 z$ ($\in R[z]$) satisfying the *interpolation* condition

$$Q_0(P_j) + r_j Q_1(P_j) = 0,\ 1 \leq j \leq n \tag{1}$$

with $Q_0 \in \mathcal{L}(A)$ and $Q_1 \in \mathcal{L}(A - G)$ for a divisor A with $\mathrm{Supp}(A) \cap \mathcal{P} = \emptyset$ such that (1) $\mathrm{deg}A < n - t$, (2) $l(A - G) > t$, where $l(A - G)$ is the dimension of the subspace $\mathcal{L}(A - G)$, and it holds that $f = -Q_0/Q_1$. Further, the polynomial Q_1 has the error locators $\mathcal{E} \subset \mathcal{P}$ as its zeros, i.e. $Q_1(P_j) = 0$, $P_j \in \mathcal{E}$.

Thus, this basic decoding algorithm can correct up to $\lfloor (n - \mathrm{deg}G - g - 1)/2 \rfloor$ errors. We present a fast method of finding a reduced Groebner basis of the error locator ideal $I(\mathcal{E}) := \{f \in I \mid f(P_j) = 0, P_j \in \mathcal{E}\}$ or rather a reduced Groebner basis of the R-module $M(f, \mathcal{E}) := \{(-fh, h) \in I \times R \mid h \in I(\mathcal{E})\}$. Consequently, we can find the exact error locators as well as the encoding function f efficiently .

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Symbolic Summation and the Evaluation of 3–loop Feynman Integrals

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In this talk we demonstrate how the summation package Sigma can be used to simplify multi–sums with the symbolic summation paradigms of telescoping, creative telescoping and recurrence solving. The underlying algorithms are based on our refined difference field theory of Karr's $\Pi\Sigma$-fields. Special emphasis is put on brand new evaluations of 2– and 3–loop massive single scale Feynman diagrams with operator insertion arising in the cooperation with Johannes Blümlein (DESY–Zeuthen; Deutsches Elektronen Synchrotron).

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Cyclicity of Systems of Polynomial Differential Equations with Nonradical Bautin Ideals

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The cyclicity of an elementary focus or center of a system (*) $\dot{x} = \sum_{j+k\leq n} a_{jk}x^j y^k$, $\dot{y} = \sum_{j+k\leq n} b_{jk}x^j y^k$ on the plane is the maximum number of limit cycles that can be made to bifurcate from the singularity under small perturbation of the coefficients of the right-hand sides. We complexify the system and let $\mathcal{B}$ denote the *Bautin ideal*, the ideal generated by the focus quantities of family (*), polynomials in the coefficients whose vanishing characterizes a center at the origin. Suppose $\mathcal{B}_K$ is the ideal generated by the first K focus quantities and generates the same variety as $\mathcal{B}$. If the ideal $\mathcal{B}_K$ is radical then the cyclicity is bounded above by K. We show how exploiting the structure of the focus quantities sometimes permits moving the ideals to a new ring in which a non-radical $\mathcal{B}_K$ can become radical. We use an example to illustrate how theorems of computational commutative algebra make all the relevant computations feasible.

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Universal Gröbner bases under monomial transformations

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We consider the problem about a correspondence between elements of universal Gröbner bases of two ideals connected by a polynomial transformation. Hoon Hong in the paper [1] proved that Gröbner basis computation commutes with polynomial composition if and only if the composition is compatible with the term ordering and nondivisibility. Nevertheless we show that partial correspondence between the elements of universal Gröbner bases of two ideals connected by a polynomial transformation can be established. Our approach is based on the conception of extended universal Gröbner basis of polynomial ideal introduced in [2]. It can be defined in terms of dimensions of crossections of ideal with linear spaces generated by multidimensional Young diagrams of the monomial lattice. Such extended universal Gröbner basis EUGB contains the standard universal Gröbner basis by Mora-Robbiano and is finite also. We study how the elements of EUGB transform under arbitrary polynomial transformations. In the case of monomial transformation our approach allow us to describe some elements of universal Gröbner bases of ideals which contains nontrivial toric ones.

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How to lose as little as possible

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Suppose Alice has a coin with heads probability q and Bob has one with heads probability $p > q$. Now each of them will toss their coin n times, and Alice will win iff she gets more heads than Bob does. Evidently the game favors Bob, but for the given p, q, what is the choice of n that maximizes Alice's chances of winning? We show that there is an essentially unique value $N(q, p)$ of n that maximizes the probability $f(n)$ that the weak coin will win, and it satisfies $\left\lfloor \frac{1}{2(p-q)} - \frac{1}{2} \right\rfloor \leq N(q, p) \leq \left\lceil \frac{\max(1-p, q)}{p-q} \right\rceil$. The analysis uses the multivariate form of Zeilberger's algorithm to find an indicator function $J_n(q, p)$ such that $J > 0$ iff $n < N(q, p)$ followed by a close study of this function, which is a linear combination of two Legendre polynomials. An integration-based algorithm is given for computing $N(q, p)$. This is joint work with Vittorio Addona and Stan Wagon, of Macalester College.

Computing Envelope and Radical of a Submodule of a Noetherian Module

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Two different definitions of radical of an ideal in ring theory correspond two different notions which are not always coincide in modules theory. Let R be a commutative ring with unity and let M be an R-module. The envelope of a submodule N of M is the module generated by the set

$$E_M(N) = \{rm : r \in R, m \in M \text{ and } r^k m \in N \text{ for some } k \in \mathbb{Z}^+\}.$$

A submodule P of M is called prime (primary) submodule if $rm \in P$ implies either $m \in P$ or $r \in P : M(r \in \sqrt{P : M})$. The radical of a submodule N of M, denoted by $\mathrm{rad}_M(N)$, is the intersection of all prime submodules containing N. It is well-known that $\langle E_M(N) \rangle \subseteq \mathrm{rad}_M(N)$. If equality hold it is said that submodule N satisfies the radical formula, or briefly strf. If every submodule of M are strf, then M is called strf. Although there many study on the subject which module are strf, there is a little effort to actually compute the envelope and the radical of a submodule.

In this work, we assume R is Noetherian and M is a finitely generated R-module. In this case every submodule of M has a primary decomposition. Our goal is to compute the envelope and the radical of a submodule N of M using this primary decomposition.

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Links between two semisymmetric graphs on 112 vertices via association schemes

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An undirected graph $\Gamma = (V, E)$ is called *semisymmetric* if it is regular (of valency k) and $Aut(\Gamma)$ acts transitively on E and intransitively on V.

We consider the unique semisymmetric graph $\mathcal{L}$ of valency 3 on 112 vertices, which is commonly called the Ljubljana graph (see, for example, [3]). Another semisymmetric graph $\mathcal{N}$ on 112 vertices of valency 15 was discovered in 1977 and described in [2]. It turns out that $\mathcal{L}$ is a spanning subgraph of $\mathcal{N}$ and moreover, $Aut(\mathcal{L})$ is a subgroup of $Aut(\mathcal{N})$.

We present an outline of a new approach to the graph $\mathcal{L}$. In particular, all embeddings of $\mathcal{L}$ into a fixed copy of $\mathcal{N}$ are considered, as well as related diverse combinatorial and geometric structures. The use of standard double covers (as in [1]) serves as a bridge between relevant concepts from topological and algebraic graph theory.

Let $\Gamma = (V, R)$ be a directed graph. Consider the *standard double cover* $\hat{\Gamma} = (\hat{V}, \hat{R})$, $\hat{V} = V \times \{1, 2\}$, $\hat{R} = \{\{(x, 1), (y, 2)\} | (x, y) \in R\}$.

Let $\Omega = V_1 = \{(x, y) | x, y \in F_8, x \neq y\}$ and let (G, Ω) be the induced transitive action of $G = A\Gamma L(1, 8)$ on Ω of degree 56. It is easy to see that (G, Ω) has rank $\frac{56-2}{3} + 2 = 20$. Thus we construct our master association scheme $\mathfrak{m} = (\Omega, 2-orb(G, \Omega))$.

Then $\mathcal{L}$ is a double cover of a 8 of the classes of Ω. Those 8 copies of $\mathcal{L}$ furthermore can be partitioned into four pairs, where the union of each pair of graphs has a unique extension to $\mathcal{N}$.

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Concert by the duo FLA-VIA

1 July 2010, 19:45

Art Kavarna of the hotel PIRAMIDA

1) J.Ch. Bach: Sonata no. 2:

- Allegro
- Menuet
- Finale

2) G. Ph. Telleman: Sonata in E-dur

- Andante con affeto
- Vivace
- Amoroso
- Presto

3) A. Stamitz: Duo in D-dur

- Allegro moderato
- Menuetto

4) G. Briccialdi: Allegro

5) G. Rossini: Duets from "The Barber of Seville"

- Cavatina (Figaro)
- Duetto (Figaro, Almaviva)
- Finale

DUO FLA-VIA

Duo Fla-Via are **Špela Kržan (flute)** and **Barbara Danko (violin)**, two young and talented musician, who despite their youth already have a varied and rich musical past. Both of them gathered their musical knowledge abroad (Vienna, Austin, Saint Petersburg, Paris) and are professionally active in Slovenia, as well as abroad. They formed the Duo Fla-Via in December 2009. Since then Duo Fla-Via has had some very successful concerts. Among the most popular were concerts at Ptuj and in Litija.. They repertoire is broad and consists of chamber works for flute and violin and soloistic pieces for both instruments. One of their strengths as musicians is also the ability to present classical music in an unclassical way and they also play different styles (Folk music, Latin music, Tango, Rock music...). Because of that Duo Fla-Via is an interesting, fresh and versatile chamber ensemble.

Mag. art. Špela Kržan (flute) completed her postgraduate studies of flute with honours at the University of Music and Dramatic Arts in Vienna, where she studied with professor Hansgeorg Schmeiser-ROM. She studied at the "Conservatoire National Superieur de Musique et de Danse de Paris" with professors Vincent Lucas, Sophie Cherrier, and Philippe Bernoldi as a part of international exchange of students - Erazmus. In September 2009 she won the international competition for the flute in Israel - Haifa International Flute Competition 2009. She also successfully participated in other international competitions, including the ARD Competition in Munich, Jeunesse Musicales in Romania, Böhm Competition in Munich, Domenico Cimarosa, Italy She attended several masterclasses with renowned professors like Luisa Sello, Aurelle Nicolet, Davide Formisano, Jan Ostry, Karl Heinz Schũtz, Natalie Rozat, AleŠ Kacjan, Gaspar Hoyos and others.

Špela Kržan continues her solo career as a concert flutist and a member of chamber ensembles with harp, organ and violin at home and abroad. She was a long-standing member of Youth Symphony Orchestra, Wiener Jeunesse Orchester, with which she attended an orchestral tour in Bombay (India) in February 2008. Occasionally she participates in the Vienna People's Opera, Vienna Volksoper. In August 2009 she held a masterclass for flute in Radlje ob Dravi. She is currently employed as a professor of flute at SGBŠ Maribor, Lenart, at a private music school in Gornja Radgona Maestro and Music School in Radlje ob Dravi. She is a co-founder of Duo Fla.-Via.

Barbara Danko, M. M. (violin) started her musical path at the age of six with the professor Zvonka Pal at the Elementary school of music in Maribor. Her talent was soon revealed and she won a second prize at the International violin competition Alpe-Adria at the age of eight.. She continued her musical training at the High school of Music and Ballet in Maribor with Ivan Pal.. At the age of sixteen she was admitted to the Academy of Music in Ljubljana as highly talented to study with professor Rok Klopčič. In 2006 she was accepted to a postgraduate program at The University of Texas at Austin, Austin, Texas, for which she also received a University grant. There she studied with Dr. Eugene Gratovich. She completed her studies with honors in May 2008. Barbara competed in several national and international competitions and achieved high rankings

She attended music festivals in Europe (Saint Petersburg, Russia; Burgos, Spain; Viana do Castelo, Portugal) and the USA (Austin, Texas), where she also had solo recitals. In 2008 she was honored by the University of Texas at Austin for an outstanding master's recital. She was one of the first members of Austin pops, the leading Austin orchestra for popular music and regularly worked with Temple Symphony Orchestra and Brazos Valley Symphony Orchestra.

During her time at the UT at Austin she was also a member of String Project, an organization for educating children in string instruments. She was also Dr. Eugene Gratovich's teaching assistant. She is currently employed as violin teacher at the Private music school in the monastery of Saint Peter and Paul in Ptuj and the Private music school Maestro in Gornja Radgona. She is also active as a solo and chamber musician and is a member of a rock group Avven. In the summer of 2009 she organized the festival Glasba v Kloštru, which had a great response. She is a co-founder of Duo Fla-Via and the project SLO-STRINGS.