# Quantum Chaos and Quantum Information

A SOCRATES Lecture Course at CAMTP,
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**Abstract**

In this course I will review some recent development in the field of Quantum Chaos, in particular in the connection to the emerging fields of Quantum Computation and Quantum Information.

I will start by defining some basic notions of Quantum Chaos in the time domain, together with some quantitative measures which characterize the complexity and stability of the quantum and underlying classical motion: dynamical entropies, Lyapunov exponents, fidelity and more general Loschmidt echoes.

Then I will continue with introducing the basic concepts of quantum information, and presenting the basic results, such as no-cloning theorem and the principle of quantum teleportation. I will discuss the statement of universal quantum computation and give examples of efficient quantum algorithms, such as quantum teleportation, quantum Fourier transformation and Grover search algorithm.

In the last part of the course I will show how understanding of stability of quantum dynamical systems can help in constructing more stable quantum algorithms, e.g. running efficient and stable quantum simulation of dynamical systems on imperfect devices. I will also discuss several candidate technologies for building a real-world quantum computer and the present state of affairs.

## Lecture 1:
## Quantum chaos in time domain and phase-space representation of quantum mechanics

In this introductory lecture I will discuss some basic definitions and principles in quantum chaos [1, 2], in particular in the time domain where dynamical chaos and complexity can directly be discussed.

I will show that the question whether genuine chaos exists in quantum dynamics of small bounded systems boils down to the interplay of timescales, in particular the so-called Heisenberg time or break time and the Ehrenfest time [3, 4]. These time-scales will be defined precisely and discussed in various situations. In order to facilitate this and later discussions I will review some phase-space formalisms of quantum mechanics, such as Wigner-Weyl picture and Moyal equation [5], (squeezed) coherent states and Husimi functions.

## Lecture 2:
## Measures of stability and complexity in classical and quantum dynamics

In the second lecture I will define and discuss basic measures of stability and complexity of classical and quantum dynamical systems. For classical dynamical systems I will consider the spectrum of Lyapunov exponents [6, 7], the Ruelle spectrum of Perron-Frobenius operator [7]), the classical Loschmidt echo or stability against systems' perturbations [9], and the dynamical entropies closely related to the algorithmic complexity [7]. I will classify different possible behaviors and present examples of each class.

Then I will consider the corresponding properties of quantum dynamical systems with examples [8, 1, 10, 11, 12]. In particular it is worth stressing that the correspondence between classical and quantum behaviors, which should hold for sufficiently small times, is typically broken for longer times, i.e. times longer than the so-called break times. Two most drastic effects in this respect, which will be explained in the lecture, are the quantum dynamical localization [3], and stability of quantum echoes against perturbations [10, 11].

## Lecture 3:
## Quantum information — basic facts

In the third lecture I will define quantum information, the notion of a qubit, and coding of quantum information. Then we shall review basic facts and theorems concerning quantum information [13, 14, 15]. For example, we shall demonstrate the quantum no-cloning theorem which forbids copying of quantum information. We shall also define quantitative measures of entanglement and show how entanglement can be used as a resource in quantum information processing.

In the second part of this lecture I will disuss the principal requirements for making a universal quantum information processor (the so-called DiVinzenco's criteria) and describe some basic technologies [13, 14, 15] which seem promising candidates for building a real-world quantum computer. For example: ion traps, solid state devices, photonics, Josephson junction arrays, etc. By listing some recent advances in experiments I will try to present the state of the art.

## Lecture 4:
## Universal quantum computation and efficient quantum algorithms — examples

In the fourth lecture we shall discuss the principles of universal quantum computation [13, 14, 15]. We shall write a simple small set of universal gates, such as Hadamard gate, Pauli gates, as examples of one-qubit gates, and controlled not gate as an example of two qubit gate.

Then we shall outline some examples of efficient quantum algorithms which process certain tasks sometimes even exponentially faster (in the number of qubits) than any classical algorithm. In particular, we shall discuss quantum protocol for performing quantum teleportation, that is a transport of an unknown quantum state through an array of qubits. Then we shall discuss quantum Fourier transform which is an efficient quantum algorithm to perform discrete Fourier transformation. This algorithm is at the heart of the most famous to-date application of quantum computing: Schor's algorithm [16] for fast factorization of integers. This algorithm has an immense potential application in cryptography, and perhaps it is interesting to mention that best current prototype quantum-computers are already able to factorize number $15 = 5 \times 3$. At last we shall also present the idea of the celebrated Grover [17] algorithm, which searches for an item in an unstructured list of

$n$ objects in $\sim \sqrt{n}$ computational steps.

## Lecture 5: Quantum computation as a dynamical system — Can chaos enhance stability or reduce decoherence of quantum computation?

In the last lecture we will present some recent developments on the connection between dynamical systems and quantum computation. In particular, one can simulate chaotic classical and quantum dynamical systems efficiently by a quantum computer. I will present an algorithm for simulating quantum kicked rotor efficiently on a quantum computer and for determining the localization length of a steady state [18].

Then we shall focus on application of recent results on the stability of dynamical systems against small variation of the Hamiltonian and the decay of quantum Loschmidt echo in the realm of quantum computation. In particular I will explain the linear response theory of fidelity which predicts that faster decay of dynamical correlations reduce fidelity of quantum computation [11, 19]. At last, if the time permits, we shall also discuss on how similar ideas can be used in dynamical system's explanation of decoherence [20], e.g. in computing the Von Neuman entropy growth in a chaotic or regular system weakly coupled to an environment.

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