Abstracts

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On the number of zeros of Abelian Integrals

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In this talk, we will introduce some new methods to estimate the lowest upper bound of the number of isolated zeros of Abelian integrals, which is called the weakened 16th Hilbert problem proposed by V. I. Arnold. Some algebraic criteria are obtained for the number of isolated zeros of Abelian integrals along energy level ovals of potential systems. As applications of our main results, we study three kinds of Abelian integrals along algebraic or non-algebraic level ovals, obtain the algebraic criteria on the Abelian integrals having Chebyshev property with accuracy one, simplify some known proof on the cyclicity of quadratic reversible centers, and give all the configurations of limit cycles from Poincaré bifurcation of two quadratic reversible systems with two centers, one of which has a nonalgebraic first integral with logarithmic function. This talk is based on the joint works with Changjian Liu.

Multiple expansions of real numbers with digits set $\{0, 1, q\}$

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For $q > 1$ we consider expansions in base q over the alphabet $\{0, 1, q\}$. Let $\mathcal{U}_q$ be the set of x which have a unique q-expansions. For $k = 2, 3, \cdots, \aleph_0$ let $\mathcal{B}_k$ be the set of bases q for which there exists x having precisely k different q-expansions, and for $q \in \mathcal{B}_k$ let $\mathcal{U}_q^{(k)}$ be the set of all such x's which have exactly k different q-expansions. In this paper we show that

$$\mathcal{B}_{\aleph_0} = [2, \infty) \quad \text{and} \quad \mathcal{B}_k = (q_c, \infty) \quad \text{for any} \quad k \geq 2,$$

where $q_c \approx 2.32472$ is the appropriate root of $x^3 - 3x^2 + 2x - 1 = 0$. Moreover, we show that for any positive integer $k \geq 2$ and any $q \in \mathcal{B}_k$ the Hausdorff dimensions of $\mathcal{U}_q^{(k)}$ and $\mathcal{U}_q$ are the same, i.e.,

$$\dim_H \mathcal{U}_q^{(k)} = \dim_H \mathcal{U}_q \quad \text{for any} \quad k \geq 2.$$

Finally, we conclude that the set of x having a continuum of q-expansions has full Hausdorff dimension.

This is a joint work with Karma Dajani, Kan Jiang and Derong Kong.

Limit cycles of piecewise linear systems

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In this talk I will introduce some results of limit cycles of piecewise linear systems, and explore the factors that limit the increase of limit cycle.

Problem of limit cycles for the Higgins-Selkov system and the Selkov system

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In this talk we research the problem of limit cycles for the Higgins-Selkov system and the Selkov system. We change the two systems into Liénard systems first. Then, we present two theorems on the nonexistence of limit cycles of Liénard systems. At last, Artés-Llibre-Valls's conjectures can be proven by these theorems and some techniques applied for Liénard systems.

Linearizability problem of planar cubic persistent centers

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The concepts of persistent and weakly persistent centers were introduced recently. Such concept was then generalized for complex planar differential systems. In the talk we extend the notion of persistent center to a linearizable persistent center and a linearizable weakly persistent center. Using the methods and algorithms of computational algebra we characterize the planar cubic differential system having linearizable persistent and linearizable weakly persistent centers at the origin.

Integrability of complex planar systems with homogeneous nonlinearities

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The problem of integrability of systems of differential equations is one of central problems in the theory of ODE's. Although integrability is a rare phenomena and a generic system is not integrable, integrable systems are important in studying various mathematical models, since often perturbations of integrable systems exhibit rich picture of bifurcations.
In this talk we discuss conditions for the existence of a local analytic first integral for a family of quintic systems having homogeneous nonlinearities studied in [1], i.e.

$$\begin{aligned} \dot{x} &= x - a_{40}x^5 - a_{31}x^4y - a_{22}x^3y^2 - a_{13}x^2y^3 - a_{04}xy^4, \\ \dot{y} &= -y + b_{5,-1}x^5 + b_{40}x^4y + b_{31}x^3y^2 + b_{22}x^2y^3 + b_{13}xy^4 + b_{04}y^5, \end{aligned} \tag{1}$$

where x, y, a_{jk}, b_{kj} are complex variables.

One of important mechanisms for integrability is the so-called time-reversibility (or just reversibility). We will describe an approach to find reversible systems within polynomial families of Lotka-Volterra systems with homogeneous nonlinearities.

References

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On persistent $p : -q$ resonant center problem

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The notion of $p : -q$ resonant center was introduced recently and studied by several authors. We generalize the notion of a persistent center to a persistent $p : -q$ resonant center and find conditions for existence of a persistent $p : -q$ resonant center for several $p : -q$ resonant systems with quadratic nonlinearities. To prove the sufficiency of the obtained conditions we use either the Darboux theory of integrability or look for a formal first integral of the required form or we use the method based on the blow-up transformation.

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Symmetries of quadratic systems of ODE's

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One of important problems arising in the investigation of the qualitative behavior of dynamical systems is determining whether a given system admits some kind of symmetry. The existence of time-reversible symmetry in a polynomial system is related closely to the integrability of the system and, rotational symmetries have a connection to the second part of Hilbert's 16th problem.

For a given family of real planar polynomial systems of ordinary differential equations depending on parameters, we consider the problem of how to find the systems in the family which become symmetric after some affine transformation. We first propose a general computational approach for quadratic systems to solve this problem, and then demonstrate its usage for the case of the family of planar quadratic systems.

Limit cycles in two-species chemical reactions

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Kinetic differential equations, being nonlinear, are capable of producing many kinds of exotic phenomena. However, the existence of multistationarity, oscillation or chaos is usually proved by numerical methods. We searched for limit cycles in two models of chemical reactions among two species where one reaction step is of the third order. In the case of the first model with three parameters, using symbolic methods we find the necessary and sufficient conditions on the parameters of the kinetic differential equation of the reaction under which a limit cycle bifurcates from the stationary point in the positive quadrant in a supercritical Hopf bifurcation. In the case of the second model with eight parameters we show that the kinetic differential equation of the reaction has two limit cycles surrounding the stationary point of focus type in the positive quadrant. The outer limit cycle is stable and the inner one is unstable. For the application of the methods we use the Wolfram language because the symbolic calculations to carry out are too complicated to do by hand.

Local integrability of two dimensional systems

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We study the integrability of two dimensional system of the form

$$\begin{aligned} \dot{x} = &\quad x - a_{10}x^2 - a_{20}x^3 - a_{11}x^2y - a_{02}xy^2 - a_{-13}y^3 \\ \dot{y} = &\ -y + b_{01}y^2 + b_{3,-1}x^3 + b20x^2y + b_{11}xy^2 + b_{02}y^3 \\ . \end{aligned} \tag{2}$$

Necessary and sufficient conditions for existence of analytic first integral of four subfamilies of system 2 are obtained. The necessary conditions are derived using algorithms of computational commutative algebra. To prove integrability we mainly used the method of Darboux. Using specific approach we obtain general integrability conditions for system 2.

References

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Structure, size and statistical properties of chaotic components in Hamiltonian systems with divided phase space

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Generic Hamiltonian dynamical systems are neither integrable nor fully chaotic. Whether the motion is chaotic or not depends on the initial condition. The phase space is divided into various invariant components. Typically, the chaotic component, known as the chaotic sea, surrounds an infinite number of KolmogorovArnold-Moser (KAM) islands [1]. Invariant fractal sets konwn as cantori may also be present and limit transport in the phase space causing the phenomenon known as stickiness [2]. In the talk we will present the statistical properties of the largest chaotic component in a family of billiards introduced in [3]. By changing the value of the billiard parameters, we may acquire anything from a fully regular (integrable) to fully chaotic (ergodic) Hamiltonian systems. We divide the phase space into a grid of cells and determine which of them belong to the chaotic component by the iteration of a chaotic orbit. We compare the dynamics of the cell filling with the so-called random model [4, 5], that assumes completely uncorrelated cell visits and accurately describes the filling of cells for ergodic systems. We will show that due to stickiness the random model fails to describe the cell filling in systems where stickiness is present. The statistics of cell recurrence times provide a way of quantifying the stickiness of the structures in the phase space.

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Multiplicity of solutions for a nonlocal nonhomogeneous elliptic equation with critical exponential growth

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We are interested in the following nonlocal nonhomogeneous elliptic equation

$$-\Delta u + V(x)u = \left(\frac{1}{|x|^{\mu}} * \frac{F(u)}{|x|^{\beta}}\right)\frac{f(u)}{|x|^{\beta}} + \varepsilon h(x) \quad \text{in} \quad \mathbb{R}^2,$$

where V is a positive continuous potential, $0 < \mu < 2$, $\beta \geq 0$, $2\beta + \mu \leq 2$, ε is a small parameter and $F(s)$ is the primitive function of $f(s)$. Suppose that the nonlinearity $f(s)$ is of critical exponential growth in the sense of Trudinger-Moser inequality, we prove the existence and multiplicity of solutions by variational methods.

Number of synchronized and segregated solutions for linear coupled elliptic systems

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We study the following linear coupled elliptic equations:

$$\begin{cases} -\varepsilon^2\Delta u + u = u^3 + \lambda v & \text{in } \Omega, \\ -\varepsilon^2\Delta v + v = v^3 + \lambda u & \text{in } \Omega, \\ u > 0,\ v > 0 & \text{in } \Omega, \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = 0 & \text{on } \partial\Omega, \end{cases} \tag{P_ε}$$

where Ω is a smooth bounded domain in R^3, ε is a small parameter and λ is a coupling constant. Due to Lyapunov-Schmidt reduction method, we prove that (P_ε) has at least $O\left(\frac{1}{\varepsilon^3|\ln\varepsilon|^3}\right)$ synchronized and $O\left(\frac{1}{\varepsilon^6|\ln\varepsilon|^6}\right)$ segregated solutions for ε sufficiently small and some λ. Moreover, for each $m \in (0, 3)$ there exist synchronized and segregated solutions for (P_ε) with energies in the order of ε^{3-m}.

Convergence Theorems for Modifified Inertial Viscosity Splitting Methods in Banach Spaces

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We study a modified viscosity splitting method combined with inertial extrapolation for accretive operators in Banach spaces and then establish a strong convergence theorem for such iterations under some suitable assumptions on the sequences of parameters. As an application, we extend our main results to solve the convex minimization problem. Moreover, the numerical experiments are presented to support the feasibility and efficiency of the proposed method.

A viscosity approximation scheme for a family of finite asymptotically nonexpansive mappings in Banach spaces

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We propose a viscosity approximation scheme to solve the general split common fixed point problem for a family of finite asymptotically nonexpansive mappings in Banach spaces. Under suitable conditions, we verify the strong convergence theorems of the scheme. As applications, we apply our main results to solve some hierarchical variational inequality problems and equilibrium problems.

The split common fixed point problem for asymptotically nonexpansive mappings in Banach spaces

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We study the split common fixed point problem and propose an iteration algorithm for finding a split common fixed point of asymptotically nonexpansive mapping in the frameworks of two real Banach spaces. Under some suitable conditions imposed on the sequences of parameters, some strong convergence theorems are proved. We also apply our main results to solve some variational inequalities. The results generalize and improve the main results of other authors.