CAMTP

The Second Workshop
Dynamical Systems and Applications

in the framework of the project FP7-PEOPLE-2012-IRSES-316338
within the 7th European Community Framework Programme

Hotel PIRAMIDA, Maribor, Slovenia
8 June 2016 - 9 June 2016

Book of Abstracts

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Organizer

CENTER FOR APPLIED MATHEMATICS AND THEORETICAL PHYSICS •
CENTER ZA UPORABNO MATEMATIKO IN TEORETIČNO FIZIKO
KREKOVA 2 • SI-2000 MARIBOR • SLOVENIA

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Prof. Dr. Valery Romanovski, CAMTP and Faculty of Natural Sciences and Mathematics

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Invited Speakers

Mag. Ana Anušić

Dr. Brigita Ferčec

Mr. Wilker Thiago Resende Fernandes

Prof. Dr. Jaume Giné

Prof. Dr. Maite Grau

Prof. Dr. Matej Mencinger

Dr. Goran Radunović

Prof. Dr. Marko Robnik

Ms. Camila Aparecida Benedito Rodrigues

Prof. Dr. Valery Romanovski

SCHEDULE

Wednesday, 8 June 2016	
09:00-09:10	**Opening**
Chairman	Maite Grau
09:10-09:50	**Jaume Giné**
09:50-10:30	**Marko Robnik**
10:30-11:00	Tea & Coffee
Chairman	Matej Mencinger
11:00-11:40	**Maite Grau**
11:40-12:20	**Goran Radunović**
12:20-13:00	**Valery Romanovski**
13:00-15:00	Lunch
Chairman	Jaume Giné
15:00-15:40	**Matej Mencinger**
15:40-16.20	**Brigita Ferčec**
16:20-16:50	Tea & Coffee
Chairman	Goran Radunović
16:50-17.20	**Wilker T. Resende Fernandes**
17:20-17.50	**Ana Anušić**
17:50-18.20	**Camila A. Benedito Rodrigues**
18:30	Dinner

Topological structure of unimodal inverse limit spaces

ANA ANUŠIĆ

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Study of inverse limit spaces gained significance in topological dynamics in 1970 when Williams showed that hyperbolic one-dimensional attractors can be represented as inverse limit spaces. We are interested in the topological structure of inverse limit spaces with one tent bonding map. Such inverse limit spaces can be embedded in the plane as global attractors of one-parameter family of planar homeomorphisms and they vary continuously in Hausdorff topology. In 1991 the problem of classifying tent map inverse limits was introduced and became known as the Ingram conjecture. After a sequence of partial results, in 2012, Barge, Bruin and timac showed that nondegenerate unimodal inverse limits are all non-homeomorphic. However, the proof crucially depends on the ray compactifying on the core, thus leaving the core version of the conjecture still open. We will discuss the recent results in the case when the critical orbit of the bonding maps is infinite and non-recurrent.

Integrability conditions of planar polynomial systems

BRIGITA FERČEC

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We present an approach to finding general conditions for integrability of a given family of two-dimensional polynomial systems using conditions computed when some parameters were fixed. We apply it to obtain integrability conditions for a LotkaVolterra planar complex quartic system having homogeneous nonlinearities. Then, we also study bifurcations of limit cycles from each component of the center variety of the corresponding quartic real system.

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First integrals of the May-Leonard asymmetric system

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We investigate the existence of first integrals in the three dimensional May-Leonard asymmetric system. Using the computational algebra systems Mathematica and Singular we first look for families of the May-Leonard asymmetric system admitting invariant surfaces of degree two. Then using these invariant surfaces and invariant planes we construct first integrals of the Darboux type identifying subfamilies of such system admitting one first integral and two independent first integrals.

Center conditions for Liénard, Cherkas and generalized Kukles systems

JAUME GINÉ

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We will study the center problem for polynomial differential systems and we will prove that any center of an analytic differential system is analytically reducible. We will study the center problem for polynomial Linard systems of degree n with damping of degree n, the centers for the Cherkas polynomial differential systems and the centers for certain generalized Kukles systems. We will also establish a conjecture about the center conditions for such systems.

Essential perturbations in planar polynomial differential systems

MAITE GRAU

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A way to study the limit cycles which bifurcate from the periodic orbits of a center of a planar polynomial differential system is to perturbate the system with the center in a certain parametric family of systems. In this context, it appears the notion of essential perturbation used for the first time by Iliev (1998) in a paper on quadratic systems. We will give its explicit definition.

Given a perturbation of a particular family of centers of polynomial differential systems of arbitrary degree for which we explicitly know its Poincaré–Liapunov constants, we find its essential perturbations. As a consequence we give the structure of its k-th Melnikov function. This result generalizes the result obtained by Chicone and Jacobs (1991) for perturbations of degree at most two of any center of a quadratic polynomial system. Moreover we study the essential perturbations for all the centers of the differential systems

$$\dot{x} = -y + P_{\mathrm{d}}(x, y), \quad \dot{y} = x + Q_{\mathrm{d}}(x, y),$$

where P_{d} and Q_{d} are homogeneous polynomials of degree d, for $\mathrm{d} = 2$ and $\mathrm{d} = 3$.

This talk is based on the joint work: ADRIANA BUICĂ, JAUME GINÉ AND MAITE GRAU, *Essential perturbations of polynomial vector fields with a period annulus*, Commun. Pure Appl. Anal., **14** no. 5 (2015) 1073–1095.
`doi:10.3934/cpaa.2015.14.1073`

The center and cyclicity problem of some maps

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We consider some dynamical properties of maps given by

$$f(x) = -x - \sum_{k=1}^{\infty} a_k x^{k+1} \tag{1}$$

arising from some polynomial equations $\Psi(x,y) = x + y + \sum_{i+j=1}^{n} \alpha_{ij} x^i y^j = 0$, where higher order terms up to degree four are present (i.e. $n \leq 4$).

Following definitions from [3,4] we consider the center-focus problems and the cyclicity of the center in some maps given by (1). We use the analogue, Φ, of first integral for

$$\begin{aligned}\dot{x} &= -y + P(x,y)\\ \dot{y} &= x + Q(x,y)\end{aligned}$$

where $P(x,y)$ and $Q(x,y)$ are polynomials starting with quadratic terms, and

$$\Phi(x) = x^2 \left(1 + \sum_{k=1}^{\infty} b_k x^k\right).$$

Furthermore, we use the analogue of focus quantities, g_{2k}, defined by

$$\Phi(f(x)) - \Phi(x) = g_2 x^4 + g_4 x^6 + \cdots + g_{2k} x^{2k+2} + \cdots .$$

For primary decompositions of ideals $\mathcal{I} = \langle g_2, g_4, g_6, ...\rangle$ we use CAS Singular [2]. In many (sub)cases the corresponding ideal $\mathcal{I}$ contains in the primary decomposition a nonradical component and we can not estimate the cyclicity simply using the Bautin method. In this cases we adopt (see [3, Th. 1]) an approach from the theory of ODEs [1, Th. 2.1] to estimate the bifurcation of limit cycles from each component of the center variety. The analogue of Theorem [1, Th. 2.1] for estimating the cyclicity, given in [3, Th. 1], will be explained and used for estimating the cyclicity of the center in two cases arising from $\Psi(x,y) = 0$ containing homogeneous terms of even degree.

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Relative fractal drums, complex dimensions and geometric oscillations

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We give an overview of the higher-dimensional theory of complex dimensions for relative fractal drums. Relative fractal drums or, in short, RFDs are a far reaching and convenient generalization of compact sets in Euclidean spaces. For such objects we associate a fractal zeta function which we call the distance (or Lapidus) zeta function. The corresponding complex dimensions of the RFD are then defined as the poles (or more general singularities) of the associated distance zeta function. These complex dimensions generalize the classical Minkowski dimension and are connected to the intrinsic geometric oscillations of the RFD. Possible application of the theory could be found in fractal analysis of bifurcations of dynamical systems. This is a joint work with Michel L. Lapidus and Darko Zubrinic.

Energy evolution in one-dimensional time-dependent Hamiltonian oscillators

MARKO ROBNIK

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Time-dependent dynamical systems are becoming of increased interest. I shall present most recent results on time-dependent one-dimensional Hamiltonian oscillators. The time-dependence describes the interaction of an oscillator with its neigbourhood. While the Liouville theorem still applies (the phase space volume is preserved), the energy of the system changes with time. We are interested in the statistical properties of the energy of an initial microcanonical ensemble with sharply defined initial energy, but uniform distribution of the initial conditions with respect to the canonical angle. We are in particular interested in the change of the action at the average energy, which is also adiabatic invariant, and is conserved in the ideal adiabatic limit, but otherwise changes with time. Thus the spreading of the energy distribution is related to the accuracy of the (non)preservation of the adiabatic invariants. We shall treat the linear and nonlinear oscillators. In the linear oscillator the value of the adiabatic invariant always increases, implying the increase of the Gibbs entropy in the mean (at the average energy). The energy universally has the arcsine distribution, independent of the driving law. In nonlinear oscillators things are different. For slow but not yet ideal adiabatic drivings the adiabatic invariant at the mean energy can decrease, just due to the nonlinearity and nonisochronicity, but nevertheless increases at faster drivings, including the limiting fastest possible driving, namely parametric kick (jump of the parameter). This so-called PR property will be analyzed, as well as the ABR property (local equivalent of PR). The WKB method is employed for the linear oscillator, and the generalized nonlinear WKB-like method for the class of homogeneous power law potentials, which allows for analytic results to be compared with the numerical calculations.

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Averaging theory of any order for computing limit cycles of planar discontinuous piecewise differential systems with many zones

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The increasing interest in the theory of nonsmooth vector fields has been mainly motivated by its strong relation with Physics, Engineering, Biology, Economy, and other branches of science. In fact, their associated differential systems are very useful to model phenomena presenting abrupt switches such as electronic relays, mechanical impact, and Neuronal networks. The extension of the averaging theory to discontinuous piecewise vector field has been the central subject of investigation of many recent papers, for example, [1-4]. In this talk we will investigate the existence of periodic solutions for an family of planar discontinuous differential systems $Z(x, y; \epsilon)$ with many zones. We show that for $|\epsilon| \neq 0$ sufficiently small the averaged functions at any order control the existence of crossing limit cycles for systems in this family. We also provide some examples dealing with nonlinear centers when $\epsilon = 0$.

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Invariants and time-reversibility in polynomial systems of ODE's

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We present some results related to the theory of invariants of ordinary differential equations. Invariants of a group of orthogonal transformations of two-dimensional systems are considered in details. An algorithm to compute a generalizing set of invariants is given and an interconnection of the invariants and time-reversibility is shown. Some generalizations to the case of three-dimensional systems are discussed as well.