

# Heunova enačba in kvazinormalen spekter

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Heunova enačba je navadna diferencialna enačba drugega reda s štirimi regularnimi singularnostmi. Med drugim jo rešujemo pri opisu črnih lukenj v asimptotskih dS/AdS prostorih. Pred kratkim so zapisali [1] eksakten analitičen izraz za termičen propagator v konformni teoriji polja preko dualnosti AdS/CFT z uporabo znane tehnike [2] za rešitev Heunove enačbe. V tem seminarju bom najprej podal kratek uvod v Heunovo enačbo, nato pa pokazal kako se račun občutno poenostavi pri iskanju kvazinormalnega spektra [3].

## Reference

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- [3] B. Bajc, O. Lisovyy in S. Grozdanov, v pripravi.

# The Heun equation and quasinormal modes

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The Heun equation is an ordinary differential equation of second order with four regular singularities. It appears among others in the description of black holes in asymptotically dS/AdS spacetimes. Recently an exact analytic closed form for the thermal CFT propagator has been written down [1] via the AdS/CFT correspondence using a known technique [2] for solving the Heun equation. In this talk I will give a short introduction into the Heun equation and show how can one simplify considerably the computation of the quasinormal modes [3].

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# Hirsch-index scaling and limits from gintropy

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It is shown that the Hirsch index, used in evaluating citation popularity of individual scientists and institutions, is proportional to the gintropy (an entropy like quantity derived from the Gini index, characterizing income inequalities by a single number) in case of a Tsallis – Pareto distribution. Several scaling assumptions follow from this, as well as limits from the maximum of gintropy. These analytical results are compared to massive data from google scholar, collecting publication and citation numbers together with the Hirsch index. Earlier linear assumptions, seen in some data, appear with a validity limited to low and medium Hirsch indices.

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# Optična manipulacija bipolaronov v sistemu z nelinearno elektronsko-fononsko sklopitvijo

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Predstavil bom kvantno mehansko evolucijo dveh elektronov, ki sta nelinearno sklopljena s kvantnimi fononi. Analiziral bom dinamični odziv sistema, ki je podvržen kratkemu prostorsko enakomernemu optičnemu impulzu, ki se sklaplja z vibracijskimi načini. Nelinearna elektronsko fononska sklopitev lahko zmehta ali okrepi fononsko frekvenco v prisotnosti elektronske gostote. V prvem primeru zunanji optični impulz, s frekvenco, ki je tik pod fononsko frekvenco, generira privlačno sklopitev med elektroni in vodi v dolgoživo vezano stanje tudi po tem, ko je optični impulz izklopljen. Privlačna sklopitev izvira iz dinamične modifikacije fononske porazdelitve, ki inducira metastabilno stanje. S povečanjem frekvence impulza se privlačna elektron-elektronska interakcija spremeni v odbojno. Dva zaporedna optična impulza z različnimi frekvencami lahko preklapljata med privlačno in odbojno interakcijo. Nazadnje pokažem, da je impulzno inducirana vezava elektronov učinkovita tudi za šibko disperzivne optične fonone, v prisotnosti anharmoničnega fononskega spektra in v dveh dimenzijah.

## Reference

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# Optical manipulation of bipolarons in a system with nonlinear electron-phonon coupling

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I will present full quantum mechanical evolution of two electrons nonlinearly coupled to quantum phonons and simulate the dynamical response of the system subject to a short spatially uniform optical pulse that couples to dipole-active vibrational modes. Nonlinear electron-phonon coupling can either soften or stiffen the phonon frequency in the presence of electron density. In the former case, an external optical pulse tuned just below the phonon frequency generates attraction between electrons and leads to a long-lived bound state even after the optical pulse is switched off. It originates from a dynamical modification of the self-trapping potential that induces a metastable state. By increasing the pulse frequency, the attractive electron-electron interaction changes to repulsive. Two sequential optical pulses with different frequencies can switch between attractive and repulsive interaction. Finally, we show that the pulse-induced binding of electrons is shown to be efficient also for weakly dispersive optical phonons, in the presence anharmonic phonon spectrum and in two dimensions.

## References

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# Attosecond Chronoscopy: From Atoms to Condensed Matter

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Observing and clocking non-equilibrium electronic dynamics in real time has developed into one of the key areas of attosecond physics [1, 2]. Attosecond chronoscopy holds the promise to provide novel information on many-electron systems complementary to conventional spectroscopy. The timing of the photoelectric effect represents one of the first breakthroughs of attosecond chronoscopy [3, 4]. Its extension to condensed matter opens up new opportunities to explore electronic band structures and topology, electron transport, and decoherence. We will illustrate the timing of electronic processes with the help of a few recent prototypical examples. They include the Eisenbud-Wigner-Smith (EWS) and transport time delay in layered materials [5, 6], the influence of the collective screening response on electron timing, the quest for identifying the speed limit of optoelectronics [7], timing of valleytronics in graphene [8], and first results on time-resolved non-linear atom-radiation interactions [9].

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# Quantum classical correspondence and finite-time dynamics

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According to Max Jammer: "In fact, there was rarely in the history of physics a comprehensive theory which owed so much to one principle as quantum mechanics owed to Bohr's correspondence principle". In particular, in the field of quantum chaos, integrable or chaotic classical dynamics is associated to Poisson or Wigner- Dyson distribution, respectively. However this correspondence fails in typical situations e.g. in presence of dynamical localization, for systems with divided phase space and for pseudo integrable systems. This failure is at the origin, for example, of conflicting results for polygonal billiards. By considering a correspondence between classical finite-time trajectories and the eigenfunctions and eigenvalues in an energy shell we provide a criterium to extend the classical quantum correspondence to all regimes of classical motion. Our criterium remains valid in the deep quantum regime.

## Reference

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# Modeli in realizacije dvodimenzionalnega nematika

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Nematski tekoči kristali imajo v idealizaciji na dve dimenziji strukturo, ki omogoča poenostavljeno matematično obravnavo. Polarni kot nematskega direktorja zadosti Laplaceovi enačbi, kar pomeni, da lahko tako direktor kot njegov polarni kot modeliramo kot kompleksno analitično funkcijo [1,2]. V tem pogledu so defekti ničle in poli analitične funkcije, večličnost vektorske reprezentacije direktorskega polja pa se ujema z večličnostjo korenskih funkcij.

Predstavljam bom značilnosti kompleksnega opisa in posplošitev ovojnega števila defektov na kompleksne vrednosti, ter algoritem, ki omogoča določanje direktorskega polja v okolici poljubnega števila majhnih delcev z različnimi robnimi pogoji [3,4].

Nazadnje bom prikazal uporabo algoritma za opis eksperimentalnih rezultatov nematika v tanki plasti ob prisotnosti mikro stebričkov.

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# Models and realizations of two-dimensional nematics

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In two-dimensional idealization, nematic liquid crystals have a structure that allows simplified mathematical modeling. The polar angle of the nematic director satisfies the Laplace equation, meaning that both the director and the polar angle can be represented by a complex analytical function [1,2]. In this representation, defects are zeroes and poles of the function, and the multivalued nature of the director field corresponds to the multivaluedness of the square root function.

I will present the features of the complex representation, a generalization of the winding number of defects to complex values, and an algorithm allowing calculation of the director field around an arbitrary set of small particles with various boundary conditions [3,4].

Finally, I will demonstrate the use of this algorithm to describe the experimental results of a thin nematic layer in the matrix of microposts.

## References

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- [4] U. I. Kara, B. Chen, S. Čopar, U. Tkalec and X. Wang, *unpublished*.

# Geometrični izvor višjih simetrij v kvantni teoriji polja

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S preučevanjem teorije strun na singularnih nekompaktnih prostorih  $X$  s postopkom rezanja in lepljenja singularnosti, ki segajo do meje  $X$ , lahko pokažemo pojav simetrij višje oblike v nastalih kvantnih teorijah polja. Proučili bomo usodo teh simetrij, ko postanejo prostori  $X$  kompaktni z nadaljnjo uporabo tehnik lepljenja. Poudarili bomo konstrukcije, ko so prostori  $X$  eliptično vlaknasti Calabi-Yaujevi mnogoterniki, ki ustrezajo F-teoriji, geometrijskemu režimu teorije strun in vključujejo nastali kvant teorije polja standardnega modela. Tam lahko te rezultate primerjamo s prejšnjimi študijami, kodiranimi v aritmetični strukturi eliptičnih krivulj, gradnikov eliptično vlaknatih prostorov.

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# Geometric Origin of Higher Symmetries in Quantum Field Theory

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By studying String Theory on singular non-compact spaces  $X$  we demonstrate, via a process of cutting and gluing of singularities that extend to the boundary of  $X$ , the appearance of higher-form symmetries in the resulting quantum field theories. We study the fate of these symmetries when spaces  $X$  become compact by further employing gluing techniques. We highlight constructions when spaces  $X$  are elliptically fibered Calabi-Yau manifolds, which correspond to F-theory, the geometric regime of String Theory, and include resulting quantum field theories of the Standard Model. There we can compare these results to previous studies, encoded in the arithmetic structure of elliptic curves, the building blocks of elliptically fibered spaces.

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# Could there be a Fifth Giant Planet in the Solar System?

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It is known from several investigations that our planetary system as it is with 8 planets and many other smaller bodies like asteroids, comets will be stable for billions of years. For the moment we ignore the 4 terrestrial planets and concentrate on the Gas Giants Jupiter, Saturn, Uranus and Neptune. What's about another Gas Giant present in the early Solar System - a question which was posed in several papers e.g. Nesvorny and Morbidelli (2012). They studied the dynamics of a hypothetical early planetary system consisting of 5 and even 6 giant planets! It turned out that of special interest is to deal with a fifth large planet. In this system in some of the many thousands numerical integrations one planet was ejected and the planetary system survive as a stable one consisting of four planets. In our numerical simulation we asked the question whether there could survive an additional giant planet like Uranus between Jupiter and Saturn on a stable orbit. Our results show in extension of older papers Dvorak and Kubala (2022) and Dvorak and Cuntz (2023) that on special initial semi-major axes between  $a=7.12$  and  $a=7.28$  such a fictive body could survive for up to 0.6 Gigayears which is 10 percent of the estimated life-time of our Solar System. We show in a series of graphs the respective results and discuss them.

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# Limitni cikli v sistemih NDE

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Najprej bom razložila Hopfovo bifurkacijo za ravninske sisteme. Opisala bom limitne cikle na centralni mnogoterosti in degenerirane Hopfove (Bautinove) bifurkacije. Nato bom predstavila enega izmed najbolj znanih problemov kvalitativne teorije navadnih diferencialnih enačb - Hilbertov 16. problem o številu limitnih ciklov dvo-dimenzionalnih sistemov

$$\dot{x} = P_n(x, y), \quad \dot{y} = Q_n(x, y)$$

( $n$  je maksimalna stopnja polinomov na desni strani sistema). Bistven del problema je problem ocenitve maksimalnega števila limitnih ciklov, ki bifurcirajo iz singularne točke tipa center ali fokus pod vplivom majhnih motenj koeficientov sistema, t.j. problem cikličnosti. Ključna značilnost našega pristopa je, da je v primeru elementarne singularne točke problem cikličnosti reduciran na algebraični problem iskanja baze določenega polinomskega ideala. Ta pristop nato uporabimo za problem cikličnosti poddružine kubičnih sistemov.

## Reference

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# Limit cycles in systems of ODE's

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First, I will discuss a Hopf bifurcation for planar systems. I will describe also limit cycles on the center manifold and degenerate Hopf (Bautin) bifurcations. Then, I will present one of the most famous problems in qualitative theory of ordinary differential equations - Hilbert's sixteenth problem on the number of limit cycles of two dimensional polynomial systems

$$\dot{x} = P_n(x, y), \quad \dot{y} = Q_n(x, y)$$

( $n$  is the maximum degree of the polynomials on the right-hand side of the system). An essential part of the problem is the problem of estimating of the maximum number of limit cycles which can bifurcate from a singular point of center or focus type under small perturbations of coefficients of the system, the so-called cyclicity problem. The key feature of our approach is that in the case of an elementary singular point the problem of cyclicity is reduced to the algebraic problem of searching for a basis of a certain polynomial ideal. We apply this approach for solving the cyclicity problem for a subfamily of cubic systems.

## References

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# Preučevanje zapletenosti dinamike v omrežjih celic beta: nauki iz fenomenoloških modelov

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Fenomenološki modeli igrajo ključno vlogo pri preučevanju kompleksnih bioloških sistemov. Z zagotavljanjem poenostavljene, vendar natančne predstavitve realnih pojavov služijo kot most med podrobnostmi bioloških sistemov in našim razumevanjem njihovih osnovnih principov. Ta abstrakcija nam omogoča izluščiti temeljne uvide, razkriti osnovne meh-anizme ter olajšati globlje razumevanje dinamike sistema. Poleg tega poenostavlja inter-pretacijo eksperimentalnih podatkov, kar omogoča prepoznavanje organizacijskih principov in vzorcev delovanja. V tem prispevku bom predstavil, kako se lahko fenomenološki modeli uporabljajo za preučevanje zapletenosti dinamike v omrežjih celic beta. V teh večceličnih enotah več sto celic beta, ki delujejo sinhrono in proizvajajo sekretorne pulze inzulina, hor-mona ključnega za nadzor presnovne homeostaze. Njihovo kolektivno ritmično aktivnost omogoča medcelična sklopitev, zanjo pa je značilna multimodalnost, ki izvira iz prepleta povratnih zank med različnimi podsistemi. Na naravo kolektivne aktivnosti vpliva tudi njihova funkcionalna heterogenost. Iz teh razlogov je večcelična dinamika populacij celic beta zelo kompleksna. Z uporabo fenomenološkega modela bom najprej pokazal, katere fiziološke determinante je treba upoštevati v modelu, da dosežemo dobro skladnost med modelom in eksperimentalnimi rezultati, ki temeljijo na večcelični konfokalni mikroskopiji. Izkazalo se je, da lahko model dobro reproducira eksperimentalno opažene kompleksne di-namične vzorce, če so vključeni kombinacija heterogenih in stimulusno-odvisnih časovnih zamikov, variabilnost ravni ekscitabilnosti celic in heterogena sklopitev [1]. V nadalje-vanju bomo model nadgradili z vključitvijo počasne oscilatorne komponente, ki jo upravlja paradigmatični Poincaréjev oscilator in odraža metabolno aktivnost beta celic. To nam omogoča preučevanje, kako kombinacija električne in metabolne sklopitve uravnava kolek-tivno ritmično aktivnost in kako se le-ta odraža na strukturi funkcionalnih mrež celic beta. Izgleda, da lahko električna sklopitev sama po sebi sinhronizira odzive, medtem ko vključitev metabolnih interakcij ne le dodatno izboljša sinhronost, temveč tudi razširi prostorski doseg interakcij, poveča število povezav znotraj funkcionalnih mrež in izboljša skladnost z eksperimentalnimi opažanji [2]. Na koncu bom izpostavil, da predlagani pristop fenomenološkega modeliranja, ki vključuje zelo malo parametrov, ni koristen le za pri-

dobivanje vpogledov v osnovne mehanizme oblikovanja večceličnih odzivov, ampak tudi opozarja na različne lastnosti, ki jih je treba upoštevati pri izgradnji celovitih in realističnih modelov omrežij celic beta.

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# Deciphering the Intricacies of Beta Cell Network Dynamics: Lessons from Phenomenological Models

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Phenomenological models play a pivotal role in the study of complex biophysical systems. By providing a simplified yet accurate representation of real-world phenomena, they serve as a bridge between the intricate details of biophysical systems and our comprehension of their essential principles. This abstraction allows us to extract fundamental insights, reveal underlying mechanisms, and facilitate a deeper understanding of the system's dynamics. Moreover, it simplifies the interpretation of experimental data, making it easier to identify patterns and organizing principles. In this contribution I will present how phenomenological models can be used to decipher the intricacies of beta cell network dynamics. In these multicellular networks several hundred of beta cells work in synchrony to produce secretory pulses of insulin, a hormone crucial for controlling metabolic homeostasis. Their collective rhythmic activity is facilitated by intercellular coupling and affected by their multimodal activity due to networked feedback interactions of various oscillatory subsystems as well as by their functional heterogeneity. Consequently, the multicellular dynamics of beta cell populations is far from elemental and is characterized by intricate patterns of activity. Using a phenomenological model, I will first show which physiological determinants need to be considered in the model to achieve good agreement between the model and experimental results based on multicellular confocal microscopy. It turned out that the model can firmly reproduce the experimentally observed complex dynamical patterns if a combination of heterogeneous and stimulus-dependent time lags, variability in excitability levels and a heterogeneous coupling are incorporated [1]. Next, we upgrade the model by incorporating the slow oscillatory component, which is governed by the paradigmatic Poincaré oscillator and reflects the metabolic activity of beta cells. This eases us to investigate how the combination of electrical and metabolic coupling generates collective rhythmicity and shapes functional beta cell networks. It appears that, while electrical coupling alone can synchronize the responses, the inclusion of metabolic interactions not only further improves coordination but also expands the spatial range of interactions, increases the number of connections within the functional networks, and enhances the alignment with experimental observations [2]. I will conclude by emphasizing that the proposed phenomenological modelling approach that involves very few parameters is not only beneficial for getting

insights into basic mechanisms that shape the multicellular responses, but also points out the various attributes that should be considered by building up comprehensive and realistic models of beta cell networks.

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# Perturbativni razvoj kvantnih spektrov in gravitacije okoli neskončnega števila dimenzij

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Perturbativna analiza spektrov linearnih operaterjev je dobro znana in neprecenljiva metoda za 'reševanje' Schrödingerjeve enačbe v kvantni mehaniki. Običajne analize razvijejo tak problem v obliki vrste v potencah sklopitvene konstante. V nekaterih primerih pa si je mogoče izmisliti tudi drugačen, nenavaden perturbativni parameter:  $1/d$ , kjer je  $d$  število prostorskih dimenzij. Tako v sistemih z gravitacijo kot tudi elektromagnetizmom je razvoj okoli  $d = \infty$  možen zaradi dejstva, da elektromagnetne in gravitacijske sile padajo kot  $1/r^d$  in se v  $d = \infty$  efektivno lokalizirajo okoli vsakega točkastega naboja ali mase. Na tem predavanju bom predstavil zgodovino te metode in pokazal, kako jo lahko uporabimo za izpeljavo energijskih nivojev atomov vodika in celo helija. Nato pa bom razložil, kako deluje isti tip razvoja v gravitaciji in kako lahko uporabimo te rezultate s pomočjo holografske dualnosti za rekonstrukcijo nekaterih spektrov močno sklopljenih kvantnih teorij polja.

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# Perturbative expansion of quantum spectra and gravity around an infinite number of dimensions

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Perturbative analysis of the spectra of linear operators is a well-known and invaluable method for ‘solving’ the quantum mechanical Schrödinger equation. Usually, this is done by expanding the problem as a power series in the coupling constant. In some cases, however, one may invent a different, unusual perturbative parameter ‘from thin air’:  $1/d$ , where  $d$  is the number of spatial dimensions. Both in systems with gravity and electromagnetism, such an expansion around  $d = \infty$  is possible due to the fact that electromagnetic and gravitational forces scale as  $1/r^d$  and in  $d = \infty$  become effectively localised around each point charge or mass. In this talk, I will present the history of this method and show how one can use it to derive the energy levels of the Hydrogen and even the Helium atom. I will then explain how one can use the same expansion in gravity and utilise it with the help of the holographic duality to reconstruct certain spectra of strongly coupled quantum field theories.

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# Stability and chaos of the duopoly model of Kopel: A study based on symbolic computations

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Since Kopel's duopoly model was proposed about three decades ago, there are almost no analytical results on the equilibria and their stability in the asymmetric case. The first objective of this talk is to analyze the asymmetric duopoly model of Kopel analytically by using several tools based on symbolic computations. We discuss the possibility of the existence of multiple positive equilibria and establish necessary and sufficient conditions for a given number of positive equilibria to exist. The possible positions of the equilibria in Kopel's model are also explored. Furthermore, if the duopolists adopt the best response reactions or homogeneous adaptive expectations, we establish rigorous conditions for the existence of distinct numbers of positive equilibria for the first time. The occurrence of chaos in Kopel's model seems to be supported by observations through numerical simulations, which, however, is challenging to prove rigorously. The second objective is to prove the existence of snapback repellers in Kopel's map, which implies the existence of chaos in the sense of Li-Yorke according to Marotto's theorem.

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# Novi koncept detektorja za pozitronsko tomografijo (PET)

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V prispevku bom predstavil razvoj novih detektorjev žarkov gama, ki znatno izboljšajo časovno ločljivost zaznavanja in s tem omogočajo nov pristop pri načrtovanju naprav za slikanje s pozitronsko tomografijo, eno izmed najpomembnejših diagonostičnih metod. V tem novem konceptu je aparatura sestavljena iz posameznih detektorskih panelov, kar prinese veliko fleksibilnosti pri njeni uporabi. V prispevku bom predstavil rezultate naših raziskav in načrte za njihovo nadaljevanje v okviru velikega evropskega EIC Pathfinder projekta PETVision.

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# Novel detector concept for positron emission tomography (PET)

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In this contribution, I will present the development of new gamma-ray detectors that significantly improve the time resolution of detection and thereby enable a new approach in the design of positron-emission tomography imaging devices, one of the most important diagnostic methods. In this new concept, the apparatus consists of individual detector panels, which brings a lot of flexibility in its use. In the paper, I will present the results of our research and plans for their continuation within the framework of the large European EIC Pathfinder project PETVision.

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# Precizijska fizika okusov pri Belle II

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Fizika okusov je ena od bistvenih Komponent Standardnega modela. Eksperimentalne študije šbkih razpadov mezonov B in D pri eksperimentih Belle in BaBar so skupaj z odkritjem Higgsovega bozona na Velikem hadronskem trkalniku LHC omogocile končno potrditev veljavnosti SM. Sedanja generacija precizijskih poskusov v fiziki okusov išče odstopanja od napovedi Standardnega modela. V prispevku bomo govorili o študijah anomalij v pri razpadih hadronov s kvarkom b in o študijah redkih razpadov, ki so zelo obetavna metoda za iskanje nove fizike.



# Precision flavour physics with Belle II

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Flavour physics is one of the essential elements of the Standard Model. Experimental studies of weak decays of B and D mesons at B factories have, together with the discovery of the Higgs boson at LHC, provided the final confirmation of the validity of the SM. The present generation of precision flavour-physics experiments is looking for departures from the SM. We discuss studies of anomalies in b hadron decays, and studies of rare decays which are a very promising method for searching for new physics.

# Advances in multimodal machine learning in healthcare

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Machine learning methods in healthcare have traditionally focused on using data from a single modality, limiting their ability to effectively replicate the clinical practice of integrating multiple sources of information for improved decision making. Here, we provide a review of recent advances in multimodal machine learning approaches in healthcare discussing different data fusion strategies and offering perspectives for the near future.

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# Complexity: From Pattern Formation to Pattern Recognition

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According to several prominent authors, including Stephen Hawking, a main part of 21st century science will be on complexity research. The intuitive idea is that global patterns and structures emerge from locally interacting elements like atoms in laser beams, molecules in chemical reactions, proteins in cells, cells in organs, neurons in brains, transistors in electronic systems etc. (Mainzer 2007). Complex pattern formation has been reported from many disciplines (e.g., physics, chemistry, biology, brain research, engineering). The causes of complex pattern formation have been analyzed from various perspectives such as Schrödinger's (1948) order from disorder, Prigogine's (1980) dissipative structure, Haken's (1983) synergetics, Langton's (1990) edge of chaos etc. But concepts of complexity are often based on examples or metaphors only. We argue for a mathematically precise and rigorous definition of local activity as the cause of complex pattern formation which can be tested in natural as well as technical sciences by constructive methods. Recently, these results of complexity research have become important for machine learning of AI (artificial intelligence) systems (e.g., neural networks, cognitive AI-systems, robots). Instead of complex pattern formation in nature, complex pattern recognition of artificial neural networks (ANN) is considered which is modeled in statistical learning theory. But, ANN-systems are only simulated on digital computers with von-Neumann architectures which run into severe problems of energy consumption (von-Neumann bottleneck) in the future - in contrast to the highly energetically efficient biological brains. Therefore, neuromorphic systems with more brain-oriented computing should be considered also for hardware realization. The research target is sustainable computing and AI with respect to energy and environment.

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# Matematično modeliranje izločanja insulina in glukagona

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Trebušna slinavka je pomemben organ, ki med drugim skrbi za izločanje insulina in glukagona kot dveh ključnih hormonov za uravnavanje glukoze in drugih metabolitov v našem telesu. Z metodo matematičnega modeliranja proučujemo procese v celicah alfa in beta trebušne slinavke in napovedujemo izločanje glukagona in insulina pri različnih pogojih. Pomembne so napovedi, ki kažejo na bolezenska stanja, ko so procesi deregulirani. Še posebej se osredotočamo na vlogo mitohondrijev kot ključnih celičnih organelov za produkcijo celične energije v obliki ATP in zagotavljanja anaplerotičnih metabolnih poti. Z modelnimi napovedmi uspešno napovemo patologije, ki so povezane z deregulacijo mitohondrijske funkcije.

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# Mathematical modeling of insulin and glucagon secretion

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The pancreas is an important organ that, among other functions, is responsible for the secretion of insulin and glucagon, two key hormones for regulating glucose and other metabolites in our body. Through the method of mathematical modeling, we study the processes in the alpha and beta cells of the pancreas and predict the secretion of glucagon and insulin under various conditions. Predictions indicating disease states, where processes are deregulated, are important. We particularly focus on the role of mitochondria as key cellular organelles for the production of cellular energy in the form of ATP and anaplerotic metabolic pathways. With model predictions, we successfully forecast pathologies associated with the deregulation of mitochondrial function.

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# Simulacija holesterika v vijačno simetrični ograditvi

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Predstavljam bom rezultate numeričnega modeliranja dvojne vijačne strukture holesteričnega tekočega kristala v vijačno simetrični ograditvi. Simulacija pojasni novo holesterno strukturo, ki je bila nedavno eksperimentalno opažena. Vijačna simetrija tri-dimenzionalni problem simulacije omeji na dve dimenziji, kar znatno poenostavi numerično reševanje. Tekoči kristal opišemo z Landau-de Gennesovim modelom v eno-konstantni aproksimaciji. Za različne oblike in različne vrednosti zvitja tekoče kristalne strukture analiziramo dobljena metastabilna stanja. Pri vrednostih parametrov, ki se ujemajo z eksperimentalnimi podatki, uspemo simulirati metastabilno stanje, ki ima dvojni zvoj z istima ročnostima in lokalizirana defektna območja v konicah vboklin v vijačni strukturi. V treh dimenzijah dobimo torej dve linijski zviti disklinaciji, s čimer potrdimo razumevanje nastale strukture.

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# Simulation of a cholesteric in a helical confinement

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Recently, a novel cholesteric structure has been experimentally observed. I will discuss the numerical modelling of this doubly helical structure in a helical confinement. Helical symmetry reduces the simulation problem to two dimensions, thus making the simulation much faster. Cholesteric is modeled within the Landau-de Gennes formalism using one-constant approximation. We simulate different possible helical shapes and differently twisted structures and analyze the resulting meta-stable states. For the shape and parameter values that correspond to the experimental data we succeed in obtaining a meta-stable state with a double twist director structure with the same sign and two localized defects in the two cusps of the twisted structure. In three dimensions that corresponds to two twist disclinations.

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# Integrable open quantum systems

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In physics, most problems are typically solved using approximations. However, there exist some rare cases where the large amount of symmetries makes them exactly solvable. Those systems are called Integrable.

In this talk, I will introduce the boost automorphism mechanism: a systematic approach for constructing new integrable spin chains characterized by a R-matrix, solution of the Yang-Baxter equation. The method consists in starting with an ansatz for the Hamiltonian and then restricting it to belong to an integrable model. This method has applications in different areas of physics, from condensed matter to high-energy physics. I will explain how we used it to provide the first systematic classification of integrable Lindbladians: open quantum system where the environmental response is Markovian. I will present one of the new models we discovered and discuss its properties: it corresponds to the first integrable next-to-nearest neighbour deformation of the Hubbard model.

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# Quantum Chromodynamics: 50 Years After Its Invention

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In the early 1960's strongly interacting elementary particles were classified along the symmetry group  $SU(3)$ . It was suggested that hadrons behave as if they were constituted by substructures, then named as quarks [1] or aces [2]. It was left open, if these subunits merely reflect symmetries or should be understood as particles.

By 1968, a SLAC-MIT study of deep-inelastic electron scattering produced first experimental evidence that the proton contains point-like constituents [3]. These findings [4,5] were awarded the Nobel prize only much later (in 1990). 'Partons' got identified with quarks, but it took some time to develop the theory of their interactions. In late 1972, Fritzsche and Gell-Mann proposed a gauge theory of  $SU(3)_C$  [6]. Its generators, the gluons, mediated the strong interactions. From then on the pertinent color gauge theory, quantum chromodynamics (QCD), has generally been assumed as the fundamental theory of the strong/hadronic force [7].

Now, 50 years later, QCD is not yet solved completely. The corresponding millenium prize is still to be earned [8]! However, many methods have been developed to gain reliable results, such as perturbation theory, effective field theories, quark models etc. I will review several of these approaches and discuss corresponding results, manifesting our present understanding of hadronic physics.

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# Kvantni mnogodelčni kaos

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Kvanti kaos (v pretežno enodelčnih sistemih) je bil zelo živahno raziskovalno področje v 1980ih in 1990ih. V zadnjih letih pa se je področje kvantnega kaosa spet zelo oživilo okrog kvantnega mnogodelčnega problema, kar je pritegnilo teoretične fizike iz zelo razno-terih skupnosti, od fizike kondenzirane materije, kvantne optike in kvantne informatike, pa do kvantne gravitacije. Ena od najbolj splošnih in robustnih lastnosti kvantno-kaotičnih sistemov je aplikativnost teorije slučajnih matrik.

Po motivacijskem uvodu v področje kvantnega kaosa se bom posvetil predvsem fascinantnemu problemu presenetljive učinkovitosti teorije slučajnih matrik za popis spektralnih fluktuacij v razsežnih kvantnih sistemih na mrežah. Pred kratkim smo identificirali razred takšnih sistemov z lokalno interakcijo, kjer lahko matematično rigorozno pokažemo uje-manje statističnih fluktuacij lastnih (kvazi)energij, karakterizirano s t.i. spektralnim oblikovnim faktorjem, ter teorijo slučajnih matrik. Takšni *dvojno unitarni sistemi* vsebu-jejo celotno ergodično hierarhijo dinamičnih režimov, od integrabilnosti, ne-ergodičnosti, do ergodičnosti, mešanja in maksimalnega (kvantnega) kaosa. Dvojno unitarni sistemi omogočajo analitičen izračun prostorsko-časovnih korelacijskih funkcij lokalnih opazljivk in so zelo primerni za eksperimentalni študij v aktualnih kvantnih simulatorjih.

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# Quantum Many-Body Chaos

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Quantum chaos (of mostly single particle problems) has been a fruitful interdisciplinary arena of research in 1980s' and 1990s'. In recent years, the field has dramatically revived around the quantum many-body problem gathering very diverse communities of theoretical physicists ranging from condensed matter, quantum optics and quantum information, to quantum gravity. One of the most general characteristics of quantum chaotic systems is the ubiquitous applicability of random matrix theory.

After a general introduction to the field, I will focus specifically on the fascinating problem of 'unreasonable effectiveness' of random matrix theory for description of spectral fluctuations in extended quantum lattice systems. A class of locally interacting spin systems has been recently identified where the fundamental measure of (quasi)energy level fluctuations, the spectral form factor, is proven to match with random matrix theory, and where spatiotemporal correlation functions of local observables as well as some measures of dynamical complexity can be calculated analytically. These, so-called dual unitary systems, include the whole 'ergodic hierarchy' of dynamics: integrable, non-ergodic, ergodic, and generically, (maximally) chaotic cases, and are ideally suited for experimental studies on the current generation of quantum simulators.

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**Kvantni kaos v mešanih sistemih:  
Potenčno pojevanje deleža mešanih stanj v semiklasični  
limiti**

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Kvantni kaos je študij lastnosti lastnih stanj, energijskih spektrov, dinamike, Wignerjevih in Husimijevih funkcij (v kvantnem faznem prostoru) sistemov, katerih klasični analogon je kaotičen, ali delno kaotičen [3]. Generični, tipični sistemi, delno kaotični, imajo v faznem prostoru v sobivanju regularna in kaotična območja. Takšna struktura klasičnega faznega prostora se zrcali v lastnostih Wignerjevih in Husimijevih funkcij v kvantnem faznem prostoru, kjer opazimo regularna stanja na invariantnih torusih, kaotična stanja v kaotičnih območjih (kjer so lahko lokalizirana), ter mešana stanja, ki so povezana z različnimi mehanizmi sklopitve regularnih in kaotičnih območij. Princip enakomerne semiklasične kondenzacije Wignerjevih in Husimijevih funkcij (Robnik 1998, [2]) napoveduje v striktni semiklasični limiti dovolj majhne efektivne Planckove konstante obstoj zgolj regularnih stanj na invariantnih torusih ter kaotičnih stanj v klasično kaotičnih območjih, medtem ko se delež mešanih stanj v semiklasični limiti zmanjšuje, kar je v skladu z Berry-Robnikovo sliko [1]. Predstavil bom stacionarni vidik ter opis kvantnih lastnih stanj v kvantnem faznem prostoru, ter pokazal prvo kvantitativno fenomenološko evidenco za potenčno pojevanje deleža mešanih stanj za primer t.i. limonastih biljardnih sistemov [4]. Ti rezultati so potrjeni tudi v drugih sistemih, kot so brcana vrtavka [5], Dickov model [6] ter Fermi-Pasta-Ulam-Tsingou sistem [7], kar kaže na univerzalnost potenčne zakonitosti, vendar z eksponenti, ki so specifični za sistem, in niso univerzalni.

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# Quantum chaos in mixed-type systems: Power-law decay of the fraction of mixed eigenstates in the semiclassical limit

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Quantum chaos is a study of properties of eigenstates, energy spectra, dynamics, Wigner and Husimi functions (in the quantum phase space) of systems, whose classical analogon is chaotic, or partially chaotic [3]. Generic, typical systems, partially chaotic, have in the phase space coexisting regular and chaotic regions. Such structure of the classical phase space is reflected in the properties of the Wigner and Husimi functions in the quantum phase space, where we observe regular states on the invariant tori, chaotic states in chaotic regions (which can be localized), and mixed states which are connected with various mechanisms of coupling between the regular and chaotic regions. The Principle of uniform semiclassical condensation (PUSC) of Wigner and Husimi functions (Robnik 1998, [2]) predicts in the strict semiclassical limit of sufficiently small effective Planck constant, the existence of merely regular states on invariant tori, and chaotic states on the classically chaotic regions, while the relative fraction of the mixed states decays, in agreement with the Berry-Robnik picture [1]. I shall present the stationary aspect and description of quantum eigenstates in the quantum phase space, and show the first quantitative phenomenological evidence for the power-law decay of the relative fraction of the mixed states in the case of so-called lemon billiards [4]. These results are confirmed also in other systems like kicked top [5], Dicke model [6] and the Fermi-Pasta-Ulam-Tsingou system [7], which indicates the universality of the power law, but with exponents which are system dependent, and thus not universal.

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# Invariante in reverzibilnost v polinomskih sistemih NDE

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Najprej proučujemo povezavo med invariantami nekaterih delovanj grup in časovno reverzibilnostjo v razredu dvorazsežnih polinomskih sistemov z 1:-1 resonantno singularnostjo v izhodišču. Časovna reverzibilnost je tesno povezana z mnogoterostjo Sibirskega, ki je vsebovana v centralni mnogoterosti, in vemo, da vsak časovno reverzibilen sistem premore lokalni analitični prvi integral v okolici izhodišča. Podamo nov algoritem za izračun množice generatorjev ideala Sibirskega in nekatere algebraične lastnosti tega ideala.

V preostanku raziskujemo posplošitev pojma časovne reverzibilnosti v tri razsežnih sistemih z  $1 : \zeta : \zeta^2$  resonančno singularnostjo v izhodišču (kjer je  $\zeta$  primitivni tretji koren enote). Preučujemo povezavo te reverzibilnosti z invariantami določenih delovanj grup v prostoru parametrov sistema in z Lawrence-ovimi ideali.

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# Invariants and reversibility in polynomial systems of ODEs

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We first investigate the interconnection of invariants of certain group actions and time-reversibility of a class of two-dimensional polynomial systems with 1:-1 resonant singularity at the origin. The time-reversibility is related to the Sibirsky subvariety of the center (integrability) variety and it is known that every time-reversible system has a local analytic first integral at the origin. We propose a new algorithm to obtain a generating set for the Sibirsky ideal of such polynomial systems and investigate some algebraic properties of this ideal.

Then, we discuss a generalization of the concept of time-reversibility in the three dimensional case considering the systems with  $1 : \zeta : \zeta^2$  resonant singularity at the origin (where  $\zeta$  is a primitive cubic root of unity) and study a connection of such reversibility with the invariants of some group actions in the space of parameters of the system and Lawrence ideals.

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# Brezmrežne simulacije večfizikalnih in večnivojskih problemov s strjevanjem

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V prispevku najprej predstavimo koncept povezanih simulacij strjevanja večsestavinskih sistemov na več merilih ob prisotnosti zunanjih polj. Pri tem so makroskopski modeli formulirani na podlagi teorije idealne mešanice, mezoskopski na podlagi celičnih avtomatov in mikroskopski na podlagi faznega polja. V modelih upoštevamo prenos mase, energije, gibalne količine in sestavin. Fazni diagram je vključen na podlagi vzvodnega pravila, kašasto področje pa s porozno plastjo. Predstavimo inovativne brez mrežne rešitve na vseh treh nivojih, ki temeljijo na konceptu močne formulacije, razdelitve področja na poddomene in uporabo različnih avtomatskih prilagoditvenih strategij. Na koncu predstavimo uporabo nekaterih razvitih modelov pri snovanju vodilnih tehnologij.

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# Meshless simulation of multiphysics and multiscale solidification problems

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In this paper, we first present the concept of coupled simulations of solidification of multi-component systems at multiple scales in the presence of external fields. The macroscopic models are formulated on the basis of the ideal mixture theory, mesoscopic models on the basis of cellular automata and microscopic models on the basis of the phase field. The models take into account the transfer of mass, energy, momentum and species. The phase diagram is included based on the lever rule, and the mushy zone with the porous layer. We present innovative meshless solutions at all three levels based on the concept of strong formulation, partitioning the domain into subdomains, and use of different automatic adaptation strategies. Finally, we present the use of some developed models in the creation of leading technologies.

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# Nonequilibrium quantum dynamics in trapped ultracold mixtures: From quenching across phase boundaries to impurity transport

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Our focus is the correlated non-equilibrium quantum dynamics for a binary mixture of ultracold trapped atoms. Three different scenarios will be addressed. First we explore the quench dynamics of a binary Bose–Einstein condensate crossing the miscibility–immiscibility threshold. Increasing the interspecies repulsion leads to the filamentation of the density of each species, involving shorter wavenumbers and longer spatial scales in the many-body approach compared to mean-field theory. These filaments appear to be strongly correlated and exhibit domain-wall structures. We simulate single-shot images to connect our findings to possible experimental realizations.

Next we explore the dynamical transport of an impurity between different embedding majority species which are spatially separated in a double well. The transfer and storage of the impurity is triggered by dynamically changing the interaction strengths between the impurity and the two majority species. We find a simple but efficient protocol consisting of linear ramps of majority-impurity interactions at designated times to pin or unpin the impurity. We analyze the dynamics in terms of single-particle densities, entanglement growth and provide an effective potential description involving mean-fields of the interacting components.

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# Numerical methods of chaos detection

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Determining the chaotic or regular nature of orbits of dynamical systems is a fundamental problem of nonlinear dynamics, having applications to various scientific fields. The most commonly employed method for distinguishing between regular and chaotic behavior is the evaluation of the maximum Lyapunov exponent (MLE)  $\sigma_1$ , because if  $\sigma_1 > 0$  the orbit is chaotic [1 and references therein]. The main problem of using this chaos indicator is that its numerical evaluation may take a long -and not known a priori- amount of time to provide a reliable estimation of the MLE's actual value. Over the years, many techniques have been developed, which try to overcome this problem (review presentations of some of them can be found in [2]).

In this talk we will focus our attention on two very efficient methods of chaos detection: the Smaller (SALI) and the Generalized (GALI) Alignment Index techniques [3-9]. We will first recall the definitions of the SALI and the GALI and will briefly discuss the behavior of these indices for conservative Hamiltonian systems and area-preserving symplectic maps, emphasizing that these methods are based on the evolution of more than one deviation vectors from the studied orbit, in contrast to the computation of the MLE where only one deviation vector is needed. Then, we will explain how one can use these methods to investigate the dynamics of time-dependent dynamical systems by considering a barred galaxy model whose parameters evolve in time [10].

We will also present some recently introduced methods to estimate the chaoticity of orbits in conservative dynamical systems from computations of Lagrangian descriptors (LDs) [11, 12] on short time scales. In particular, we will consider methods based on the difference and the ratio of the LDs of neighboring orbits, as well as a quantity related to the finite-difference second spatial derivative of the LDs [13, 14]. These indicators can correctly identify the chaotic or regular nature of orbits to better than 90% agreement with results obtained by the SALI method. These findings indicate the capability of LDs to efficiently identify chaos in conservative dynamical systems without knowing the variational equations (tangent map) of continuous (discrete) time systems needed by traditional chaos indicators.

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# Spin resonance without a spin: A microwave analog

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Following a suggestion of Joyner et al. [1] we previously realized a microwave network with a particular symmetry simulating a spin 1/2 [2,3]. In this work the analogy is promoted even further towards a microwave analog of a magnetic resonance experiment. The network consists of two identical subgraphs coupled by a pair of bonds with a length difference corresponding to a phase difference of  $\pi$  for the waves traveling through the two bonds. As a consequence all eigenvalues appear as Kramers doublets. Detuning the length difference from the  $\pi$  condition Kramers degeneracy is lifted, which may be interpreted as a Zeeman splitting of a spin 1/2 in a magnetic field. The lengths of another pair of bonds are modulated periodically with frequencies of some 10 MHz by means of diodes, thus emulating a magnetic radiofrequency field. Features well-known from nuclear magnetic resonance such as the transition from the laboratory to the rotating frame, or Lorentzian shaped resonance curves can thus be realized.

This is a joint work together with Tobias Hofmann, Finn Schmidt, Philipps University of Marburg, and Ulrich Kuhl, Université Côte d'Azur of Nice.

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# Fazni prehodi z zlomom ergodičnosti

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Kvantna termalizacija in ergodičnost sta lastnosti, ki veljata za večino fizikalnih sistemov. Kljub temu pa se nekateri sistemi ne podrejata tem zakonitostim in kažejo drugačne lastnosti. To vodi do prehoda med dvema različnima fazama: tisto, v kateri sistem termalizira, in tisto, v kateri ne termalizira. Prehod lahko opazimo v različnih lastnostih sistema, denimo v lastnostih lastnih stanj in spektra, naravi transporta naboja in energije, in tako dalje. V termodinamski limiti te prehode imenujemo fazni prehodi z zlomom ergodičnosti.

V seminarju se bomo osredotočali na to, kako zaznati fazni prehod z zlomom ergodičnosti skozi vidik neravnovesne kvantne dinamike. Eden od načinov študije dinamike je osredotočanje na skalirno invarianco, ker pomeni, da sistem ne spremeni svojega obnašanja ob spremembi časovne ali prostorske skale. Skalirna invarianca je v ergodičnih sistemih uveljavljen pojav ob dolgih časih, kot je denimo pojav t.i. prečke (ang. *ramp*) v spektralnem oblikovnem faktorju. Pokazali bomo, da se skalirna invarianca v sistemih na prehodu med ergodično in neergodično fazo pojavi že ob precej krajših časih. Osredotočali se bomo na dinamiko dveh količin, t.i. preživetvene verjetnosti začetnega stanja (ang. *survival probability*) in spektralnega oblikovnega faktorja, ter obravnavali tako kvadratne modele kot modele z interakcijo.

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# Ergodicity breaking phase transitions

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Quantum thermalization and ergodicity are principles that most physical systems obey. However, some systems may deviate from these principles and show a different behavior. This can cause a transition between two contrasting phases: one where the system thermalizes and one where it does not. The transition can be observed in various aspects of the system, such as the properties of its eigenstates and spectra, the nature of charge and energy transport, and so on. In the thermodynamic limit, these transitions are known as ergodicity breaking phase transitions.

I will talk about how to detect the ergodicity breaking phase transitions based on their nonequilibrium dynamics. One way to study the dynamics is to look at the scale invariance, which means that the system does not change its behavior when the scale of time or space is changed. Scale invariance is common at long times in ergodic systems as shown, e.g., by the ramp feature in the spectral form factor. However, we find that scale invariance also appears at shorter times in systems that are at the transition point between the ergodic and nonergodic phases. We investigate the scale invariant dynamics of two quantities: the survival probability and the spectral form factor, and we study both quadratic and interacting systems.

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# Kvantni in kvantno-navdihnjeni algoritmi za analitiko grafov

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Grafne nevronske mreže so zmogljivo orodje za analitiko grafov s strojnimi učenjem. Njihova široka uporabnost pri analizi bioloških in družbenih sistemov, kemijskih formul, pa tudi besedil in slik, je v znanstveni skupnosti vzbudila veliko zanimanje za razvoj hitrejših, natančnejših in učinkovitejših algoritmov. Ključna pomanjkljivost sedanjih algoritmov je, da so zaradi prepletanja interakcij dolgega in kratkega dosega zahtevni za procesiranje na trenutnih CPU/GPU in posledično računsko kompleksni. Nedavni pristop k temu problemu je uporaba kvantnega računalništva, ki naslavlja prepletanje med interakcijami dolgega in kratkega dosega s pomočjo kvantne prepletenosti. Po predstavitvi osnov strojnega učenja na kvantnih računalnikih bom predstavil nekatere nedavne rezultate na tem področju in zaključil s predstavitvijo študije primera uporabe kvantnih grafnih nevronskih mrež za segmentacijo.

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- [2] S. Thabet, R. Fouilland, M. Djellabi, I. Sokolov, S. Kasture, L.-P. Henry and L. Henriot, *arXiv preprint arXiv:2310.20519* (2023).

# Recent developments in developing quantum and quantum-inspired algorithms for graph analytics

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Graph neural networks are a powerful tool for analysis of graphs with machine learning. Their widespread usefulness in analysis of biological and social systems, chemical compositions, as well as text and images, has sprouted large interest from scientific community in developing faster, more accurate and more efficient algorithms. The key downside of current algorithms is that they are notoriously computationally difficult, due to the interplay between long-range and short-range interactions, which makes the processing ill-suited for current CPUs/GPUs. A recent approach to the problem is the use of quantum computing to address the interplay between long and short-range interactions through quantum entanglement. After introducing the basics of machine learning on quantum computers, I will discuss some recent results in the field and finish with a presentation of a case study of a quantum graph neural network applied to classification.

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# Emergence of unitary symmetry of microcanonically truncated operators in chaotic quantum systems

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We study statistical properties of matrix elements entering the eigenstate thermalization hypothesis by studying the observables written in the energy eigenbasis and truncated to small microcanonical windows. We put forward a picture, that below certain energy scale collective statistical properties of matrix elements exhibit emergent unitary symmetry. In particular, below this scale the spectrum of the microcanonically truncated operator exhibits universal behavior for which we introduce readily testable criteria. We support this picture by numerical simulations and demonstrate existence of emergent unitary symmetry scale for all considered operators in chaotic many-body quantum systems. We discuss operator and system-size dependence of this energy scale and put our findings into context of previous works exploring emergence of random-matrix behavior in narrow energy windows.

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# Power-law decay of the fraction of the mixed eigenstates in different quantum systems

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The properties of the mixed eigenstates in a generic quantum system with a classical counterpart that has mixed-type phase space, although important to understand various fundamental questions that arise in both theoretical and experimental studies, are still less known. Here, we perform an analysis of the features of mixed eigenstates in two different quantum systems, namely kicked-top model and Dicke model. By using the phase-space overlap index, we show that the mixed eigenstates appear due to various tunneling processes between different phase-space structures, while the regular and chaotic eigenstates are, respectively, associated with invariant tori and chaotic components in phase space. We investigate how the probability distribution of the phase-space overlap index evolves with increasing system size. In particular, we demonstrate that the relative fraction of the mixed eigenstates shows a power-law decay as the system size increases, indicating that only purely regular and chaotic eigenstates are left in the strict semiclassical limit. Our results provide further verification of the principle of uniform semiclassical condensation of Husimi function and confirm the correctness of the Berry-Robnik picture.

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- [2] Q. Wang and M. Robnik, *ArXiv: 2309.11740* (2023).
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# Every raindrop has an exceptional history

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Rainfall from ice-free clouds requires collisions of very large numbers of microscopic droplets to create every raindrop, and the collision rate for the first few droplet coalescences is very low, typically less than one per hour [1]. The onset of rain showers can be surprisingly rapid, much faster than the mean time required for a single collision. One possible explanation [2,3] is that every raindrop is the result of a sequence of exceptionally rare events, where the first few collisions happen unusually quickly. The talk will discuss how large-deviation theory [4] can give a quantitative justification of this suggestion. The calculation [5] shows that a rain shower can occur on a timescale which is *less* than the mean time for the first of the approximately one million collisions which must occur to make each raindrop. Recent results [6] on the growth history of those ‘lucky’ droplets will be presented. I shall also comment on the whether the calculations are relevant to planet formation, which is usually regarded as another example of a runaway aggregation process.

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# Spacing ratios of mixed-type systems

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The distribution of the consecutive level-spacing ratio is now widely used as a tool to distinguish regular from chaotic quantum spectra, mostly due to its avoiding of the numerical spectral unfolding. Similar to the use of the Rosenzweig-Porter approach to obtain the Berry-Robnik distribution of level-spacings in mixed-type systems [1], in this work we extend this approach [2] to derive analytically the distribution of spacing ratios [3], for random matrices comprised of independent Poisson blocks and GOE blocks. One application of this distribution is that one can use it to characterize the relative size of the integrable part of phase space, which we have verified numerically in quantum maps such as quantum kicked rotor and quantum kicked top, and in continuous system such as quantum three-particle FPUT [4].

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# Ko se površnost izplača: nehermitskost in anomalna relaksacija

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V kvantni mehaniki smo običajno vajeni delati z unitarno evolucijo in pripadajočimi hermitskimi generatorji. A kadar nas zanima le del opazljivk, to naravno privede do nehermitskosti. Na konkretnem primeru Markovskega procesa bom pokazal, kako lahko dinamika v takem primeru, v nasprotju z običajno folkloro, ki pravi, da je relaksacijski čas dan s končno spektralno režo, privede to fantomske relaksacije, ki ni določena z nobeno lastno vrednostjo. To nas bo pripeljalo do koncepta psevdospektra in spoznanja, da se pri delu z nehermitskimi operatorji lahko zgodi, da je eksakten rezultat napačen, mičkeno napačen pa pravilen.

## Reference

- [1] M. Žnidarič, *Solvable non-Hermitian skin effect in many-body unitary dynamics*, arXiv:2205.01321 (2022).
- [2] L. N. Trefethen and M. Embree, *Spectra and Pseudospectra* (Princeton, 2005).



# When being sloppy helps: Non-Hermiticity and anomalous relaxation

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Quantum mechanics is ruled by Hermitian generators inducing unitary propagation, nevertheless, when tracing out some degrees of freedom one can end up with non-Hermitian operators. An example is entanglement or out-of-time-ordered correlation functions in random circuits, where the relaxation dynamics is described by a non-Hermitian Markovian matrix with a finite gap. Naturally one would expect that the relaxation, i.e., the rate of generating entanglement, will be given by this finite spectral gap. However, this is not the case. Rather, in the thermodynamic limit the rate is given by a phantom eigenvalue – an "eigenvalue" that is not in the spectrum. Resolution of this puzzle will lead to a pseudospectrum and a realization that when dealing with non-Hermitian matrices being exact can actually be wrong, while being slightly wrong is correct.

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# Informiranje javnosti o (astro)fizikalnih raziskovalnih rezultatih

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Informiranje javnosti je dolžnost, za katero bi morala poskrbeti vsaka raziskovalna skupina, ki je javno financirana. Ob tem koristi tudi pri pridobivanju novih kadrov. Ko se lotimo take naloge, si je dobro zamisliti, kaj želimo sporočiti, pri tem pa upoštevati, komu govorimo. V predstavitvi se bom osredotočil na nekaj praktičnih navodil, kaj delati in česa ne, kaj so pasti in kaj prednosti. Končen zaključek bo, da tak angažma zahteva precej razmisleka in posledično tudi časa. Povedano bom ilustriral z nedavnimi raziskovalnimi rezultati in praktičnimi primeri, ki segajo od motivacije osnovnošolskih otrok do nastopov v elektronskih medijih in pisanja učbenika za študente.

# Public outreach on research results in (astro)physics

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Public outreach can be considered a duty which is to be honored by any publicly financed research group. Besides, it brings visibility which may have its benefits in attracting good students. When planning for such a task it is important to define the message we want to convey and to take into account the type of target audience. I shall focus on some practical advice on what to do and what to avoid, what can be problematic and what is not. An obvious conclusion will be that this type of activity demands good planning and therefore quite a bit of time. I will use examples of recent research results which have been presented to different audiences, from talks to school children to presentations for general public in electronic media and to writing of a textbook for students.